Sequential design of experiment on a stochastic multi-fidelity simulator to estimate a probability of exceeding a threshold

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Abstract:

One of the main goals in fire safety science is to assess the conformity of various infrastructures, like building or watercraft. A method consists in simulating fire using computer models, and comparing quantities of interest (temperature, visibility,...)to security regulatory thresholds. The probability of conformity of the structure is evaluated as the probability that the outputs remain below or above these thresholds. Fire Dynamics Simulator (FDS) is a famous highly predictive stochastic fire simulator. Consider for simplicity that FDS returns a stochastic scalar output Z from a vector of inputs x. Denote by z^{crit} the security threshold associated to Z. Our aim is to estimate the probability that the output exceeds this threshold

$$\mathbb{P}\left(Z > z^{\mathrm{crit}}\right) = \int_{\mathbb{X}} \mathbb{P}\left(Z > z^{\mathrm{crit}} \middle| x\right) f_{\mathbb{X}}\left(x\right) \mathrm{d}x, \tag{1}$$

where $f_{\mathbb{X}}$ is the probability distribution of inputs. However, FDS is a complex expensive-to-evaluate simulator. One accurate simulation can last up to several weeks. Consequently, direct methods like Monte-Carlo cannot be used to estimate the probability of failure.

An important feature of FDS is the possibility to change the fidelity of the simulations. FDS proceeds by finite difference methods to solve equations of fluid dynamics (see Guillaume [2015]). The size of the mesh t rules the accuracy and the duration of the simulation. A coarse meshing gives quickly a rough result, while a fine meshing consumes a lot of time before returning an accurate result. The main idea is to consider the mesh size as an additional input variable, and to combine simulations at different levels of fidelity to estimate the probability of failure. The probability of exceeding the threshold is assessed at the highest level of fidelity t_{HF}.

We propose a threefold method of estimation. First, build a multi-fidelity Bayesian model of the output Z of the simulator, based on Gaussian process regression. Then, use this model to compute the posterior distribution of the probability of failure, conditionally to observations $\chi_n = (x_i, t_i; z_i)_{1 \le i \le n}$. Finally, choose new observation points in order to improve the estimation of the probability of exceeding the threshold.

We use the Bayesian model described in Stroh et al. [2016]. It supposes that the output Z at (x, t) follows a normal distribution, with mean $\xi\left(x, t\right)$ and variance $\lambda\left(t\right)$. Then, different prior distributions, adapted from Picheny and Ginsbourger [2013] and Tuo et al. [2014], are added hierarchically to this model to describe the effect of fidelity t on the output distribution. This model allows us to compute the posterior distribution of our probability of failure, conditionally to observations χ_n.

The main contribution of this work is an algorithm of sequential design of experiment. Once the posterior distribution computed, we would like to add new observation points to improve the estimation. According to the Stepwise Uncertainty Reduction (SUR) principle (see Bect et al. [2012]; Vazquez and Bect [2009]), the selected next observation is the observation which gives the most uncertainty reduction on our quantity of interest. As our quantity is a probability of exceeding a threshold, we use a measure of uncertainty of this probability, adapted to the case of a stochastic simulator

$$H_n = \int_{\mathbb{X}} \mathbb{V}\mathrm{ar}\left[\Phi\left(\frac{\xi\left(x, t_{HF}\right) - z^{\mathrm{crit}}}{\sqrt{\lambda\left(t_{HF}\right)}}\right)\middle|\chi_n\right] f_{\mathbb{X}}\left(x\right) \mathrm{d}x. \tag{2}$$

Moreover, we also propose a sequential design of experiment in the multi-fidelity framework. Thus, we consider a cost function $C\left(t\right)$, supposed known, which measures the cost of observing the level t. Similarly to the sequential design of experiment proposed by Le Gratiet and Cannamela [2015], our methodology is based on the comparison between a single-level criteria and the cost to observe the level. By combining the two principle, the next observation point (x_{n+1}, t_{n+1}) is the input maximizing the rate of uncertainty reduction

$$(x_{n+1}, t_{n+1}) = \arg\max_{(x,t)} \left\{\frac{H_n - \mathbb{E}\left[H_{n+1}|\chi_n, (X_{n+1}, T_{n+1}) = (x,t)\right]}{C\left(t\right)}\right\}. \tag{3}$$

We propose the first results of design of experiment on a multi-fidelity stochastic simulator, in order to evaluate a probability of exceeding a threshold. The results are compared with a non-sequential design. The methodology is illustrated on a fire safety case study to assess conformity of a building

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Short biography – Rémi Stroh received the Engineer's Degree (equivalent to a master's degree in Electrical Engineering) from Supelec in 2014, with a specialization in applied mathematics. Since February 2015, he is a full-time PhD student at LNE and L2S (CentraleSupélec/ Univ. Paris-Sud/CNRS/Université Paris-Saclay).