Reference prior construction for Bayesian inference applied to seismic fragility curve estimation

Ecole Thématique sur les Incertitudes en Calcul Scientifique

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Seismic Probabilistic Risk Assessment framework

Seismic fragility curves

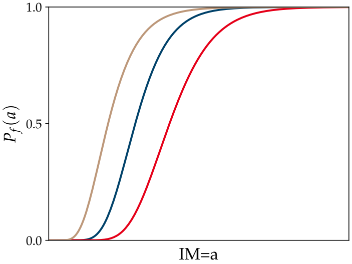

Figure: Fragility curve: Probability('failure') = f('ground-motion').

Intensity Measure (IM):
Derived from the ground-motion signal.
Most common ones:

- Peak Ground Acceleration (PGA)
- Pseudo Spectral Acceleration (PSA)

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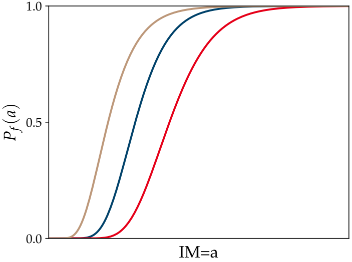

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Log-normal model:

$$\mathbb{P}(\text{'failure'}|IM = a) = \Phi\left(\frac{\log a - \log \alpha}{\beta}\right)$$

with Φ being the c.d.f. of a standard Gaussian variable.

- We look for an estimation of the parameter $\theta = (\alpha, \beta)$.

Bayesian framework

Motivation: The data scarcity in the SPRA context makes uncertainty quantification challenging. This motivates the use of the Bayesian framework.

Dataset $\mathbf{y} = (\mathbf{a}, \mathbf{z}) = (a_1, z_1), \dots, (a_k, z_k)$ $a = IM$ and $z \in \{0, 1\}$ ($z = 0$ if safe, $z = 1$ if fails).

Model: **likelihood** $\ell(y|\theta) = \Phi(\beta^{-1} \log \frac{a}{\alpha})^z (1 - \Phi(\beta^{-1} \log \frac{a}{\alpha}))^{1-z}$, $y = (a, z)$;

$\ell_k(\mathbf{y}|\theta) = \prod_{i=1}^k \ell(y_i|\theta)$.

- Bayesian viewpoint: θ is an r.v. with distribution π, the **prior**

  The **marginal** p.d.f.: $p_{\mathbf{Y}}(\mathbf{y}) = \int_\Theta \ell_k(\mathbf{y}|\theta)\pi(\theta)d\theta$.

  The **posterior** p.d.f.: $p(\theta|\mathbf{y}) = \pi(\theta)\ell_k(\mathbf{y}|\theta)/p_{\mathbf{Y}}(\mathbf{y})$

  $\propto \pi(\theta) \prod_{i=1}^k \ell(y_i|\theta)$

  $\longrightarrow$ Generation of samples of *a posteriori* θ via MCMC methods

  $\longrightarrow$ How to choose the prior π ?

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An example of subjective prior from the literature[1]: $\pi^{SK}(\theta) \propto \frac{1}{\alpha\beta}\exp\left(-\frac{(\log\alpha - \mu)^2}{2\sigma^2}\right)$.

[1] D. Straub and A. Der Kiureghian. "Improved seismic fragility modeling from empirical data". Struct. Saf. 2008

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A main goal is to find an objective criterion to choose the prior π.

Reference prior theory

Notation: Data $\mathbf{y}$ lives in a measure space $(\mathcal{Y}^k, \mathscr{Y}^{\otimes k}, \mu^{\otimes k})$; θ lives in a measurable space $(\Theta, \mathscr{T})$.

The mutual information is a measure of the divergence between the prior and the posterior.

Definition: mutual information

$$I(\pi|k) = \int_{\mathcal{Y}^k} KL(p(\cdot|\mathbf{y})||\pi)p(\mathbf{y})d\mu^{\otimes k}(\mathbf{y})$$

[2] J. M. Bernardo. “Reference Posterior Distributions for Bayesian Inference”. J. R. Stat. Soc. B. 1979
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$\Longrightarrow$ i.e. Choose π to maximize the influence of the observations on the posterior distribution.

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Theorem[3]

Under appropriate assumptions, asymptotically w.r.t. k, the Jeffreys prior maximizes the mutual information.

Reminder: The Jeffreys prior has density J w.r.t. the Lebesgue measure $J(\theta) \propto |\mathcal{I}(\theta)|^{1/2}$ with $\mathcal{I}$ denoting the Fisher information matrix.

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Note: Fubini-Lebesgue's theorem provides

$$\int_{\mathcal{Y}^k} KL(p(\cdot|\mathbf{y})||\pi)p(\mathbf{y})d\mu^{\otimes k}(\mathbf{y}) = \int_{\Theta} KL(\ell_k(\cdot|\theta)||p_{\mathbf{Y}})d\pi(\theta)$$

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which can be written:

$$\int_{\Theta} KL(\ell_k(\cdot|\theta)||p_{\mathbf{Y}})d\pi(\theta) = \mathbb{E}_{\theta\sim\pi}[KL(\mathbb{P}_{\mathbf{Y}|\theta}||\mathbb{P}_{\mathbf{Y}})]$$

With $\mathbb{P}_{\mathbf{Y}|\theta}$ and $\mathbb{P}_{\mathbf{Y}}$ resp. denoting the distributions associated with the densities $\ell_k(\cdot|\theta)$ and $p_{\mathbf{Y}}$ w.r.t. $\mu^{\otimes k}$.

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Connection with global sensitivity analysis indices

Global sensitivity indices

Sensitivity analysis framework: $Y = \eta(X_1, \dots, X_p)$ is observed.

Definition: global sensitivity index[4]

If D is a dissimilarity measure, the sensitivity index w.r.t. D measures the impact of X_i to Y :

$$S_i = \mathbb{E}_{X_i}[D(\mathbb{P}_Y || \mathbb{P}_{Y|X_i})]$$

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Naturally, a similar generalization for the mutual information comes:

Definition: general mutual information

$$I_D(\pi|k) = \mathbb{E}_{\theta\sim\pi}[D(\mathbb{P}_{\mathbf{Y}} || \mathbb{P}_{\mathbf{Y}|\theta})]$$

[4] S. Da Veiga. “Global sensitivity analysis with dependence measures”. J. Stat. Comput. Simul. 2015

A result with f-divergences
f-divergence

Motivation: We focus on a class of dissimilarity measure D with the objective of stating the associated reference prior (the one which maximizes I_D).

Definition: f-divergence[5]

Let p, q be probability measure densities w.r.t. ν the f-divergence between p and q is

$$d_f(p||q) = \int f\left(\frac{q}{p}\right) p\, d\nu$$

Remark: if $f = -\log$ we obtain the Kullback-Leibler divergence.

[5] I. Csiszár. “Information-type measures of difference of probability distributions and indirect observation”. Stud. Sci. Math. 1967

A result with f-divergences

Our theorem

We work on $\Theta \subset \mathbb{R}^d$ compact and suppose the likelihood to be regular enough.

Assumptions on f

f is locally bounded and for some $\beta < 1$, α, γ,

$$f(x) \underset{x\to 0^+}{=} \gamma + \alpha x^\beta + o(x^\beta), \qquad f(x) \underset{x\to+\infty}{=} O(x).$$

Theorem

Consider $\mathcal{P}$: the class of priors admitting a continuous and positive density on Θ w.r.t. the Lebesgue measure.

For any prior in $\mathcal{P}$ with density π: $\displaystyle \lim_{k\to\infty} k^{d\beta/2} I_{d_f}(\pi|k) = \alpha C_\beta \int_\Theta \pi(\theta)^{1+\beta} |\mathcal{I}(\theta)|^{-\beta/2} d\theta.$

Moreover, if $\alpha(\beta+1) > 0$, then: $\displaystyle \lim_{k\to\infty} k^{d\beta/2} (I_{d_f}(J|k) - I_{d_f}(\pi|k)) \geq 0,$

where $J(\theta) = |\mathcal{I}(\theta)|^{1/2} / \int_\Theta |\mathcal{I}(\tilde\theta)|^{1/2} d\tilde\theta$ is the Jeffreys prior p.d.f. The equality stands iff $\pi = J$.

[6] A. Van Biesbroeck et al. “Connections between reference prior theory and global sensitivity analysis, an illustration with f-divergences”. JDS 2023

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Example of suitable f: α-divergence

$$f(x) = \frac{x^\alpha - \alpha x - (1-\alpha)}{\alpha(\alpha-1)} \quad ; \quad \alpha \in (0,1)$$

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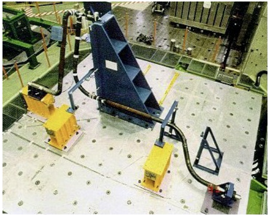
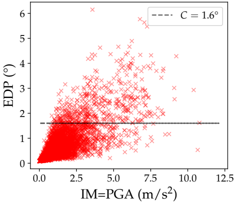
Application

Case study presentation

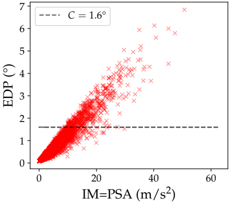


Piping system from nuclear reactor[7]

- Submitted to real seismic signals on shaking table; and simulated numerically. 10^4 computations carried out for benchmark.
- Theoretical failure of the equipment: excessive rotation of the first elbow.



PGA



PSA

Figure: Scatter plot of elbow rotation as a function of PGA (left), PSA (right).

[7] F. Touboul, P. Sollogoub, and N. Blay. "Seismic behaviour of piping systems with and without defects". N. E. D. 1999

[8] S. Rezaeian and A. Der Kiureghian. "Simulation of synthetic ground motions…" Earthq. En. St. Dyn. 2010

Jeffreys' prior computation

Our model characteristics:

- Data $y=(a,z)$, $\mathcal{Y}=\mathbb{R}_+\times\{0,1\}$
- Likelihood $\ell(a,z|\theta)=\Phi\left(\beta^{-1}\log\frac{a}{\alpha}\right)^z\left(1-\Phi\left(\beta^{-1}\log\frac{a}{\alpha}\right)\right)^{1-z}$ (with respect to $\mathbb{P}_A$, $d\mathbb{P}_A(a)=p(a)da$)

We derive $J(\theta)=\sqrt{\det\mathcal{I}(\theta)}$ with

$$\mathcal{I}(\theta)=\begin{pmatrix}\int_{\mathcal{Y}}\frac{\partial^2}{\partial\alpha^2}\log\ell(a,z|\theta)\ell(a,z|\theta)p(a)dadz & \int_{\mathcal{Y}}\frac{\partial^2}{\partial\alpha\partial\beta}\log\ell(a,z|\theta)\ell(a,z|\theta)p(a)dadz\\ \int_{\mathcal{Y}}\frac{\partial^2}{\partial\alpha\partial\beta}\log\ell(a,z|\theta)\ell(a,z|\theta)p(a)dadz & \int_{\mathcal{Y}}\frac{\partial^2}{\partial\beta^2}\log\ell(a,z|\theta)\ell(a,z|\theta)p(a)dadz\end{pmatrix}.$$

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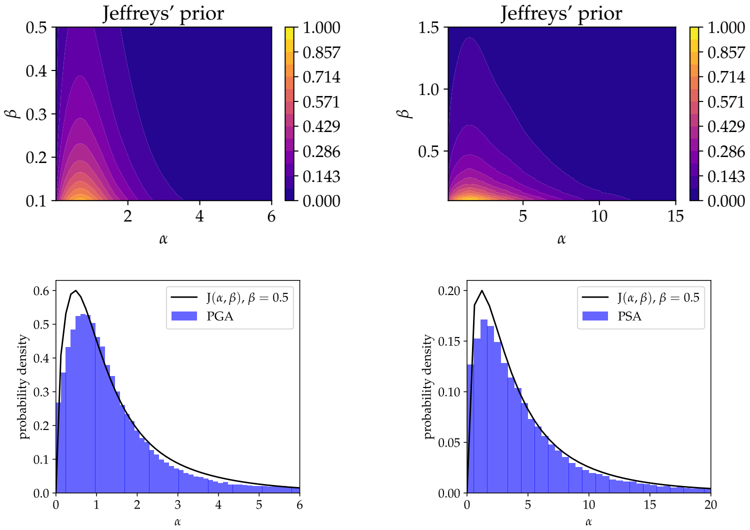

PGA PSA

Figure: Jeffreys' prior (top) and IM distribution (bottom). For PGA (left) and PSA (right).

Competing approaches

- Bayesian approaches with subjective prior, for instance[9]

$$\pi^{SK}(\theta) \propto \frac{1}{\alpha\beta} \exp\left(-\frac{(\log\alpha - \mu)^2}{2\sigma^2}\right).$$

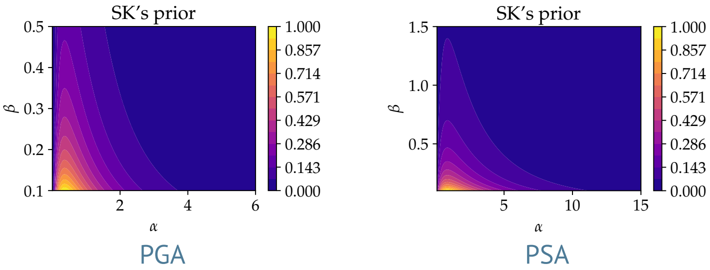

Figure: Straub & Der Kiureghian's prior for PGA (left) and PSA (right).

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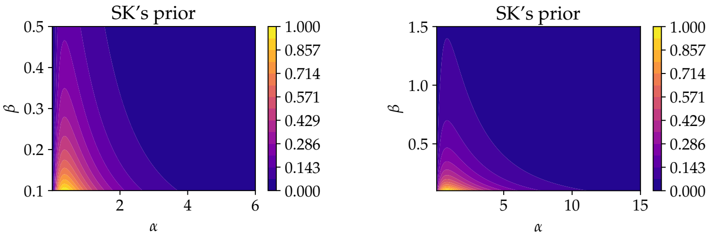

PGA PSA

Figure: Straub & Der Kiureghian's prior for PGA (left) and PSA (right).

- Classical Maximum Likelihood estimation with bootstrapping

$$\hat{\theta}_k^{MLE} = \underset{\theta\in(0,\infty)^2}{\arg\max}\, \ell_k(\mathbf{z}, \mathbf{a}|\theta) = \underset{\theta\in(0,\infty)^2}{\arg\max} \prod_i \ell(z_i, a_i|\theta).$$

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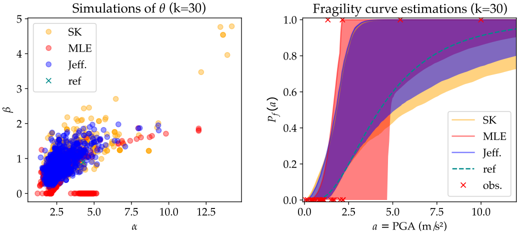
Numerical results

- Sampling of *a posteriori* θ using MCMC.
- Comparison with P_f^{ref}: a reference derived from a 10^4-items validation data-set.
- Empirical computation of a quadratic error w.r.t. the dataset size: $\mathcal{E}^{Q|\mathbf{a},\mathbf{z}} = \mathbb{E}\big[\|P_f^{|\mathbf{a},\mathbf{z}} - P_f^{\mathrm{ref}}\|_{L^2}^2 | \mathbf{a},\mathbf{z}\big]$.



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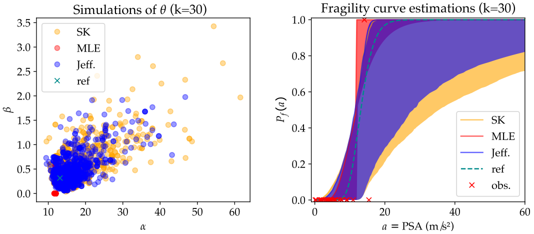

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[10] A. Van Biesbroeck et al. "Influence of the choice of the seismic intensity measure on fragility curves estimation in a Bayesian framework based on reference prior". UNCECOMP 2023

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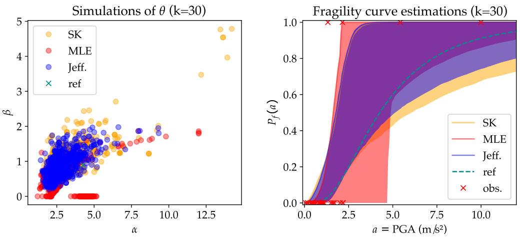

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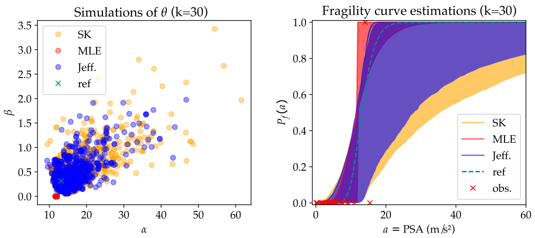

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Comments: ▶ Irregularity of MLE;

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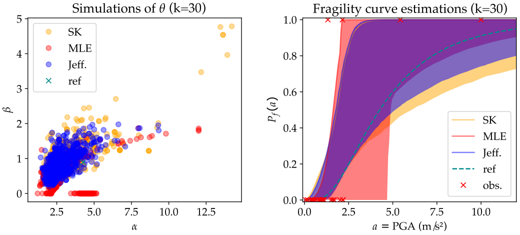

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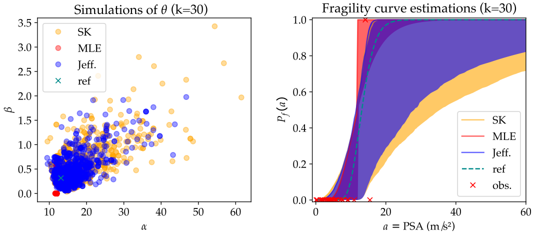

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Comments:
- ▶ Irregularity of MLE;
- ▶ Less outliers generation with Jeffreys' prior.

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Theoretical results

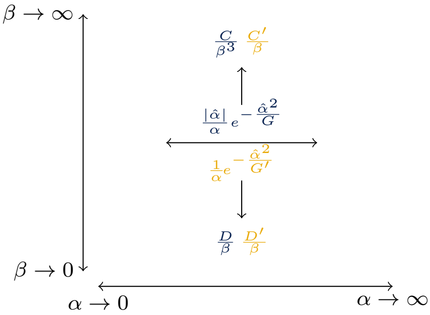

Figure: Jeffreys' prior and SK prior asymptotics ($\hat\alpha = \log\alpha$).

[11] A. Van Biesbroeck et al. "Reference prior for Bayesian estimation of seismic fragility curves". arxiv.2302.06935. 2023

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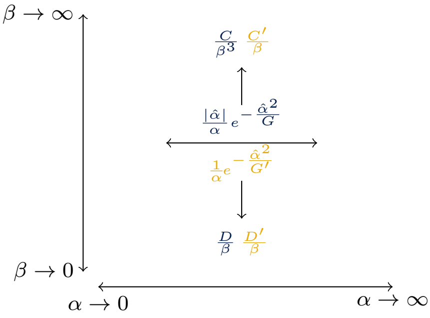

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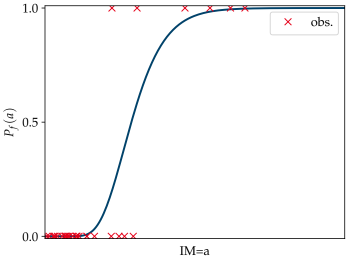


Figure: Example with non-degenerate data case.

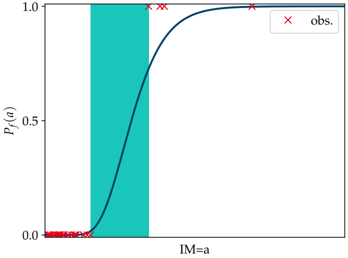


Figure: Example with degenerate data case.

[11] A. Van Biesbroeck et al. "Reference prior for Bayesian estimation of seismic fragility curves". arxiv.2302.06935. 2023

Theoretical results

Non-degenerate case

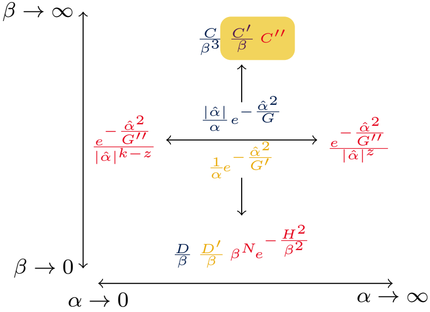


Figure: Jeffreys' prior, SK prior and likelihood asymptotics ($\hat{\alpha} = \log \alpha$).

SK's posterior improper when $\beta \to \infty$.

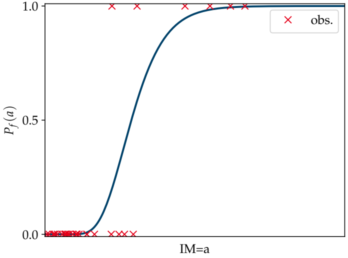


Figure: Example with non-degenerate data case.

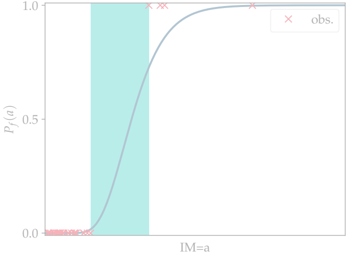


Figure: Example with degenerate data case.

[11] A. Van Biesbroeck et al. "Reference prior for Bayesian estimation of seismic fragility curves". arxiv.2302.06935. 2023

Theoretical results

Degenerate case

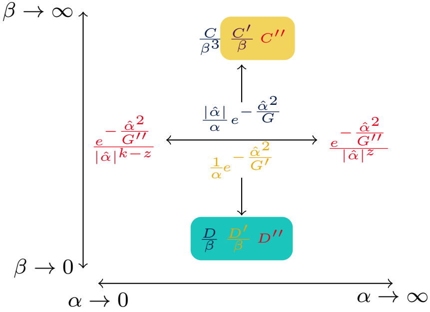


Figure: Jeffreys' prior, SK prior and likelihood asymptotics ($\hat\alpha = \log \alpha$).

SK's posterior improper when $\beta \to \infty$.

Both posteriors improper when $\beta \to 0$.

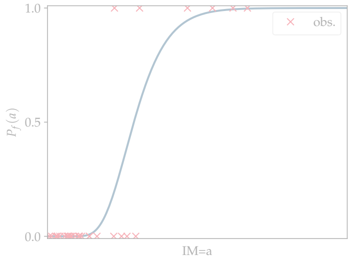


Figure: Example with non-degenerate data case.

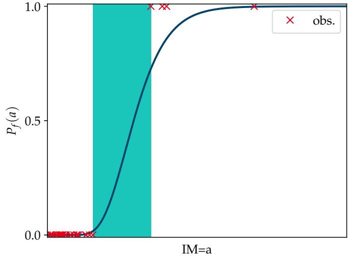


Figure: Example with degenerate data case.

[11] A. Van Biesbroeck et al. "Reference prior for Bayesian estimation of seismic fragility curves". arxiv.2302.06935. 2023

Conclusion & Perspectives

Conclusion:

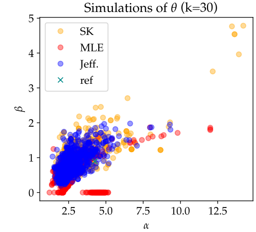
- We defend the necessity of a auditable methodology to build the prior in practical Bayesian studies.
- We enriched and generalized the reference prior theory, giving insights for the prior choice.
- The Jeffreys' prior selection is supported; its implementation in practice is proven robust and highlights the necessity of a proper design of the prior.

Perspectives:

- Further study of the reference prior theory: consideration of other dissimilarity measures, consideration of constraints.
- Development of computational methods to derive the reference prior.
- Construction of a design of experiments method based on reference prior to solve the degenerate phenomena of the data-set.

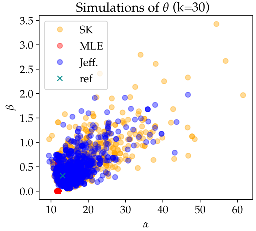
Thank You for Your Attention!

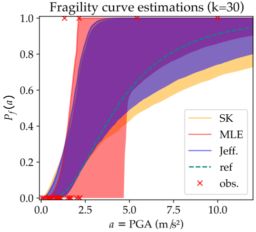
Appendix: Numerical results

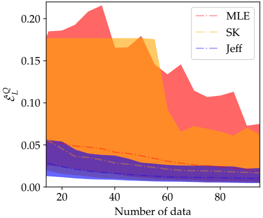


- Sampling of *a posteriori* θ using MCMC.
- Comparison with P_f^{ref}: a reference derived from a 10^4-items validation data-set.
- Empirical computation of a quadratic error w.r.t. the dataset size:

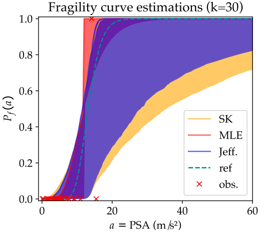
$$\mathcal{E}^{Q|\mathbf{a},\mathbf{z}} = \mathbb{E}\big[\|P_f^{|\mathbf{a},\mathbf{z}} - P_f^{\mathrm{ref}}\|_{L^2}^2|\mathbf{a},\mathbf{z}\big].$$

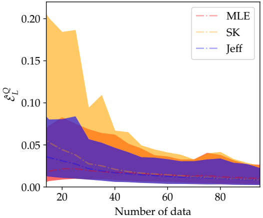






PGA





PSA

Comments: ▶ Irregularity of MLE; ▶ Less outliers generation with Jeffreys' prior;
▶ More degenerative results on small data samples or with PSA.

[12] A. Van Biesbroeck et al. "Influence of the choice of the seismic intensity measure on fragility curves estimation in a Bayesian framework based on reference prior". UNCECOMP 2023

Appendix

Coefficients of variation PGA/PSA