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Abstract:

We study the initial boundary value problem for the stochastic nonlocal Allen-Cahn equation :

$$\begin{aligned}&\partial_t u = div(\nabla\sigma(\nabla(u))) + f(u) - \fint_D f(u) + \dot{W}(t)\\&\frac{\partial u}{\partial \nu} = 0\\&u(x,0) = u_0(x)\end{aligned} \tag{1}$$

on a smooth domain $D \subset R^n$ of area $|D|$, where ν is outward normal vector to ∂D and where $n \geq 1$ is the space dimension.

- σ is a smooth function from $R^n \to R$ which is strictly convex.
- The reaction term f is the derivative of a double-well potential F. More precisely we assume that $f(u) = u - u^3$.
- $\dot{W}(t)$ is the additive white noise in time.
- $$\fint_D f(u) := \frac{1}{|D|}\int_D f(u(x))dx$$

The deterministic problem in the case of the Laplacian was first introduced by Rubinstein and Sternberg [4] as a model for phase separation in a binary mixture. The existence, uniqueness and regularity of the solution of the mass-conserved Allen-Cahn were proved by [2].We are interested in the well-posedeness of the nonlocal initial boundary value problem with the additional stochastic term together with the more general nonlinear diffusion term.
Our aim is to prove the existence and the uniqueness of the solution of the Problem (1).

The first step is to perform the change of unknown function $v(t) = u(t) - W(t)$ which yields the mass conservation property

$$\int_D v(x,t)dx = \int_D v_0(x)dx \quad \text{for all} \quad t > 0$$

where $v_0(x) = u_0(x)$.
We apply a Galerkin method, and search for a priori estimates which lead us to bound uniformly the approximate solution in $L^\infty([0,T]; L^2(D))$, $L^4([0,T] \times D)$ and $L^2(0,T,H^1(D))$. We deduce that the approximate weak solution v_m weakly converges along a subsequence to a limit $\bar{v}$ as

$m \to \infty$. The main problem is then to identify the limit of the term $f(v_m + W)$ as $m \to \infty$, which we do by means of the so-called monotonicity method [3].
We also prove the uniqueness of the weak solution which in turn implies the convergence of the whole sequence $\{v_m\}$.

Similar methods permits to prove the existence and uniqueness of the solution of the problem studied by [1]:

$$\begin{aligned} &\partial_t u = \Delta(u) + f(u) - \fint_D f(u) + \dot{W}(x,t) \\ &\frac{\partial u}{\partial \nu} = 0 \\ &u(x,0) = u_0(x) \end{aligned} \tag{2}$$

- $\dot{W}(x,t)$ is smooth in space and white in time; more precisely $W(x,t)$ is a Q-Wiener process defined by :

$$W(x,t) = \sum_{k=1}^{\infty} \sqrt{\lambda_k}\beta_k(t)e_k(x),$$

where $\{e_k\}_{k\geq 1}$ is an orthormal basis of $L^2(D)$, $\{\lambda_k\}_{k\geq 1}$ are the eigenvalues of a nonnegative definite symmetric operator Q on $L^2(D)$ such that Tr Q $< +\infty$ and $\{\beta_k\}$ is a sequence of independent Brownian motions.

Our next purpose is to extend our study to the case of a multiplicative noise.
This is joint work with T. Funaki, D. Hilhorst and K. Lee.

References

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Short biography – I am a second year PhD student at the university Paris-Sud Paris-Saclay, where I was also a master 2 student in the study direction " Partial Differential Equations and Scientific Computations". In October 2015, I started my doctoral study on the topics called:" Stochastic and deterministic partial differential equations in biology and medecine", under the guidance of Danielle Hilhorst,Research Director at CNRS. This work is supported by a public grant as part of the "Investissement d'avenir project, reference ANR-11-LABX-0056-LMH, LabEx LMH".