Aircraft trajectory optimization in uncertain environments : a test case for stochastic optimization algorithms

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Ph.D. expected duration: 2014-2017

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Abstract: We consider the stochastic aircraft trajectory optimization problem:

$$\begin{aligned} &\text{Find } u^\star = \arg\min_u \mathbb{E}_\omega \left(g(x(t_f), \omega, t_f) + \int_{t_0}^{t_f} -\dot{m}(x(s), u(s), \omega) ds \right) \\ &s.t.\ \forall \omega,\ \forall t > t_0\ \dot{x}(t) = f(x(t), u(t), \omega) \\ &\qquad x(t_0) = x_0 \\ &\qquad d(t_f) = d_f, \end{aligned}$$

where x is the state of the aircraft, m its mass, $\dot{m}$ its instantaneous fuel consumption, d the ground distance it has flown over, u the path control, f the instantaneous dynamic, g the terminal cost function and ω a random variable representing the environment. Estimates of the cost of trajectories are usually obtained through numerical integration of the flight dynamic equations represented by f. Most representative formulations of f rely on interpolation of experimental local measurements and cannot be integrated analytically. Moreover the relation between cost and trajectory control parameters cannot reasonably be assumed to be convex. At last, the cost estimation relies on some predicted flight conditions including atmospheric ones. Hence, real-flight costs can deviate substantially from their predictions and some uncertainty propagation method must be applied to obtain an accurate estimate of the expected flight costs. Finally, the computational efficiency is a key ingredient as it must be performed only a few hours before the planned flight.

Commercial aircraft trajectories are however highly constrained and the search space of admissible controls u can be significantly reduced. For example, as displayed on Figure 1, a vertical path has to be made of a sequence of flight segments at constant altitudes called steps. Hence we only have to consider the vectors of positions of the steps and the vectors of steps' altitudes denoted respectively x and h on Figure 1 as optimization variables.

This problem can thus be formulated as a specific case of the following stochastic optimization problem:

$$\text{Find} \quad x^\star = \arg\min_{x \in E} \mathbb{E}_\omega(U(x, \omega)), \tag{*}$$

where E is a finite space, $U : E \to [0, M] \subset \mathbb{R}_+$ is a bounded cost function, ω a random variable of law $\mathcal{P}(x)$. More specifically, U and thus $\mathbb{E}_\omega(U(x, \omega))$ have no analytical forms and are evaluated through numerical experimentations. Typically, $\mathbb{E}_\omega(U(x, \omega))$ is approximated by a Monte-Carlo sampling of size N: $\widehat{\mathbb{E}}_{\omega,N}(U(x, \omega)) := \frac{1}{N} \sum_{i=1}^N U(x, \omega_i)$, where $(\omega_i)_{i=1..N}$ are N independent realizations of ω. A classical way of evaluating the performance of algorithms solving problem (*)

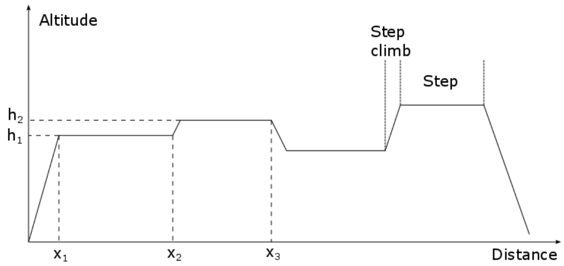

Figure 1: Aircraft trajectory, structure of the vertical path

is to study the number of cost function evaluations they require to retrieve ϵ-optimal solutions x_ϵ with probability $1-\delta$, i.e., $\mathbb{P}(U(x_\epsilon) \leq U(x^\star) + \epsilon) \geq 1-\delta$. We denote this property as $\epsilon - \delta$ convergence. Obviously, the mini-batch size N is a critical element to control in order to ensure good performance. Intuitively the sample size should, as the required cost estimation accuracy, increase with the iterations of the algorithm. It is in fact the rate of increase that is of major interest.

In a recent work [1], we provided an upper bound on the number of cost function evaluations ensuring $\epsilon - \delta$ convergence for a modified version of the simulated annealing adapted to problem (*). We have shown that the size should increase polynomially with the number of algorithm iterations, a quadratic increase being optimal. This theoretical result is supported by numerical experimentations highlighting that under-polynomial mini-batch size increase cannot ensure the convergence. This result improves the results of [3].

In this present work, we extend the numerical experimentations about the mini-batch size rate of increase requirement to the bandit and expected improvement approaches. The choice of the simulated annealing was indeed motivated by the specificity of our main application. Simulated annealing is known to be efficient in contexts where cost function evaluations are quite fast and the computation of the cost of an aircraft trajectory takes about a second. We pointed out, this could be prohibitive for the expected improvement as it requires heavy computations to choose which point to evaluate next. The bandit approach has become very popular to solve stochastic problems as (*). We highlighted in [1] that the performance of bandit approaches is highly dependent on the size of E. For the aircraft trajectory optimization problem in our setting, the search space is a discretized subspace of $\mathbb{R}^{10}$.

We study the mini-batch version of the adaptive treed bandit algorithm of [2] and the EGO from [4]. We provide an extensive numerical study of the practical performances of these algorithms using both test cases provided by the authors of the original algorithms and our specific test case. This benchmark validates the choice of the noisy simulated annealing for our application.

References

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Short biography – Clément Bouttier - graduated from ISAE-SUPAERO and University of Toulouse III - is working on a Ph.D. about Aircraft trajectory Optimization under Uncertainty. It is funded by Airbus and co-directed by S. Gadat, S. Gerchinovitz from the Institute of Mathematics of Toulouse and F. Nicol from ENAC. Current methodologies in that field are mainly dealing with the deterministic setting, i.e. when flight conditions are assumed to be deterministic and known in advance. The main goal of this Ph.D is to evaluate the benefit of a stochastic approach and propose consequent optimization procedures.