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Nested polynomial trends for the improvement of Gaussian process-based predictors

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Outline

Context of uncertainty quantification

- For the last decades, the use of simulation has kept increasing for the analysis of complex and non-linear physical systems.
- Computational models are introduced:
  - to **optimize** the system or its maintenance w.r.t. constraints (geometry, cost, certification criteria...)
  - to **explore** the design space (prototyping, better understanding of the phenomena...)
  - to **evaluate** its robustness and its reliability.

Context of uncertainty quantification

When interested in a complex phenomenon $\boldsymbol{x} \mapsto y(\boldsymbol{x})$, two kinds of problems can be distinguished:

Forward problems

Given information about $\boldsymbol{x}$, we would like to:

1. **infer** the distribution of $\boldsymbol{x}$,
2. **compute** some statistical quantities of y, such as:
   - its mean and its variance,
   - probabilities of exceeding threshold,
   - its full density.

Backward problems

Given information about $\boldsymbol{x}$ and y, we would like to:

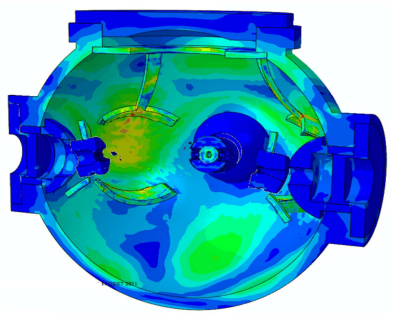
1. **Construct** a parametric model $\mathcal{M}(\cdot; \boldsymbol{\beta})$ for $\boldsymbol{x} \mapsto y(\boldsymbol{x})$,
2. **calibrate** parameters $\boldsymbol{\beta}$ and validate model $\mathcal{M}$,
3. **classify** the influence of each input on the variability of y.

The information about $\boldsymbol{x}$ and $\boldsymbol{y}$ is generally a set of experiments (or code evaluations).

Problematic

Most of the methods associated with these problems are based on the post-processing of large sets of simulations. When the numerical cost associated with one simulation is very high (for instance $\sim$100h on 100 cores in parallel), we need to know how to

- build **cheap mathematical approximations** of the quantities of interest,
- **control the relevance** of these approximations,
- **use these approximations** to solve the problem we are confronted to (by optimized sequential designs for instance).



Outline

Gaussian process-based regression (GPR)

- Let $\mathcal{S}$ be a physical system, which response depends on a d-dimensional input vector $\boldsymbol{x} = (x_1, \ldots, x_d)$, and which performance can be evaluated from the computation of a quantity of interest, $y(\boldsymbol{x})$.
- Function y is a **deterministic** mapping that is assumed to be an element of $L^2(\mathcal{D}^d, \mathbb{R})$, where $\mathcal{D}^d$ is a compact set.
- We suppose that the maximal available information about y is a set of N code evaluations, $\left\{(\boldsymbol{x}^{(1)}, y(\boldsymbol{x}^{(1)})), \ldots, (\boldsymbol{x}^{(N)}, y(\boldsymbol{x}^{(N)}))\right\}$.
- Each computation of $y(\boldsymbol{x})$ is computationally **expensive**, such that N is relatively small compared to the complexity of y.
- Given $\mathcal{F}_N$, the σ-algebra associated with this information, we are interested in the identification of the *best* predictor $y^\star$ of y:

$$\|y - y^\star\|_{L^2}^2 \;\leq\; \|y - \widehat{y}\|_{L^2}^2\,, \quad \widehat{y} \in L^2(\mathcal{D}^d, \mathbb{R}).$$

$$(u, v)_{L^2} := \int_{\mathcal{D}^d} u(\boldsymbol{x}) v(\boldsymbol{x}) d\boldsymbol{x}, \quad \|u\|_{L^2}^2 := (u, u)_{L^2}\,, \quad u, v \in L^2(\mathcal{D}^d, \mathbb{R}).$$

Gaussian process-based regression (GPR)

Hypothesis [Sacks et al., 1989],[Santner et al., 2003]

$\boldsymbol{x} \mapsto y(\boldsymbol{x})$ is supposed to be a realization of a Gaussian process (GP),

$$Y \sim \mathsf{GP}(\mu, C).$$

If the mean and the covariance functions μ and C are known, then :

$$Y \mid \mathcal{F}_N \sim \mathsf{GP}(\mu_N, C_N),$$

- $\mu_N(\boldsymbol{x}) = \mu(\boldsymbol{x}) + \boldsymbol{r}(\boldsymbol{x})^T [C]^{-1} (\mathbb{Y} - \boldsymbol{\mu})$,
- $C_N(\boldsymbol{x}, \boldsymbol{x}') = C(\boldsymbol{x}, \boldsymbol{x}') - \boldsymbol{r}(\boldsymbol{x})^T [C]^{-1} \boldsymbol{r}(\boldsymbol{x}')$,

and $\mathbb{E}[Y \mid \mathcal{F}_N] = \mu_N$ is the **best predictor** of y in the L^2 sense.

$\mathbb{Y} = \left(y(\boldsymbol{x}^{(1)}), \ldots, y(\boldsymbol{x}^{(N)})\right)$, $\boldsymbol{\mu} = \left(\mu(\boldsymbol{x}^{(1)}), \ldots, \mu(\boldsymbol{x}^{(N)})\right)$,
$[C]_{n,m} = C(\boldsymbol{x}^{(n)}, \boldsymbol{x}^{(m)})$, $\boldsymbol{r}(\boldsymbol{x}) = \left(C(\boldsymbol{x}, \boldsymbol{x}^{(1)}), \ldots, C(\boldsymbol{x}, \boldsymbol{x}^{(N)})\right)$.

Gaussian process-based regression (GPR)

Parametric formulation

Assuming that μ and C belong to classes that are parameterized by $\boldsymbol{\beta}$ and $\boldsymbol{\Theta}$, such that:

$$Y \mid \boldsymbol{\beta}, \boldsymbol{\Theta} \ \sim \ \mathrm{GP}(\mu(\boldsymbol{\beta}), C(\boldsymbol{\Theta})),$$

for all $\boldsymbol{x}$ in $\mathcal{D}^d$, the best predictor of $y(\boldsymbol{x})$ is given by:

$$\mathbb{E}[Y(\boldsymbol{x}) \mid \mathcal{F}_N] \ = \ \int_{\boldsymbol{\beta},\boldsymbol{\Theta},y} y \times \pi[Y(\boldsymbol{x}) = y \mid \boldsymbol{\beta}, \boldsymbol{\Theta}, \mathcal{F}_N] \, \pi[\boldsymbol{\beta}, \boldsymbol{\Theta} \mid \mathcal{F}_N] \, d\boldsymbol{\beta} d\boldsymbol{\Theta} dy,$$

where:

- $\pi[Y(\boldsymbol{x}) = y \mid \boldsymbol{\beta}, \boldsymbol{\Theta}, \mathcal{F}_N]$ is Gaussian,
- $\pi[\boldsymbol{\beta}, \boldsymbol{\Theta} \mid \mathcal{F}_N]$ is the **posterior** joint distribution of $(\boldsymbol{\beta}, \boldsymbol{\Theta})$.

Gaussian process-based regression (GPR)

Linearized plug-in approach [Bect et al., 2012], [Bichon et al., 2008]

- As a good compromise between complexity, efficiency, and errors control, the "linearized plug-in" approach (or "Universal Kriging") is generally preferred to this "full-Bayesian" approach.
- This method consists in :
  1. computing the maximum likelihood estimates of $\boldsymbol{\beta}$ and $\boldsymbol{\Theta}$, which are denoted by $\boldsymbol{\beta}^\star$ and $\boldsymbol{\Theta}^\star$,
  2. linearizing the mean function around $\boldsymbol{\beta}^\star$, such that:

$$\mu(\boldsymbol{x};\boldsymbol{\beta}) \approx c_0(\boldsymbol{x}) + \langle \boldsymbol{f}(\boldsymbol{x}), \boldsymbol{\beta} \rangle \,,$$
$$c_0(\boldsymbol{x}) := \mu(\boldsymbol{x};\boldsymbol{\beta}^\star) - \langle \boldsymbol{f}(\boldsymbol{x}), \boldsymbol{\beta}^\star \rangle \,, \quad \boldsymbol{f}(\boldsymbol{x}) := \frac{\partial \mu}{\partial \boldsymbol{\beta}}(\boldsymbol{x};\boldsymbol{\beta}^\star),$$

  3. reparameterizing: $\boldsymbol{f}(\boldsymbol{x}) \leftarrow (c_0(\boldsymbol{x}), \boldsymbol{f}(\boldsymbol{x}))$, $\boldsymbol{\beta} \leftarrow (1, \boldsymbol{\beta})$,
  4. assuming that $\boldsymbol{\beta} \sim \mathcal{U}_{\mathbb{R}^{|\boldsymbol{\beta}|_0}}$, and conditioning all the results by $\boldsymbol{\Theta}^\star$.

Gaussian process-based regression (GPR)

Linearized plug-in approach [Bect et al., 2012], [Bichon et al., 2008]

It comes:

$$Y \mid \boldsymbol{\Theta}^\star, \mathcal{F}_N \;\sim\; \mathsf{GP}(\widehat{\mu}_N, \widehat{C}_N),$$

$$\widehat{\mu}_N(\boldsymbol{x}) = \left\langle \boldsymbol{f}(\boldsymbol{x}), \widehat{\boldsymbol{\beta}} \right\rangle + \boldsymbol{r}(\boldsymbol{x})^T [C]^{-1} (\mathbb{Y} - [F]\widehat{\boldsymbol{\beta}}),$$

$$\widehat{C}_N(\boldsymbol{x}, \boldsymbol{x}') = C(\boldsymbol{x}, \boldsymbol{x}') - \boldsymbol{r}(\boldsymbol{x})^T [C]^{-1} \boldsymbol{r}(\boldsymbol{x}') + \boldsymbol{u}(\boldsymbol{x})^T ([F]^T [C]^{-1} [F])^{-1} \boldsymbol{u}(\boldsymbol{x}').$$

$$[F] = [\boldsymbol{f}(\boldsymbol{x}^{(1)}) \;\cdots\; \boldsymbol{f}(\boldsymbol{x}^{(N)})]^T, \; \widehat{\boldsymbol{\beta}} = ([F]^T [C]^{-1} [F])^{-1} [F]^T [C]^{-1} \mathbb{Y},$$
$$\boldsymbol{u}(\boldsymbol{x}) = [F]^T [C]^{-1} \boldsymbol{r}(\boldsymbol{x}) - \boldsymbol{f}(\boldsymbol{x}).$$

An analytic expression is found back for the best predictor of $y(\boldsymbol{x})$ in any non-computed point $\boldsymbol{x} \in \mathcal{D}^d$:

$$\mathbb{E}[Y(\boldsymbol{x}) \mid \mathcal{F}_N] \approx y^{\text{lin-plug}}(\boldsymbol{x}) := \widehat{\mu}_N(\boldsymbol{x}).$$

Gaussian process-based regression (GPR)

One of the main advantage of this approximated approach comes from the fact that the relevance of the "linearized plug-in" predictor of y can "easily" be computed **without any additional evaluation** of y from **cross-validation** procedures
[Miller, 1974, Dubrule, 1983, Blatman and Sudret, 2011, Bachoc, 2013]:

$$\left\| y - y^{\text{lin-plug}} \right\|_{L_2}^2 \approx \epsilon_{\text{LOO}}^2 := \frac{1}{N} \sum_{n=1}^{N} \left(y(\boldsymbol{x}^{(n)}) - y_{(-n)}^{\text{lin-plug}}(\boldsymbol{x}^{(n)}) \right)^2 ,$$

$$y(\boldsymbol{x}^{(n)}) - y_{(-n)}^{\text{lin-plug}}(\boldsymbol{x}^{(n)}) = \frac{([R]\mathbb{Y})_n}{[R]_{nn}},$$

$$[R] = [C]^{-1} \left\{ [I] - [F]([F]^T[C]^{-1}[F])^{-1}[F]^T[C]^{-1} \right\}.$$

Outline

Optimization of the GPR

To optimize the results associated with such a plug-in approach, two directions can be explored:

- work on the **parameterization of the covariance function** (stationary or not, regularity...),
- work on the **parameterization of the mean function** (linear or not with respect to β, sparse representations...).

Optimization of the GPR - covariance function

- By definition, for all $\boldsymbol{x}, \boldsymbol{x}'$ in $\mathcal{D}^d$:

$$C(\boldsymbol{x}, \boldsymbol{x}') := \mathbb{E}\left[\left(Y(\boldsymbol{x}) - \mathbb{E}\left[Y(\boldsymbol{x})\right]\right) \times \left(Y(\boldsymbol{x}') - \mathbb{E}\left[Y(\boldsymbol{x}')\right]\right)\right],$$

  such that C is *a priori* any symmetric and non-negative definite kernel that is defined on $\mathcal{D}^d \times \mathcal{D}^d$.
- Remembering that the actual goal is to predict function y, which is defined on $\mathcal{D}^d$ only, it is clear that only "simple" parametric expressions of C can be considered (from a limited number of evaluations of y, we do not pretend to be able to precisely identify C).

Optimization of the GPR - covariance function

A very commonly used representation for C is the following tensorized Matern expression:

$$C(\boldsymbol{x}, \boldsymbol{x}') = \sigma^2 \prod_{i=1}^{d} \frac{1}{2^{\nu-1}\Gamma(\nu)} \left(2\sqrt{\nu} h_i\right)^{\nu} \mathcal{B}_{\nu}^{III}\left(2\sqrt{\nu} h_i\right),$$

where:

- $h_i = g_i\left(\frac{|x_i - x_i'|}{\ell_i}\right)$,
- $\nu \leftrightarrow$ regularity of y,
- $\sigma^2 \leftrightarrow$ *a priori* uncertainty,
- $\ell_i \leftrightarrow$ correlation lengths,
- $g_i \leftrightarrow$ scaling function (to take into account the fact that the model can be non-stationary with respect to x_i).

Optimization of the GPR - covariance function

- All the parameters associated with covariance function C have to be identified from the available evaluations of y, or be *a priori* fixed (from expert judgment for instance...).
- With only very few information about y and its regularity is available, it is generally accepted that the **Matern-5/2** class is a good *a priori* choice for C, which corresponds to the case when the realizations of Y are twice mean-square differentiable:

$$C(\boldsymbol{x}, \boldsymbol{x}') = \sigma^2(1 + \sqrt{h} + (5/3)h^2) \times \exp(-\sqrt{5}\ h),$$

$$h = \sum_{i=1}^{d} |x_i - x_i'|/\ell_i.$$

  - Fixed parameters: $\nu = 5/2$, $\gamma = \sqrt{5}$, $g(h) = h$,
  - Free parameters: σ^2, $\{\ell_1, \ldots, \ell_d\}$.

Optimization of the GPR - mean function

- By definition, for all $\boldsymbol{x}$ in $\mathcal{D}^d$:

$$\mu(\boldsymbol{x}; \boldsymbol{\beta}) := \mathbb{E}\left[Y(\boldsymbol{x})\right],$$

  such that μ is *a priori* any function that is defined on $\mathcal{D}^d$.
- Any parametric expression can therefore be used to characterize μ.
- For instance, one can think to linear or non-linear functions such as polynomials (which can be orthogonal or not in $L^2(\mathcal{D}^d, \mathbb{R})$), cos and sin, neural networks, Low Rank representations...

Optimization of the GPR - mean function

- Once again, without information about y, polynomials are generally chosen for $\boldsymbol{f}$. Indeed, the set $\left\{m_{\boldsymbol{\alpha}},\ \boldsymbol{\alpha} \in \mathbb{N}^d\right\}$, with

$$m_{\boldsymbol{\alpha}}(\boldsymbol{x}) := x_1^{\alpha_1} \times \cdots \times x_d^{\alpha_d}, \quad \boldsymbol{x} \in L^2(\mathcal{D}^d, \mathbb{R}),$$

  defines a basis of $L^2(\mathcal{D}^d, \mathbb{R})$.
- $\boldsymbol{\beta} \mapsto \mu(\boldsymbol{\beta})$ is therefore chosen **linear**, and for any given value of M, the number of polynomials we want to consider for the representation of μ, the idea is to identify the best M-dimensional subset of $\left\{m_{\boldsymbol{\alpha}},\ \boldsymbol{\alpha} \in \mathbb{N}^d\right\}$ to minimize $\|y - \mathbb{E}\left[Y \mid \mathcal{F}_N\right]\|_{L^2}$.
- At last, convergence analyses based on the formerly introduced LOO error can be carried out to identify the most accurate value of M.

Optimization of the GPR - mean function

- In practice, this very complex optimization problem is replaced by an optimization over a **finite** dimensional subset of $\left\{m_{\boldsymbol{\alpha}},\ \boldsymbol{\alpha} \in \mathbb{N}^d\right\}$.
- Different truncation schemes have thus been proposed to choose such a relevant subset, which are mostly based on the assumption that the most influential elements of $\left\{m_{\boldsymbol{\alpha}},\ \boldsymbol{\alpha} \in \mathbb{N}^d\right\}$ correspond to the elements of **lowest total polynomial order**:

$$\mathcal{P}(r,d) := \left\{ m_{\boldsymbol{\alpha}} \mid \boldsymbol{\alpha} \in \mathbb{N}^d,\ \sum_{i=1}^{d} |\alpha_i| \leq r \right\}.$$

Optimization of the GPR - mean function

- A **penalization technique**, such as the $\ell - 1$ or the $\ell - 2$ penalizations or the Least Angle Regression (LAR) method [Hastie et al., 2002, Efron et al., 2004, Blatman and Sudret, 2011] are generally used to find these M most significant terms in $\mathcal{P}(r, d)$.
- Cross validation procedures are once again used to avoid overfitting.
- Such an approach will be referred as "LAR+UK" approach in the following [Kersaudy et al., 2015].

Optimization of the GPR - conclusions

Finally, given $\mathcal{F}_N$, the "LAR+UK" approach follows the following steps:

1. identify the M most interesting polynomials to represent y, which are gathered in the vector-valued function $\boldsymbol{f} = (f_1, \ldots, f_M)$, such that:

$$\mu(\boldsymbol{\beta}) := \langle \boldsymbol{f}, \boldsymbol{\beta} \rangle ,$$

2. choose an *a priori* adapted parametric expression for $C(\boldsymbol{\Theta})$,
3. assume that y is a particular realization of $Y \sim \mathrm{GP}\left(\mu(\boldsymbol{\beta}), C(\boldsymbol{\Theta})\right)$,
4. identify $\boldsymbol{\Theta}^\star$ the maximum likelihood estimation of $\boldsymbol{\Theta}$,
5. compute the posterior distribution of Y that is conditioned by $\mathcal{F}_N$, $\boldsymbol{\Theta}^\star$ and $\boldsymbol{f}$.

The optimized GPR is thus given by the **mean function** of this conditioned Gaussian process, $\mathbb{E}\left[Y(\boldsymbol{x}) \mid \mathcal{F}_N, \boldsymbol{\Theta}^\star, \boldsymbol{f}\right]$.

Optimization of the GPR - conclusions

$$\mathbb{E}\left[Y(\boldsymbol{x}) \mid \mathcal{F}_N, \boldsymbol{\Theta}^\star, \boldsymbol{f}\right] = y^{\text{trend}}(\boldsymbol{x}) + y^{\text{cond}}(\boldsymbol{x}),$$

$$y^{\text{trend}}(\boldsymbol{x}) := \left\langle \boldsymbol{f}(\boldsymbol{x}), \widehat{\boldsymbol{\beta}} \right\rangle$$
$$y^{\text{cond}}(\boldsymbol{x}) := \boldsymbol{r}(\boldsymbol{x})^T [C(\boldsymbol{\Theta}^\star)]^{-1} (\mathbb{Y} - [F]\widehat{\boldsymbol{\beta}}).$$

Comments

- when N tends to infinity, the role of y^{trend} becomes negligible,
- on the contrary, when N is relatively small compared to the complexity of y, the role of y^{trend} is crucial.

Outline

Nested polynomial trends

- When N, the number of code evaluations, is low compared to the complexity of y, such approaches are limited by the fact that **only low values** of M, the dimension of the projection family, can be considered to avoid extra-fitting.
- In order to be able to deal with higher values of M, without increasing the number of unknown parameters to be identified, we can propose a new parameterization of the polynomial trend, which is based on a **nested structure**.
- It will then be shown to what extent such an approach shows very promising results when only very limited information is available ($\leftrightarrow$ N is small).

Nested polynomial trends

Main ideas

- a **composition of two polynomials** for the mean function, μ, of the GP associated with $\boldsymbol{x} \mapsto y(\boldsymbol{x})$ is proposed,
- the composition of polynomials being polynomials, it is thus possible to span a **large subset** of $L^2(\mathcal{D}^d, \mathbb{R})$ from a **small number** of independent parameters.

$$\mu(\boldsymbol{x}; \boldsymbol{b}_1, \boldsymbol{b}_2) := \left\langle \boldsymbol{m}^{(p_2)} \left(\left\langle \boldsymbol{m}^{(p_1)}(x_1), \boldsymbol{b}_1 \right\rangle \right), \boldsymbol{b}_2 \right\rangle,$$

where $\boldsymbol{m}^{(p)}$ gathers all the polynomials, which total degree is less than $p \in \mathbb{N}^*$.

- $\boldsymbol{b}_1 \leftrightarrow$ parameterization of the inner polynomials,
- $\boldsymbol{b}_2 \leftrightarrow$ parameterization of the outer polynomials.

Nested polynomial trends

Main difficulties

1. the fonction

$$\boldsymbol{b}_1 \mapsto \left\langle \boldsymbol{m}^{(p_2)} \left(\left\langle \boldsymbol{m}^{(p_1)}(x_1), \boldsymbol{b}_1 \right\rangle \right), \boldsymbol{b}_2 \right\rangle,$$

   is strongly non-linear.

2. different values of $(\boldsymbol{b}_1, \boldsymbol{b}_2)$ could lead to the same nested-representation.

In that prospect, it has been proposed in [Perrin et al., 2016] to work on:

- a **minimal parametrization** of this nested structure to avoid redundancies,
- a **linearization** of the mean around the maximimun likelihood estimates of $(\boldsymbol{\Theta}, \boldsymbol{b}_1, \boldsymbol{b}_2)$,
- **iterative algorithms** to solve the maximum likelihood maximization.

Nested polynomial trends

Comments on the proposed nested representation

- contrary to the "LAR+UK" approach that looks for **sparse** representations of μ, the nested representation looks for **"full"** polynomial representations, which are however characterized by a very **limited number of independent coefficients**. This is particularly interesting for the modeling of complex phenomena with very limited information.
- For $d > 1$, it allows us to model separately the **dependency** structure between the different input parameters, and the **individual** actions of each input parameter. Hence, analyzing the final structure of the mean function can give us information about the structure of y (is the model additive up to a transformation of its input parameters or are the dependencies more complex?).

Nested polynomial trends

For $d = 1$ and $\mathcal{D}^d = [-1, 1]$, three analytic examples are proposed:

- case 1: $y(x) = P_2^{(4)} \circ P_1^{(4)}(x)$,
- case 2: $y(x) = \sin((x+1)^3)$,
- case 3: $y(x) = \sin(20x)\cos(2x)$,

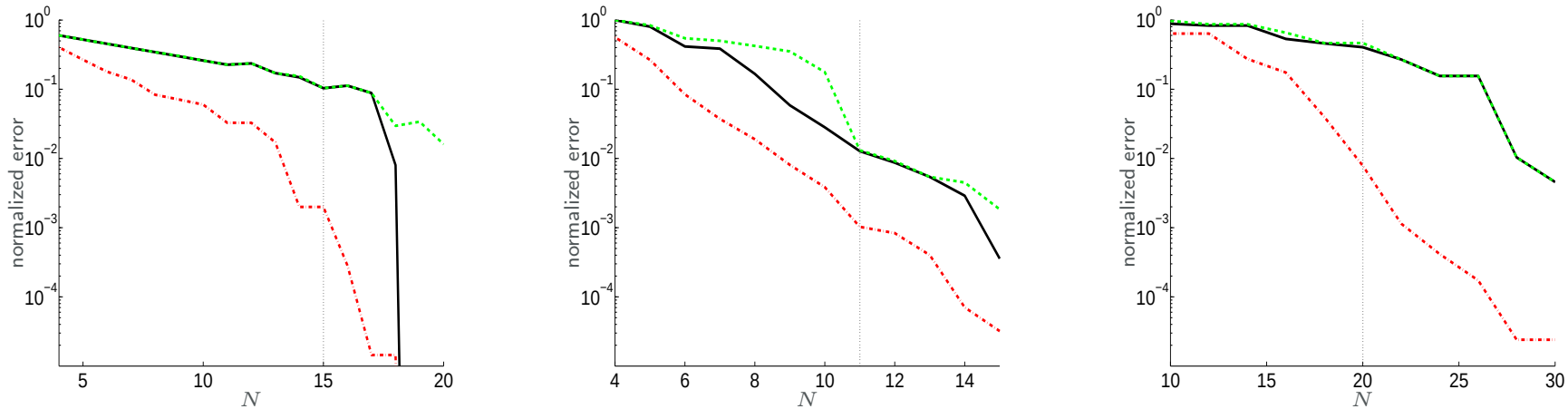


Figure: Mean value of the errors associated with these 10 repetitions. Solid black line: error associated with the LAR+UK approach. Red dotted line: error associated with the nested approach. Green dotted line: UK with linear trend

Nested polynomial trends

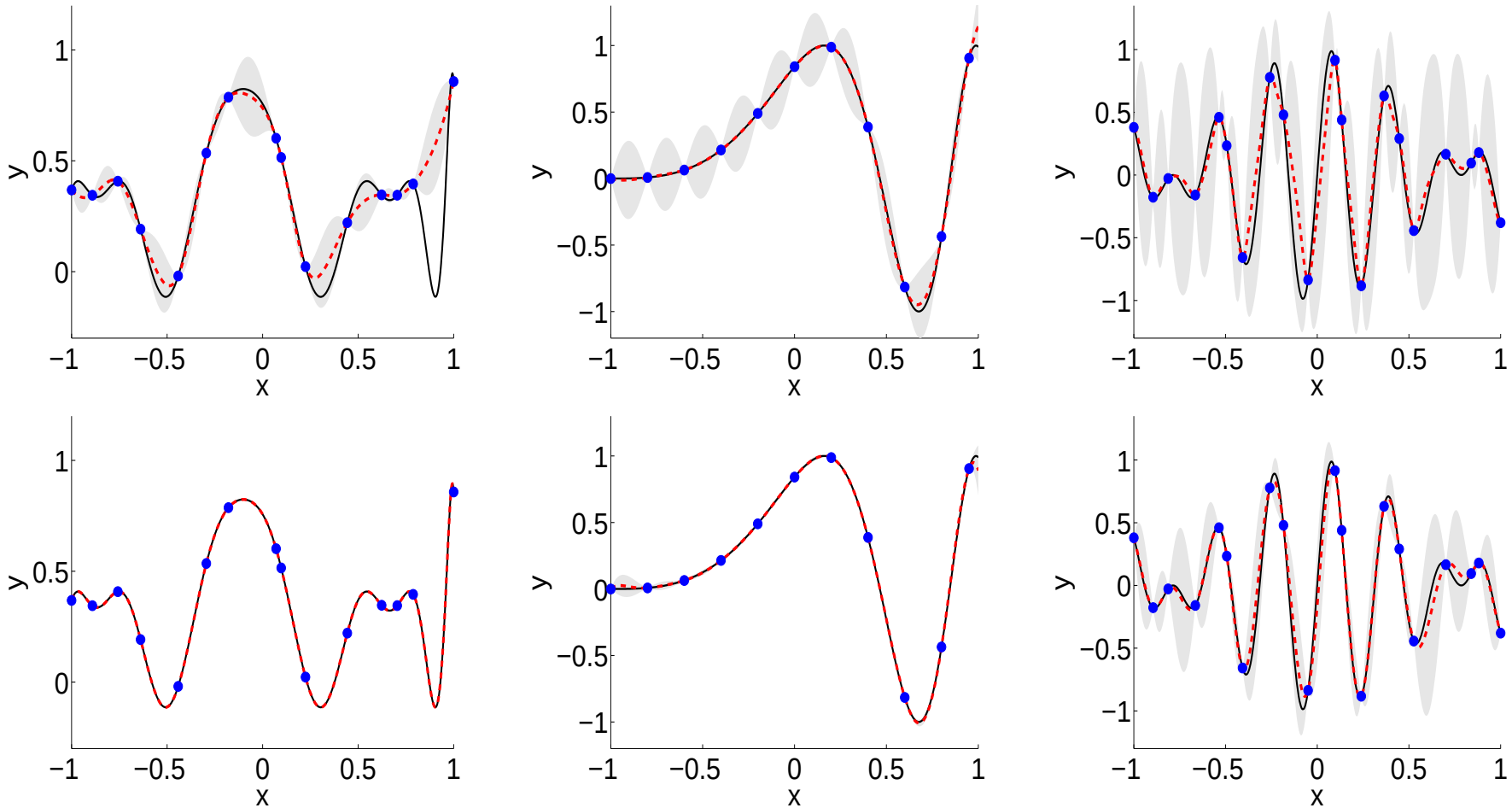


Figure: black $\leftrightarrow y_2$, blue points $\leftrightarrow$ observations, red $\leftrightarrow \widehat{\mu}_N$. top : the LAR+UK approach, bottom : the proposed approach, grey areas $\leftrightarrow 95\%$ confidence region for the prediction. Left: case 1, middle: case 2, right: case 3.

Nested polynomial trends

For $d > 1$, the same kinds of results are found:

- case 1: $y(\boldsymbol{x}) = (1 - x_1^2)\cos(7x_1) \times (1 - x_2^2)\sin(5x_2)$,
- case 2: $y(\boldsymbol{x}) = \sin(x_1) + 7\sin(x_2)^2 + 0.1\sin(x_1)x_3^4$ (**Ishigami**),
- case 3: $y(\boldsymbol{x}) = g^{(1)} \circ g^{(2)}(\boldsymbol{x})$, $g^{(1)}(\boldsymbol{z}) = 0.1\cos\left(\sum_{i=1}^{6} z_i\right) + \sum_{i=1}^{6} z_i^2$, $\boldsymbol{g}^{(2)}(\boldsymbol{x}) = (\cos(\pi x_1 + 1), \cos(\pi x_2 + 2), \ldots, \cos(\pi x_6 + 6))$.

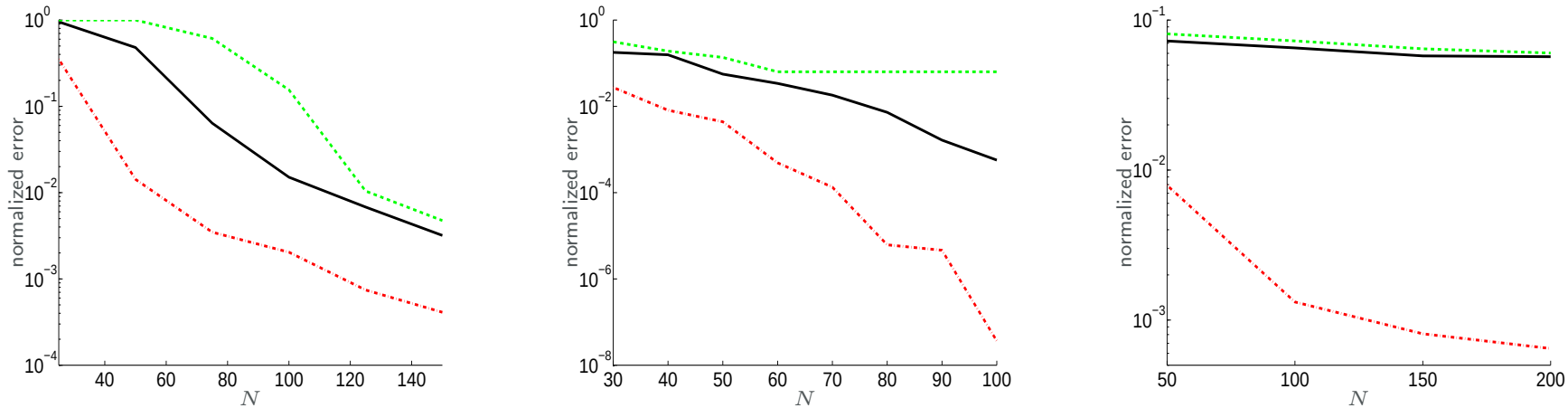


Figure: Mean value of the errors associated with these 10 repetitions. Solid black line: error associated with the LAR+UK approach. Red dashed line: error associated with the nested approach. Green dotted line: UK with linear trend.

Outline

Conclusions and prospects

- A lot of methods in uncertainty quantification and computer experiments require **many code evaluations**.
- When the numerical cost associated with each evaluation is high, **surrogate models** are generally introduced.
- When interested in deterministic mappings (when the dimension of the input space is not too large), one of the most used method is the **GPR**.
- **Working on the polynomial trend** can strongly improve the relevance of the GPR.
- When little information about the code is available, considering **nested polynomial trends** shows promising results.
- Enabling these techniques to deal with **high dimensional problem** ($d \gg 1$), even if a lot of code evaluations ($N \gg 1$) are available is still an open questions.

Conclusion

Thank you for your attention.

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