ETICS 2021: Propagation of epistemic uncertainties and global sensitivity analysis in seismic risk assessment

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Introduction and state of the art

Seismic probabilistic risk assessment

Seismic probabilistic risk assessment (SPRA) is dedicated in estimating the safety of a mechanical structure subjected to seismic ground motions and consists in three main steps [1]:

- Seismic hazard probability distribution on a given site: $dh(a) = p(a)d\mu$
- Seismic fragility curve estimation $\Psi(a)$. By definition the conditional probability of failure of the structure given a seismic intensity of level $A = a$.
- Our QoI: Final probability of failure:

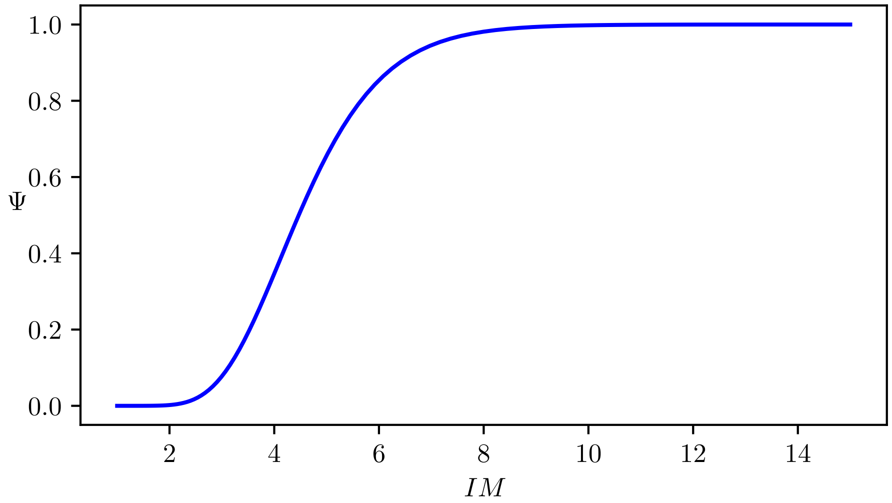
$$\Upsilon = \int \Psi(a)dh(a)$$

[1] Robert P. Kennedy. Risk based seismic design criteria. *Nuclear Engineering and Design*, 1999

Seismic fragility curve

$$\Psi(a) = \mathbb{P}(Y > C | A = a)$$

- Y: mechanical demand of the structure, obtained using time-consuming numerical simulations.
- C: critical level where the structure is considered in a failure state.
- A: *Intensity Measure* of a seismic ground motion. Scalar value representing the intensity of the temporal seismic signal.



Generation of seismic ground motions

- In pratice there is not enough seismic ground motions for fragility curve estimation.

[2] NN Ambraseys, P Smit, R Berardi, D Rinaldis, F Cotton, and C Berge. Dissemination of european strongmotion data. cd-rom collection. european commission, directorate-general xii, environmental and climate programme.
Technical report, ENV4-CT97-0397, Brussels, Belgium, 2000

[3] Sanaz Rezaeian and Armen Der Kiureghian. Simulation of synthetic ground motions for specified earthquake and site characteristics.
Earthquake Engineering & Structural Dynamics, 2010

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- Example: European Strong-Motion Database[2]. 97 real seismic signals with magnitude $5.5 < M < 6.5$ and distance to the epicenter $0 < D < 20$ km.

[2] NN Ambraseys, P Smit, R Berardi, D Rinaldis, F Cotton, and C Berge. Dissemination of european strongmotion data. cd-rom collection. european commission, directorate-general xii, environmental and climate programme. Technical report, ENV4-CT97-0397, Brussels, Belgium, 2000

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Generation of seismic ground motions

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- Example: European Strong-Motion Database[2]. 97 real seismic signals with magnitude $5.5 < M < 6.5$ and distance to the epicenter $0 < D < 20$ km.
- Modelization by a filtered and modulated white noise stochastic process (good "space filling" properties).[3]

[2] NN Ambraseys, P Smit, R Berardi, D Rinaldis, F Cotton, and C Berge. Dissemination of european strongmotion data. cd-rom collection. european commission, directorate-general xii, environmental and climate programme.
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[3] Sanaz Rezaeian and Armen Der Kiureghian. Simulation of synthetic ground motions for specified earthquake and site characteristics.
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The choice of the seismic intensity

- The seismic intensity is a scalar quantity derived from the temporal seismic accelerogram.
- There are criteria in Earthquake Engineering to choose a "good" seismic intensity[4]:
  - *Efficiency*: $Y|A \approx Y|\text{Seismic hazard}$.
  - *Sufficiency*: $Y|A$ has a small variance.
- We consider the two most classical intensity measures: the peak ground acceleration (PGA) and the spectral acceleration.
- *Spectral acceleration*: Maximal acceleration of a linear oscillator at a chosen natural frequency and damping ratio (often very high sufficiency).

[4] M. Grigoriu and A. Radu. Are seismic fragility curves fragile? *Probabilistic Engineering Mechanics*, 2021

Challenge of statistical estimation with computer code

- The mechanical demand Y is usually obtained from time-consuming numerical simulations.

- This motivates the use of statistical metamodel[5] or active learning strategy[6].

[5] Bertrand Iooss and Loïc Le Gratiet. Uncertainty and sensitivity analysis of functional risk curves based on gaussian processes.
Reliability Engineering & System Safety, 2019

[6] Rémi Sainct, Cyril Feau, Jean-Marc Martinez, and Josselin Garnier. Efficient methodology for seismic fragility curves estimation by active learning on support vector machines.
Structural Safety, 2020

Main objectives of the thesis

- Developing **active learning strategies** to alleviate the computational burden of SPRA[7].
- Modeling, propagating and assessing the impact of **epistemic uncertainties** on SPRA's related quantites of interests (QoIs) such as fragility curves, probability of failures...

[7] Clement Gauchy, Cyril Feau, and Josselin Garnier. Adaptive importance sampling for seismic fragility curve estimation, 2021.
arXiv: 2109.04323

Propagation of epistemic uncertainties

Industrial motivations

- Uncertainties are divided in two groups:
  - *Aleatory*: Natural variability of a physical phenomenon.
  - *Epistemic*: Comes from the Greek word $\epsilon\pi\iota\sigma\tau\eta\mu\eta$ (knowledge). Uncertainties resulting from a lack of knowledge

[8] Armen Der Kiureghian and Ove Ditlevsen. Aleatory or epistemic? does it matter? *Structural Safety*, 2009

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- This division is purely subjective [8].
- For SPRA in the nuclear industry, epistemic uncertainties are mostly the mechanical parameters of the structure (natural frequency, damping ratio,...). Aleatory uncertainties come from the seismic ground motions' stochasticity.

[8] Armen Der Kiureghian and Ove Ditlevsen. Aleatory or epistemic? does it matter? *Structural Safety*, 2009

Fragility curve with epistemic uncertainties

Our goal is to assess the effect of mechanical model parameters uncertainties (e.g. epistemic uncertainties) on the Quantity of Interest (QoI).

- For X a random vector of parameters, $\Psi(a, X) = \mathbb{P}(Y > C | A = a,\ X)$
- The random function $\Psi(., X)$ could be seen as a functional QoI.
- Due to computational burden, we replace the costly mechanical simulation by a Gaussian Process surrogate of the mechanical response Y.

Framework of Gaussian process regression

- Let $Z(x),\ x \in \mathbb{R}^d$ a random process.
- A random process is a **Gaussian process** if for a tuple $(x_1, \dots, x_n)$, the random vector $(Z(x_1), \dots, Z(x_n))$ is Gaussian.
- The probability distribution of a Gaussian process is completely defined by:
  - Its mean function $\mu(x) = \mathbb{E}[Z(x)]$.
  - Its covariance function $k(x, \tilde{x}) = \mathbb{E}[(Z(x) - \mu(x))(Z(\tilde{x}) - \mu(\tilde{x}))]$
- we note $Z(x) \sim \mathcal{GP}(\mu(x), k(x, \tilde{x}))$

Prediction with Gaussian process

- Thanks to Gaussian conditioning theorem we have that:

$$(Z(x_*)|Z(x_1) = z_1, \dots, Z(x_n) = z_n) \sim \mathcal{GP}(\mu_c(x_*), k_c(x_*, \tilde{x}))$$

where:

$$\mu_c(x) = \mu(x) + r(x)^T K^{-1}(\mathbf{z} - \mu)$$

$$k_c(x, \tilde{x}) = k(x, \tilde{x}) - r(x)^T K^{-1} r(x)$$

$\mu = (\mu(x_1), ..., \mu(x_n))^T$ the experiment design mean vector.

$\mathbf{z} = (z_1, ..., z_n)^T$ the value of the model output.

$r(x) = (k(x, x_1), ..., k(x, x_n))^T$ covariance between a point x and the experimental design.

$K = (k(x_i, x_j))_{1 \leqslant i,j \leqslant n}$ covariance matrix of the experimental design.

We have an analytical expression about the probability distribution of $Z(x_*)$ given data.

Fragility curve estimation

- Gaussian Process regression for functional risk curves estimation have been previously studied [9].
- We propose the following regression model:

$$Y = \beta_0 + \beta_1 \log(A) + Z(X) + \varepsilon,$$

where Z a centered Gaussian Process with a tensorized 5/2 Matèrn kernel on the log-epistemic variables $\log(X)$, $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ and Y a mechanical response (e.g. peak displacement), submitted to a nonlinear transform (e.g. logarithm or Box Cox transformation).

- β_0, β_1, the Matèrn covariance hyperparameters and the nugget effect are calibrated by the posterior mode on a training dataset. The parameters are distributed by weakly informative priors.

- Given a dataset $\mathcal{D}_n = (A_i, X_i, Y_i)_{1 \leqslant i \leqslant n}$, the predictive distribution is $(Y|\mathcal{D}_n) \sim \mathcal{GP}\left(\mu,\ \Sigma + \sigma_\varepsilon^2 Id\right)$ with μ and Σ respectivly the conditional mean and covariance matrix of the Gaussian Process.

[9] Bertrand Iooss and Loïc Le Gratiet. Uncertainty and sensitivity analysis of functional risk curves based on gaussian processes.
Reliability Engineering & System Safety, 2019

Propagation of the epistemic uncertainties

- The GP surrogate model allow us to compute rapidly the fragility curves $\Psi(a, X) = \mathbb{P}(Y > C | A = a, X)$.

- The fragility curve $\Psi(a, X)$ can be approximated using the predictive distribution of the GP by $\check{\Psi}(a, X)$:

$$\check{\Psi}(a, X) = \Phi\left(\frac{\mu(a, X) - C}{\sqrt{\Sigma(X, X) + \sigma_\varepsilon^2}}\right) ,$$

- It is also possible to propagate the GP surrogate uncertainty in the fragility curve, by only integrating the Gaussian noise ε:

$$\tilde{\Psi}(a, X) = \Phi\left(\frac{\tilde{Y}_{GP}(a, X) - C)}{\sigma_\varepsilon}\right) .$$

where $\tilde{Y}_{GP}(A, X)$ is the Gaussian Process prediction at (A, X) without the Gaussian noise ε.

Propagation of the GP uncertainty on the mean fragility curve.

- The mean fragility curve $\Psi(a) = \mathbb{P}_X(Y > C|A = a)$ is approximated using our Gaussian process $\Psi_{GP}(a) = \mathbb{P}_X(Y_{GP}(a, X) > C|A = a)$. Thus, $\Psi_{GP}(a)$ is tainted with the uncertainties coming from the Gaussian process regression.

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- $\Psi(a)$ can be approximated by the mean value $\check{\Psi}(a)$ of $\Psi_{GP}(a)$:

$$\check{\Psi}(a) = \mathbb{E}_X\left[\Phi\left(\frac{\mu(a, X) - C)}{\sqrt{\Sigma(X, X) + \sigma_\varepsilon^2}}\right)\right]$$

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- GP uncertainty is propagated using Monte Carlo on a dataset $(X_k)_{1\leqslant k\leqslant N}$ and L Gaussian process samples $(y^{(l)}(a, X_k))_{1\leqslant \ell\leqslant L}$:

$$\Psi_{GP}^{(\ell)}(a) = \frac{1}{N}\sum_{k=1}^{N} \mathbb{1}_{(y^{(\ell)}(a,X_k)>C)}$$

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- $\Psi_{GP}^{(\ell)}(a)$ is thus a Monte Carlo approximation of a realization of the random variable $\Psi_{GP}(a)$.

Global Sensitivity Analysis on fragility curves

- Sobol indices are variance-based importance measures coming from the Hoeffding-Sobol decomposition of the variance.

- Aggregated first order and total order Sobol indices for functional risk curves have been previously studied [10], raising the following expressions:

$$S_i = \frac{\mathbb{E}[\|\bar{\Psi} - \Psi_{X^i}\|_{L^2}^2]}{\mathbb{E}[\|\bar{\Psi} - \Psi_X\|_{L^2}^2]} \ ,$$

$$T_i = \frac{\mathbb{E}[\|\bar{\Psi} - \Psi_{X^{-i}}\|_{L^2}^2]}{\mathbb{E}[\|\bar{\Psi} - \Psi_X\|_{L^2}^2]} \ .$$

with $\bar{\Psi} = \mathbb{E}[\Psi(.,X)]$, $\Psi_{X^i} = \mathbb{E}[\Psi(.,X)|X^i]$ and $X^{-i} = (X^1, ..., X^{i-1}, X^{i+1}, ..., X^p)$.

[10] Bertrand Iooss and Loïc Le Gratiet. Uncertainty and sensitivity analysis of functional risk curves based on gaussian processes.
Reliability Engineering & System Safety, 2019

Pick freeze estimation

- **Lemma:** Denote $\tilde{X}^i$ the random vector such that $(\tilde{X}_j)_i = X^i$ if $i = j$ and $(\tilde{X}_j)_i = \tilde{X}^i$ if $i \neq k$ where $\tilde{X}^i$ is an independent copy of X^i. Thus:

$$S_i = \frac{\mathbb{E}[\langle \Psi_X, \Psi_{\tilde{X}^i} \rangle_{L^2}]}{\mathbb{E}[\|\bar{\Psi} - \Psi_X\|_{L^2}^2]} \ ,$$

$$T_i = 1 - \frac{\mathbb{E}[\langle \Psi_X, \Psi_{\tilde{X}^{-i}} \rangle_{L^2}]}{\mathbb{E}[\|\bar{\Psi} - \Psi_X\|_{L^2}^2]} \ .$$

Where $\langle \Psi_X, \Psi_{\tilde{X}^i} \rangle_{L^2} = \int (\Psi_X(a) - \bar{\Psi}(a))(\Psi_{\tilde{X}^i}(a) - \bar{\Psi}(a)) da.$

- We propose a Monte Carlo estimator of the aggregated Sobol indices using the lemma above.

GP uncertainty propagation on the aggregated Sobol indices.

- The GP uncertainty on the aggregated Sobol indices can be propagated (previously done for the case of scalar QoI [11]).

- Principle: replace $\Psi(., X)$ by $\tilde{\Psi}(., X)$. Thus we have to simulate GP trajectories $(\tilde{Y}_{GP}(a, X_k))_{1 \leqslant k \leqslant N}$ for N large, which is computationally challenging. One can rely on approximation methods such as spectral methods or Karhunen-Loeve decomposition.

[11] Loic Le Gratiet, Claire Cannamela, and Bertrand Iooss. A bayesian approach for global sensitivity analysis of (multifidelity) computer codes.
SIAM/ASA Journal on Uncertainty Quantification, 2014

Applications and numerical results

Numerical application: Nonlinear oscillator

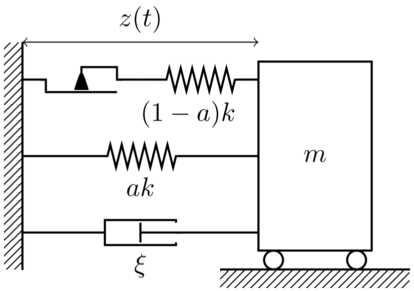


Figure: Elasto-plastic mechanical oscillator with kinematic hardening. The linear parameters are the mass m, stiffness k, damping ratio ξ. The non linearity is controlled by the yield limit ℓ_Y and the post-yield stiffness a.

$$\ddot{z}(t) + 2\beta\omega_L\dot{z}(t) + f_{NL}(t) = -s(t) \ ,$$

Table: Epistemic uncertainties on the elasto-plastic oscillator

parameter	distribution	mean	c.o.v
m	Lognormal	1	30%
k	Lognormal	900	30%
ξ	Lognormal	0.02	50%
ℓ_Y	Lognormal	5×10^{-3}	30%
a	Lognormal	0.2	30%

Nonlinear oscillator

- The intensity measure chosen is the *Spectral acceleration* at 5Hz and 2% damping ratio.
- For a seismic signal $t \to s_i(t)$, the oscillator response $t \to z_i(t)$ is computed, then the failure state is defined by:

$$\max_{t \in [0,T]} |z_i(t)| > 2\ell_Y$$

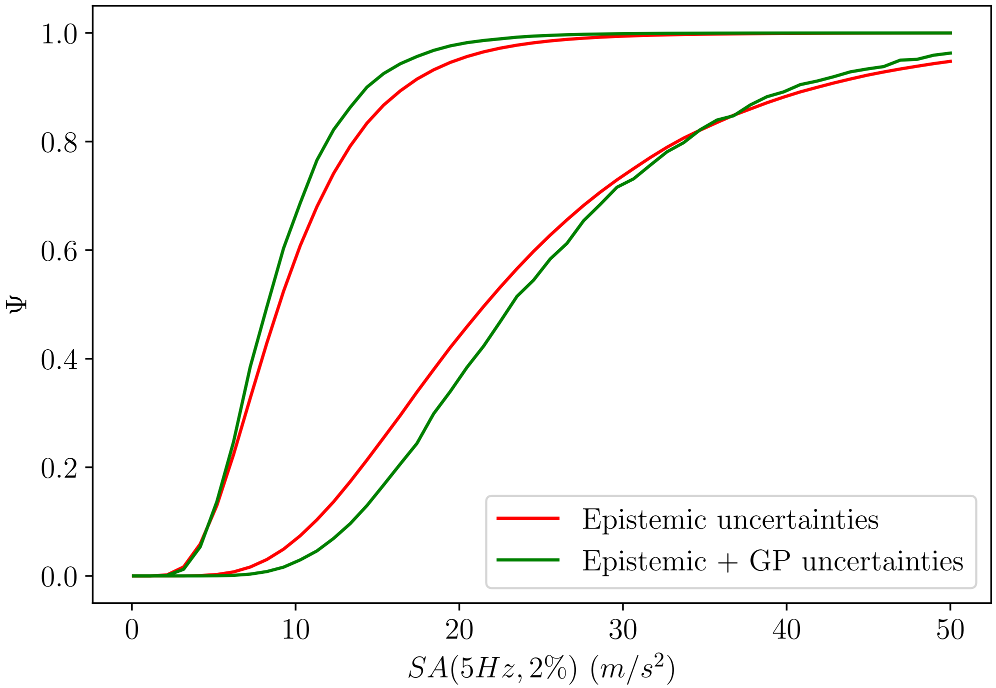
.

Design of experiments and performance of the metamodel

- The metamodel is trained on an optimized LHS with according probability distribution for each variable using **`maximin`** criterion (**`PyDOE`** package in Python) of 500 datapoints.
- The input variables are standardized (centered and scaled).
- Performance of the metamodel on the mechanical demand using the mean Q^2 predictivity coefficient computed with 5-fold cross validation on an optimized LHS dataset of size 500.

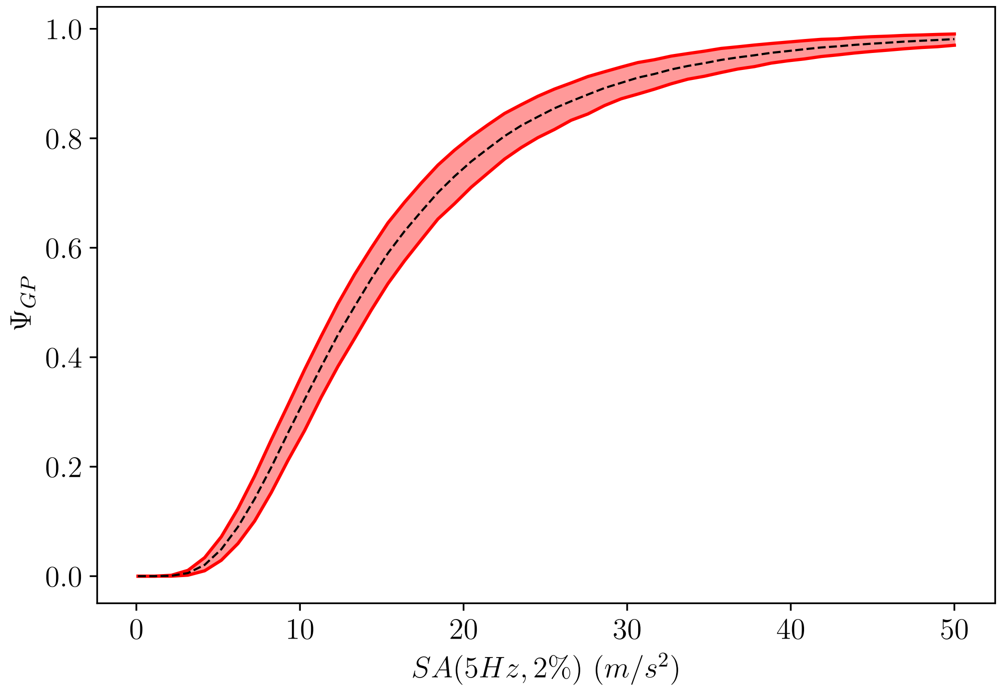
$$Q^2 = 0.86$$

Epistemic and GP uncertainty propagation



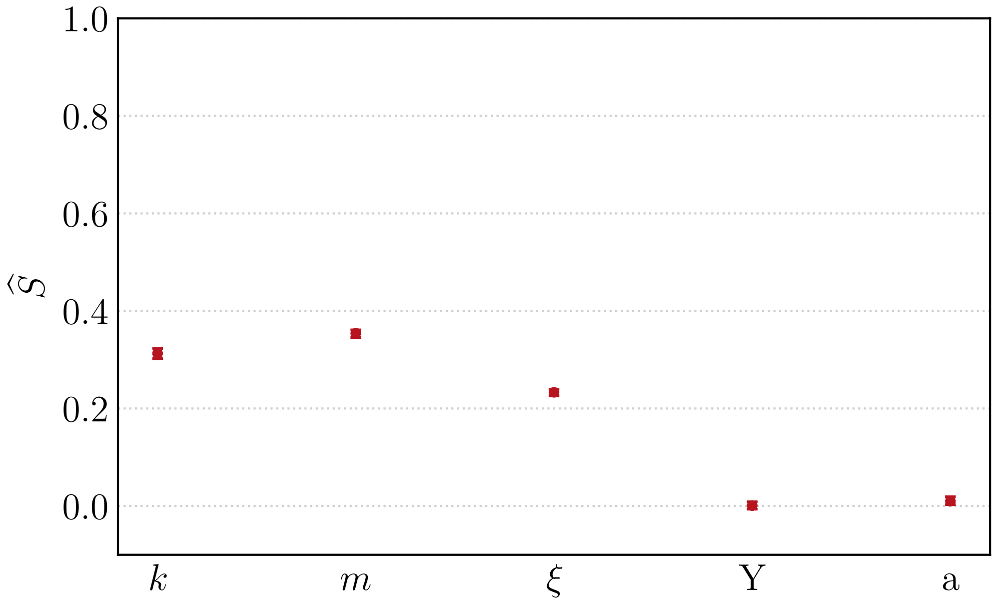
90% and 10% quantiles fragility curves estimated on a Monte Carlo of size $N = 10000$ tainted with epistemic and GP uncertainties for the nonlinear oscillator. The metamodel is trained with 80 observations.

Mean fragility curve with GP uncertainties



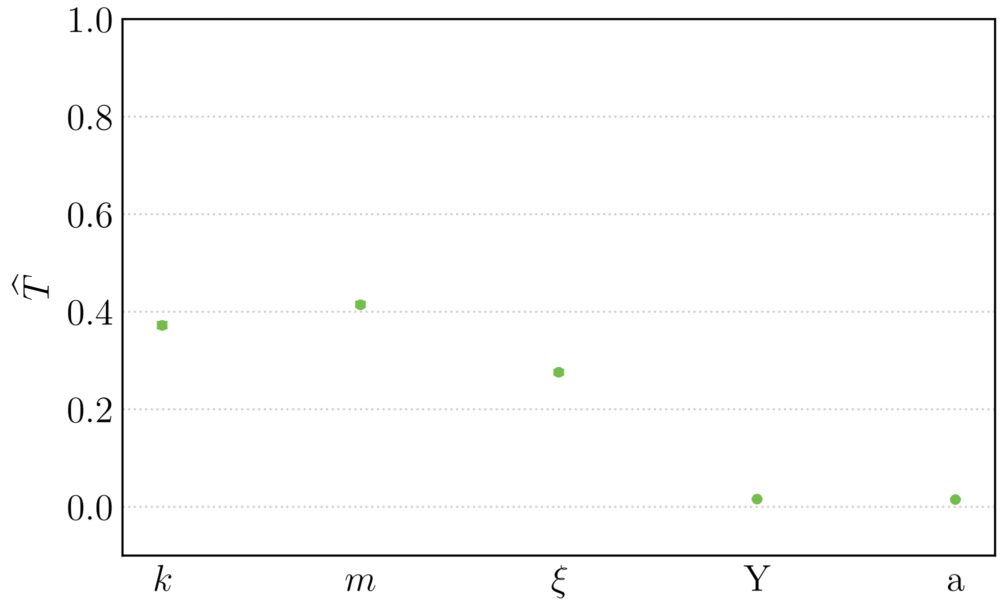
90% and 10% quantiles for the mean fragility curves estimated on a Monte Carlo of size $N = 5000$ and $L = 3000$ GP trajectories drawn. The metamodel is trained with an optimized LHS of 50 observations.

Aggregated Sobol indices estimation with posterior mean of a GP metamodel



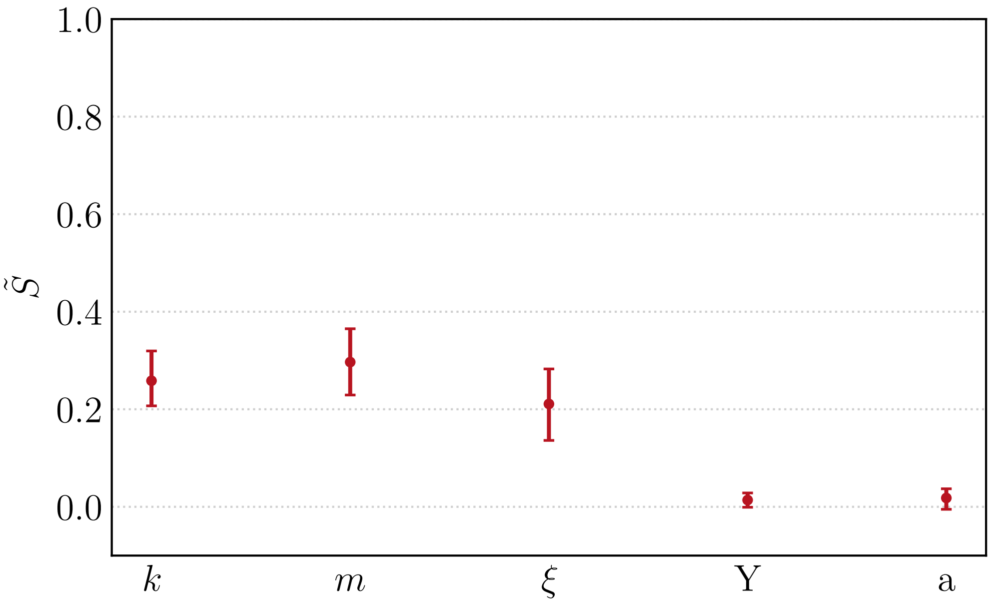
First order Sobol indices estimation on $B = 20$ bootstrap samples of size $N = 15000$. GP metamodel is trained on an optimized LHS of 50 observations

Aggregated Sobol indices estimation with GP uncertainties



Total order Sobol indices estimation on $B = 20$ bootstrap samples of size $N = 15000$. GP metamodel is trained on an optimized LHS of 50 observations.

Aggregated Sobol indices estimation with a GP metamodel



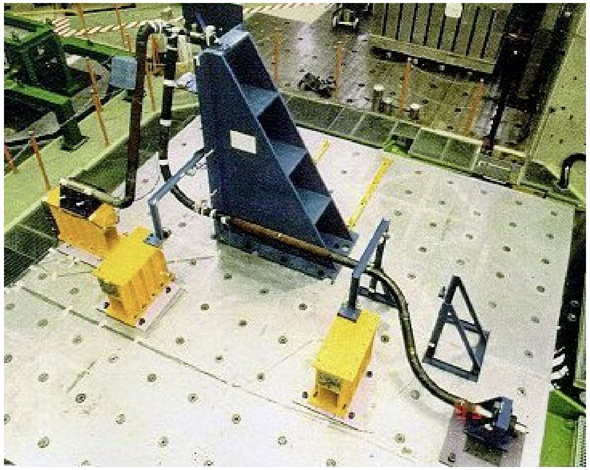
First order Sobol indices estimation on $B = 30$ bootstraps samples of size $N = 15000$ and $M = 50$ draws of the GP prediction $\tilde{Y}_{GP}(A, X)$. GP metamodel is trained on a optimized LHS of 50 observations.

Industrial test case: Safety water pipe of a French PWR

- The following test case corresponds to a piping system which is a simplified part of a secondary line of a French Pressurized Water Reactor (ASG program).

- The CAST3M numerical model was validated based on seismic tests performed on the shaking table Azalee of the EMSI laboratory of CEA Saclay.

- The failure state is defined by the out-of-plane rotation of the elbow near the clamped end of the pipe. Failure is defined by a rotation that exceeds 1°.



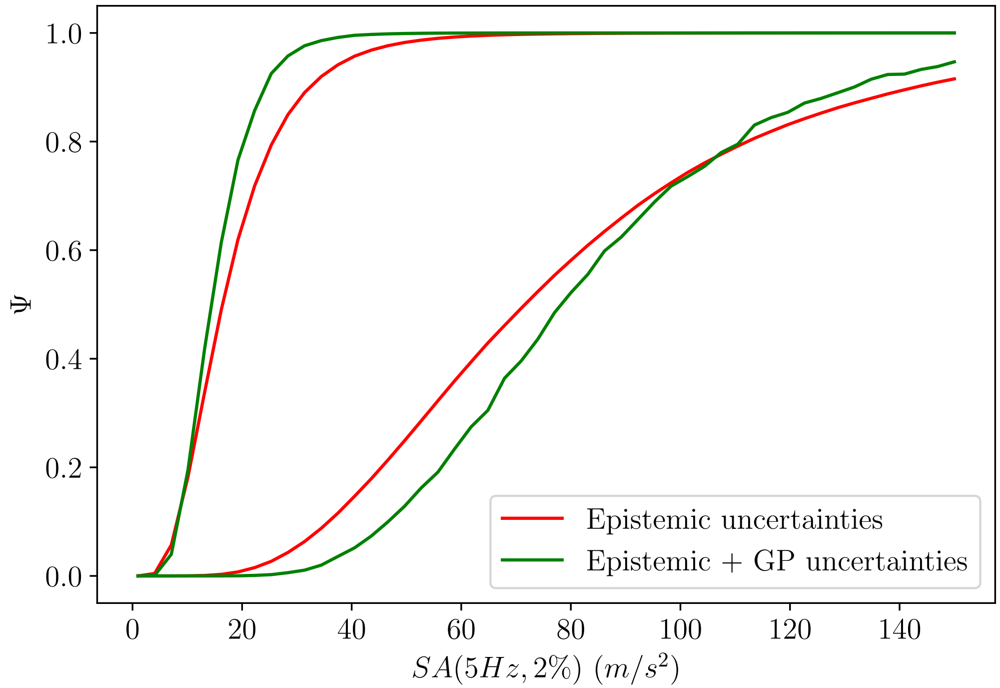
Overview of the ASG mock-up on the shaking table

Industrial test case: Epistemic uncertainties

- Uncertainties on the boundary conditions has been introduced to simulate the interaction between the simplified piping system and the overall structure. We also suppose the piping structure is inside a fictious building with unknown mechanical properties (a natural frequency and a damping ratio).

- The 12 uncertain parameters have uniform distribution with a coefficient of variation of 15%.

Epistemic and GP uncertainty propagation



90% and 10% quantiles fragility curves estimated on a Monte Carlo of size $N = 10000$ tainted with epistemic and GP uncertainties for the ASG piping structure simulation (linear analysis). The metamodel is trained with 200 observations.

Conclusion & perspectives

- Uncertainty propagation framework for seismic risk assessment using Gaussian process regression, taking into account the metamodel uncertainties.

- Global Sensitivity Analysis framework on fragility curves.

- **What's next ?:**
  - Multifidelity metamodel for the ASG piping system.
  - Defining insightful QoIs for SPRA.
  - Kernel based GSA (e.g. MMD sensitivity indices).

Thank you for your attention !

Références

NN Ambraseys, P Smit, R Berardi, D Rinaldis, F Cotton, and C Berge. Dissemination of european strongmotion data. cd-rom collection. european commission, directorate-general xii, environmental and climate programme. Technical report, ENV4-CT97-0397, Brussels, Belgium, 2000.

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Merci pour votre attention !

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Sommaire