Model order reduction of parametrized nonlinear conservation laws via the Lasserre hierarchy

Research Context. The general context of this internship is the numerical solution of parameter-dependent nonlinear conservation laws, where the parameters (initial condition, model parameters...) may be affected by uncertainties (in uncertainty quantification problems) or are unknown quantities that have to be determined (in optimization or inverse problems). A classical example is the Navier Stokes equation where the viscosity coefficient is not known exactly. Proposing numerical methods for solving efficiently these problems for many values of the parameters remains a challenge, the solutions being highly irregular, sometimes discontinuous, with discontinuities that are difficult to capture with classical numerical schemes.

Goals. The goal of this internship is to tackle these problems via the Lasserre hierarchy, surveyed in [2], which is a powerful tool to solve nonlinear and nonconvex optimization problems whose data are expressed with polynomials. It turns out that entropy weak solutions to conservation laws can be written as the solutions of such optimization problems. This reformulation of conservation laws led to a new approach for their numerical solution [3, 4]. The main goal of this internship is to extend this approach to parameter-dependent conservation laws, for the construction of reduced order models that can be used for uncertainty quantification or inverse problems. Practical experiments shall be performed through implementing a tool within the Matlab library `Gloptipoly3` [1] or the Julia library `MomentOpt`.

Working Context. The internship will be co-advised by Swann Marx (CNRS LS2N), Anthony Nouy (Laboratoire de Mathématiques de Jean Leray, Nantes) and Nicolas Seguin (IRMAR, Rennes). The Master student will be hosted by the Control team in the LS2N laboratory, located in Nantes.

Required Skills. Motivated candidates should hold a Bachelor degree and have a solid background in **either** optimization, partial differential equations, control, real algebraic geometry or computer algebra. Good programming skills are also required.

Application. The candidates are kindly asked to send an e-mail with "M2 candidate" in the title, a CV and motivation letter to `swann.marx@ls2n.fr`, `anthony.nouy@ec-nantes.fr` and `nicolas.seguin@univ-rennes1.fr`. Knowledge of French does not constitute a prerequisite.

A related PhD topic can be foreseen.

References

[1] D. Henrion, J.-B. Lasserre, and J. Lofberg. GloptiPoly 3: moments, optimization and semidefinite programming. *Optimization Methods and Software*, 24(4-5):761–779, 2009.

[2] J.B. Lasserre Moments, positive polynomials and their applications. World Scientific, 2010.

[3] S. Marx, T. Weisser, D. Henrion, and J.B. Lasserre A moment approach for entropy solutions to nonlinear hyperbolic PDEs. *Mathematical Control and Related Fields*, 10(1), pp. 113-140, 2020

[4] S. Marx, E. Pauwels, T. Weisser, D. Henrion, and J.B. Lasserre Semi-algebraic approximation using Christoffel-Darboux kernel. to appear in *Constructive Approximation*