



# Reference prior construction for Bayesian inference applied to seismic fragility curve estimation

Ecole Thématique sur les Incertitudes en Calcul Scientifique

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## Introduction: Seismic Probabilistic Risk Assessment framework

### Bayesian framework

#### Prior choice criterion

- Reference prior theory

- Connection with Global Sensitivity Analysis

- Result on generalized reference priors

#### Application

- Case study presentation

- Numerical results

- Theoretical results

### Conclusion & perspectives

# Seismic Probabilistic Risk Assessment framework

## Seismic fragility curves

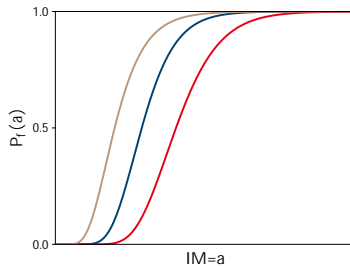


Figure: Fragility curve: Probability('failure') = f('ground-motion').

Intensity Measure (IM):

Derived from the ground-motion signal.

Most common ones:

- Peak Ground Acceleration (PGA)
- Pseudo Spectral Acceleration (PSA)

# Seismic Probabilistic Risk Assessment framework

## Seismic fragility curves

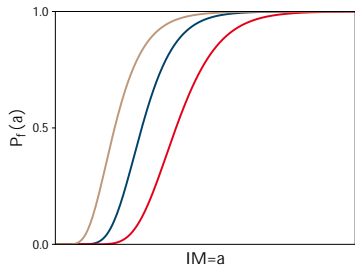


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Log-normal model:

$$P(\text{'failure'} | IM = a) = \frac{\log a}{\log}$$

with being the c.d.f. of a standard Gaussian variable.

! We look for an estimation of the parameter  $\theta = (\mu; \sigma)$ .

# Bayesian framework

**Motivation:** The data scarcity in the SPRA context makes uncertainty quantification challenging. This motivates the use of the Bayesian framework.

Dataset  $\mathbf{y} = (\mathbf{a}; \mathbf{z}) = (a_1; z_1); \dots; (a_k; z_k)$   $a = IM$  and  $z \in \{0, 1\}$  ( $z = 0$  if safe,  $z = 1$  if fails).

Model: **likelihood**  $p(\mathbf{y} | \mathbf{a}) = \prod_{i=1}^k (1 - \frac{a_i}{M})^{z_i} (\frac{a_i}{M})^{1 - z_i}$ ,  $\mathbf{y} = (\mathbf{a}; \mathbf{z})$ ;  
 $p_k(\mathbf{y} | \mathbf{a}) = \prod_{i=1}^k p(y_{ij} | a_i)$ .

| Bayesian viewpoint:  $\mathbf{a}$  is an r.v. with distribution  $p(\mathbf{a})$ , the **prior**

The **marginal** p.d.f.:  $p_Y(\mathbf{y}) = \int_{\mathbb{R}} p_k(\mathbf{y} | \mathbf{a}) p(\mathbf{a}) d\mathbf{a}$ .

The **posterior** p.d.f.:  $p(\mathbf{a} | \mathbf{y}) = \frac{p(\mathbf{a}) p_k(\mathbf{y} | \mathbf{a})}{\int_{\mathbb{R}} p(\mathbf{a}) p_k(\mathbf{y} | \mathbf{a}) d\mathbf{a}}$

! Generation of samples of  $\mathbf{a}$  *a posteriori* via MCMC methods

! How to choose the prior ?

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| Bayesian viewpoint:  $\theta$  is an r.v. with distribution  $\pi(\theta)$ , the **prior**

The **marginal** p.d.f.:  $p_{\mathbf{Y}}(\mathbf{y}) = \int_{\mathbb{R}} \ell_k(\mathbf{y} | \theta) \pi(\theta) d\theta$ .

The **posterior** p.d.f.:  $p(\theta | \mathbf{y}) = \frac{\ell_k(\mathbf{y} | \theta) \pi(\theta)}{\int_{\mathbb{R}} \ell_k(\mathbf{y} | \theta) \pi(\theta) d\theta} = \frac{\ell_k(\mathbf{y} | \theta) \pi(\theta)}{p_{\mathbf{Y}}(\mathbf{y})}$

! Generation of samples of  $\theta$  *a posteriori* via MCMC methods

! How to choose the prior  $\pi$  ?

An example of subjective prior from the literature<sup>1</sup>:  $\pi(\theta) \propto \frac{1}{\theta} \exp\left(-\frac{(\log \frac{1}{\theta})^2}{2\sigma^2}\right)$  :

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 $\propto p(\theta) \prod_{i=1}^k p(y_{ij} | \theta)$

! Generation of samples of  $\theta$  *a posteriori* via MCMC methods

! How to choose the prior  $p(\theta)$  ?

A main goal is to find an objective criterion to choose the prior  $p(\theta)$ .

## Reference prior theory

**Notation:** Data  $\mathbf{y}$  lives in a measure space  $(Y^k; \mathcal{Y}^k; \mu^k)$ ;  $\theta$  lives in a measurable space  $(\Theta; \mathcal{T})$ .

The mutual information is a measure of the divergence between the prior and the posterior.

**Definition:** mutual information

$$I(\theta; \mathbf{y}) = \int_{Y^k} KL(p(\theta | \mathbf{y}) || p(\theta)) p(\mathbf{y}) d\mu^k(\mathbf{y})$$

<sup>2</sup>J. M. Bernardo. "Reference Posterior Distributions for Bayesian Inference". J. R. Stat. Soc. B. 1979

<sup>3</sup>B. S. Clarke and A. R. Barron. "Jeffreys' prior is asymptotically least favorable under entropy risk". J. Stat. P. I.. 1994



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**Reference prior principle:** Look for  $\theta \in \Theta$   $\arg \max_{\theta \in \Theta} I(\theta; \mathbf{y})$ .<sup>2</sup>

=) i.e. Choose  $\theta$  to maximize the influence of the observations on the posterior distribution.

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### Theorem<sup>3</sup>

Under appropriate assumptions, asymptotically w.r.t.  $k$ , the Jeffreys prior maximizes the mutual information.

**Reminder:** The Jeffreys prior has density  $J$  w.r.t. the Lebesgue measure  $J(\theta) \propto |I(\theta)|^{1/2}$  with  $I$  denoting the Fisher information matrix.

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**Definition:** mutual information

$$I(jk) = \int_{\mathcal{Y}^k} KL(p(j|\mathbf{y})|p(j))p(\mathbf{y})d\mu^k(\mathbf{y})$$

**Note:** Fubini-Lebesgue's theorem provides

$$\int_{\mathcal{Y}^k} KL(p(j|\mathbf{y})|p(j))p(\mathbf{y})d\mu^k(\mathbf{y}) = \int KL(p_k(j)|p_Y(\mathbf{y}))d\mu(\mathbf{y})$$

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### Definition: mutual information

$$I(j; k) = \int_{\mathcal{Y}^k} KL(p(\mathbf{y}|j) || p(\mathbf{y})) d\mu^k(\mathbf{y})$$

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$$\int_{\mathcal{Y}^k} KL(p(\mathbf{y}|j) || p(\mathbf{y})) d\mu^k(\mathbf{y}) = \int_{\mathcal{J}} KL(\eta_k(j) || p_{\mathbf{Y}}) d\mu(j)$$

which can be written:

$$\int_{\mathcal{J}} KL(\eta_k(j) || p_{\mathbf{Y}}) d\mu(j) = E [KL(P_{\mathbf{Y}|j} || P_{\mathbf{Y}})]$$

With  $P_{\mathbf{Y}|j}$  and  $P_{\mathbf{Y}}$  resp. denoting the distributions associated with the densities  $\eta_k(j)$  and  $p_{\mathbf{Y}}$  w.r.t.  $\mu^k$ .

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# Connection with global sensitivity analysis indices

## Global sensitivity indices

Sensitivity analysis framework:  $Y = (X_1; \dots; X_p)$  is observed.

### Definition: global sensitivity index<sup>4</sup>

If  $D$  is a dissimilarity measure, the sensitivity index w.r.t.  $D$  measures the impact of  $X_i$  to  $Y$  :

$$S_i = E_{X_i}[D(P_{Y|j}P_{Y|X_i})]$$

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<sup>4</sup>S. Da Veiga. "Global sensitivity analysis with dependence measures". J. Stat. Comput. Simul. 2015

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Reminder: mutual information expression:

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Naturally, a similar generalization for the mutual information comes:

### Definition: general mutual information

$$I_D(j, k) = E [D(P_{Y_j} P_{Y_k})]$$

<sup>4</sup>S. Da Veiga. "Global sensitivity analysis with dependence measures". J. Stat. Comput. Simul. 2015

# A result with $f$ -divergences

## $f$ -divergence

**Motivation:** We focus on a class of dissimilarity measure  $D$  with the objective of stating the associated reference prior (the one which maximizes  $I_D$ ).

### **Definition:** $f$ -divergence<sup>5</sup>

Let  $p; q$  be probability measure densities w.r.t.  $Z$  the  $f$ -divergence between  $p$  and  $q$  is

$$d_f(p||q) = \int_Z f\left(\frac{q}{p}\right) p d$$

**Remark:** if  $f = \log$  we obtain the Kullback-Leibler divergence.

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<sup>5</sup>I. Csiszár. "Information-type measures of difference of probability distributions and indirect observation". Stud. Sci. Math. 1967



# A result with $f$ -divergences

## Our theorem

We work on  $\mathbb{R}^d$  compact and suppose the likelihood to be regular enough.

### Assumptions on $f$

$f$  is locally bounded and for some  $\alpha < 1$ ,  $\beta, \gamma$ ,

$$f(x) \underset{x \rightarrow 0^+}{=} \alpha + \beta x + o(x^\gamma); \quad f(x) \underset{x \rightarrow +1}{=} O(x^\alpha):$$

### Theorem

Consider  $\mathcal{P}$ : the class of priors admitting a continuous and positive density on  $Z$  w.r.t. the Lebesgue measure.

For any prior in  $\mathcal{P}$  with density  $j$ : 
$$\lim_{k \rightarrow 1} k^{d-2} I_{d_f}(jk) = C \int_Z (j(\cdot))^{1+\alpha} j(\cdot)^{-2d} d\cdot$$

Moreover, if  $(\alpha + 1) > 0$ , then: 
$$\lim_{k \rightarrow 1} k^{d-2} (I_{d_f}(Jjk) - I_{d_f}(jk)) = 0;$$

where  $J(j) = \int_{\mathbb{R}^d} j(\cdot)^{1-2\alpha} j(\cdot)^{1-2\alpha} d\cdot$  is the Jeffreys prior p.d.f. The equality stands iff  $j = J$ .

<sup>6</sup>A. Van Biesbroeck et al. "Connections between reference prior theory and global sensitivity analysis, an illustration with  $f$ -divergences". JDS 2023

# A result with f-divergences

## Our theorem

We work on  $\mathbb{R}^d$  compact and suppose the likelihood to be regular enough.

Example of suitable  $f$ :  $f$ -divergence

$$f(x) = \frac{x}{(1-x)^2} ; \quad x \in (0;1)$$

## Theorem

Consider  $\mathcal{P}$ : the class of priors admitting a continuous and positive density on  $Z$  w.r.t. the Lebesgue measure.

For any prior in  $\mathcal{P}$  with density  $j$  : 
$$\lim_{k \rightarrow 1} k^{d-2} I_{d_f}(jk) = C \int_Z (j)^{1+} j^{d-2} d :$$

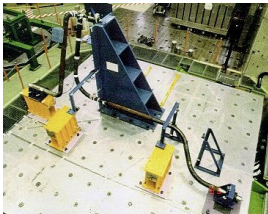
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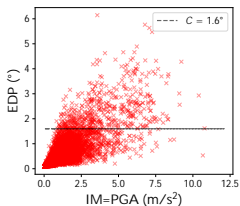
# Application

## Case study presentation

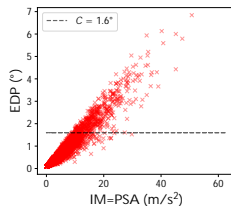


### Piping system from nuclear reactor<sup>7</sup>

- Submitted to real seismic signals on shaking table; and simulated numerically.  $10^4$  computations carried out for benchmark.
- Theoretical failure of the equipment: excessive rotation of the first elbow.



PGA



PSA

Figure: Scatter plot of elbow rotation as a function of PGA (left), PSA (right).

<sup>7</sup>F. Touboul, P. Sollogoub, and N. Blay. "Seismic behaviour of piping systems with and without defects". N. E. D. 1999

<sup>8</sup>S. Rezaeian and A. Der Kiureghian. "Simulation of synthetic ground motions..." Earthq. En. St. Dyn. 2010

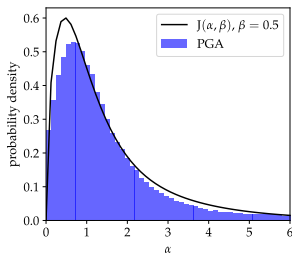
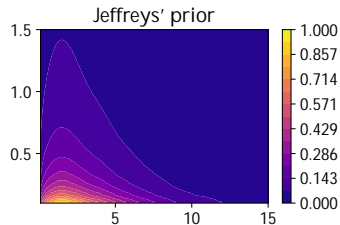
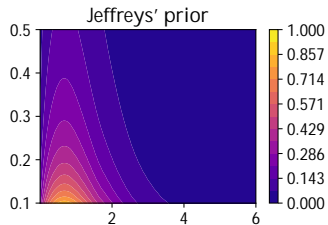
# Jeffreys' prior computation

Our model characteristics: Data  $y = (a; z)$ ,  $Y = \mathbb{R}_+ \times \{0, 1\}^g$   
 Likelihood  $\ell(a; z_j) = \prod_{i=1}^g \log \frac{a}{a+z} z_i^{1-z_i}$  (with respect to  $\mathbb{P}_A$ ,  $d\mathbb{P}_A(a) = p(a) da$ )

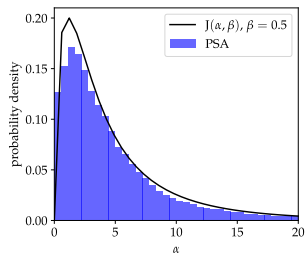
We derive  $J(\theta) = \sqrt{\det I(\theta)}$  with

$$I(\theta) = \begin{pmatrix} \int_Y \frac{\partial^2}{\partial a^2} \log \ell(a; z_j) \ell(a; z_j) p(a) da dz & \int_Y \frac{\partial^2}{\partial a \partial z} \log \ell(a; z_j) \ell(a; z_j) p(a) da dz \\ \int_Y \frac{\partial^2}{\partial z \partial a} \log \ell(a; z_j) \ell(a; z_j) p(a) da dz & \int_Y \frac{\partial^2}{\partial z^2} \log \ell(a; z_j) \ell(a; z_j) p(a) da dz \end{pmatrix}$$

# Jeffreys' prior computation



PGA



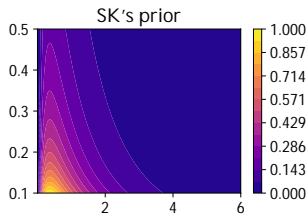
PSA

Figure: Jeffreys' prior (top) and IM distribution (bottom). For PGA (left) and PSA (right).

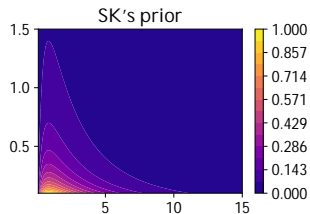
# Competing approaches

- Bayesian approaches with subjective prior, for instance<sup>9</sup>

$$sK(\cdot) \propto \frac{1}{\sigma^2} \exp\left(-\frac{(\log \frac{\cdot}{\sigma})^2}{2\sigma^2}\right) :$$



PGA



PSA

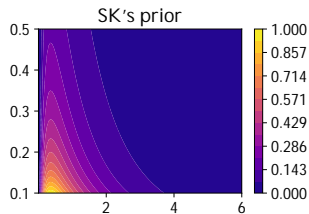
Figure: Straub & Der Kiureghian's prior for PGA (left) and PSA (right).

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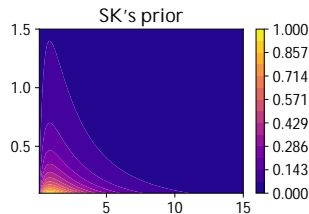
## Competing approaches

- Bayesian approaches with subjective prior, for instance<sup>9</sup>

$$SK(\cdot) \propto \frac{1}{2^2} \exp\left(-\frac{(\log \frac{\cdot}{2})^2}{2}\right) :$$



PGA



PSA

Figure: Straub & Der Kiureghian's prior for PGA (left) and PSA (right).

- Classical Maximum Likelihood estimation with bootstrapping

$$\hat{\theta}_k^{MLE} = \arg \max_{\theta \in (0;1)^2} \ell_k(\mathbf{z}; \mathbf{a}_j) = \arg \max_{\theta \in (0;1)^2} \sum_i^Y \ell(z_i; a_{ij}) :$$

<sup>9</sup>D. Straub and A. Der Kiureghian. "Improved seismic fragility modeling from empirical data". Struct. Saf. 2008

# Numerical results

- Sampling of a posteriori using MCMC.
- Comparison with  $P_f^{\text{ref}}$  : a reference derived from a  $10^4$ -items validation data-set.
- Empirical computation of a quadratic error w.r.t. the dataset size:  $E^{Q_{j_a; z}} = E \left[ k P_f^{j_a; z} - P_f^{\text{ref}} \right]^2_{L_2} j_a; z :$

PGA

PSA

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<sup>10</sup>A. Van Biesbroeck et al. Influence of the choice of the seismic intensity measure on fragility curves estimation in a Bayesian framework based on reference prior . UNCECOMP 2023



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Comments: | Irregularity of MLE;

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- Comments:
- | Irregularity of MLE;
  - | Less outliers generation with Jeffreys' prior.

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# Theoretical results

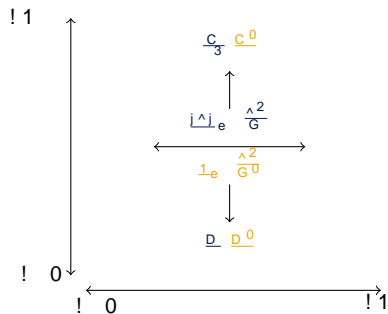


Figure: Jeffreys' prior and SK prior asymptotics ( $\wedge = \log$  ).

<sup>11</sup>A. Van Biesbroeck et al. Reference prior for Bayesian estimation of seismic fragility curves . arxiv.2302.06935. 2023

# Theoretical results

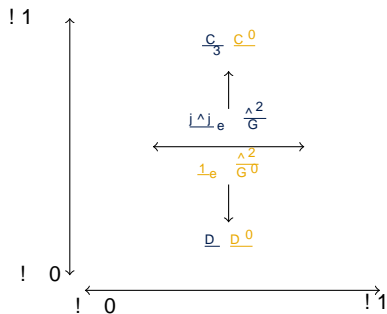


Figure: Example with non-degenerate data case.

Figure: Jeffreys' prior and SK prior asymptotics ( $\lambda = \log$  ).

Figure: Example with degenerate data case.

<sup>11</sup>A. Van Biesbroeck et al. Reference prior for Bayesian estimation of seismic fragility curves . arxiv.2302.06935. 2023

# Theoretical results

## Non-degenerate case

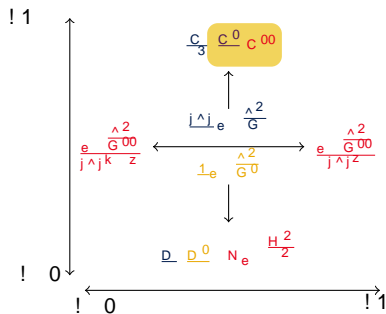


Figure: Example with non-degenerate data case.

Figure: Jeffreys' prior, SK prior and likelihood asymptotics ( $\wedge = \log$  ).

SK's posterior improper when  $! 1$  .

<sup>11</sup>A. Van Biesbroeck et al. Reference prior for Bayesian estimation of seismic fragility curves . arxiv.2302.06935. 2023

# Theoretical results

## Degenerate case

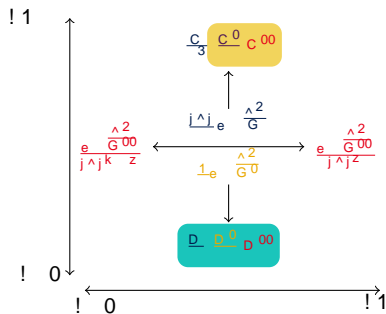


Figure: Jeffreys' prior, SK prior and likelihood asymptotics ( $\wedge = \log$  ).

SK's posterior improper when  $! 1$  .

Both posteriors improper when  $! 0$ .

Figure: Example with degenerate data case.

<sup>11</sup>A. Van Biesbroeck et al. Reference prior for Bayesian estimation of seismic fragility curves . arxiv.2302.06935. 2023

# Conclusion & Perspectives

## Conclusion:

- We defend the necessity of a auditable methodology to build the prior in practical Bayesian studies.
- We enriched and generalized the reference prior theory, giving insights for the prior choice.
- The Jeffreys' prior selection is supported; its implementation in practice is proven robust and highlights the necessity of a proper design of the prior.

## Perspectives:

- Further study of the reference prior theory: consideration of other dissimilarity measures, consideration of constraints.
- Development of computational methods to derive the reference prior.
- Construction of a design of experiments method based on reference prior to solve the degenerate phenomena of the data-set.

Thank You for Your Attention!



# Appendix: Numerical results

- Sampling of a posteriori using MCMC.
- Comparison with  $P_f^{\text{ref}}$ : a reference derived from a  $10^4$ -items validation data-set.
- Empirical computation of a quadratic error w.r.t. the dataset size:

$$E^{Q_{j a ; z}} = E \left[ k P_f^{j a ; z} - P_f^{\text{ref}} \right]_L^2_{j a ; z} :$$

PGA

PSA

Comments: | Irregularity of MLE; | Less outliers generation with Jeffreys' prior;  
| More degenerative results on small data samples or with PSA.

<sup>12</sup>A. Van Biesbroeck et al. Influence of the choice of the seismic intensity measure on fragility curves estimation in a Bayesian framework based on reference prior. UNCECOMP 2023

# Appendix

## Coefficients of variation PGA/PSA

