Spatial Quantile and Expectile Predictions for Elliptical Random Fields

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Abstract:

Kriging, introduced by Krige [4], and formalized by Matheron [6], aims at predicting the conditional mean of a random field $(Z_t)_{t\in T}$ given the values $Z_{t_1},...,Z_{t_N}$ of the field at some points $t_1,...,t_N \in T$, where typically $T \subset \mathbb{R}^d$. When using Kriging techniques, for any $x \in T$, the conditional mean of Z_x given $Z_{t_1},...,Z_{t_N}$ is approximated by a linear combination of $Z_{t_1},...,Z_{t_N}$ where the weight vector is the solution of a least square minimization problem (see Ligas and Kulczycki [5] for example). It seems natural to predict, in the same spirit as Kriging, other functionals by linear combinations. In this work, we focused on quantiles and expectiles (see Maume-Deschamps et al. [8] and Maume-Deschamps et al. [7]).

In 1978, Koenker and Bassett proposed a conditional quantile estimation as an affine combination of $Z_{t_1},...,Z_{t_N}$, called Quantile Regression (cf. Koenker and Bassett [3]). More recently, some papers propose an Expectile Regression, using the same approach (see Yang et al. [10] or Sobotka and Kneib [9], for example). The weight vector is the solution of a minimization problem, with an asymmetric loss function. In the expectile case with $\alpha = \frac{1}{2}$, it corresponds exactly to the conditional mean regression, or Kriging. Otherwise, it is more difficult to get explicit formulas. The Quantile (and Expectile) Regression approach usually requires time consuming simulations to compute expectations. Moreover, in a non-gaussian setting, the conditional quantile and expectile may not be expressed as a linear combination of the covariates, thus the consistency of the prediction by regression is not guaranteed.
In this work, we focus on elliptical random fields. Elliptical distributions, formalized by Cambanis [1], have the advantage of being stable under affine transformations. Therefore, explicit formulas for the quantile and expectile regression predictors may be obtained for consistent elliptical distributions (cf. Kano [2]). Nevertheless, the regression predictor is generally not equal to the theoretical value and the difference may be large, especially for extreme levels of α. This is why we propose a new dedicated prediction that is adapted to extremal levels of quantiles or expectiles.

The presentation is organized as follows. In a first time, we give some definitions, properties and examples of elliptical distributions satisfying the consistency property. For these models, we give formulas for conditional quantiles and expectiles. The next section is devoted to quantile and expectile regression predictors for consistent elliptical random fields: explicit formulas or iterative algorithms are obtained, and the distributions of the predictors are given. Then, we propose some extremal predictions and prove asymptotic equivalences when the level α is close to 0 or 1. To conclude, we propose a numerical study (see Figure 1). In particular, we emphasize the fact that regression is generally not appropriate, especially for extreme levels of α, justifying the use of extreme predictors. We illustrate this point on several examples.

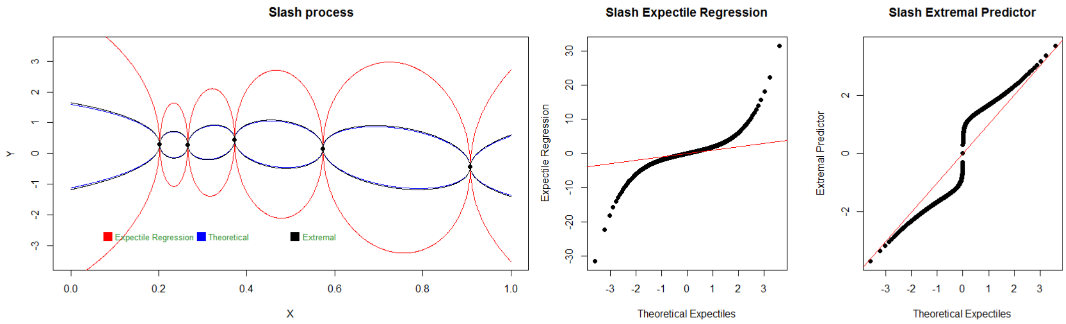

Figure 1: Spatial expectile predictions for a Slash process observed in 5 points. On the left, Expectile Regression Predictors are in red, Theoretical quantiles in blue, and Extremal Predictors in black, for extreme levels $\alpha = 0.995$ and 0.005. On the right, two $EE-$plots: Expectile Regression Predictor vs Theoretical expectiles and Extremal Predictor vs Theoretical expectiles

References

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Short biography – I obtained a Master of Research in Actuarial Science in 2015. My Master's thesis on black-box optimization for Economic Scenario Generators leaded me to spatial interpolation in my PhD thesis, funded by Université Lyon 1. I am also currently working on an application of this work for temperature random fields.