DE LA RECHERCHE À L'INDUSTRIE

Multi-fidelity modelling strategy based on a system approach including NURBS metamodels

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Introduction

Introduction

Context

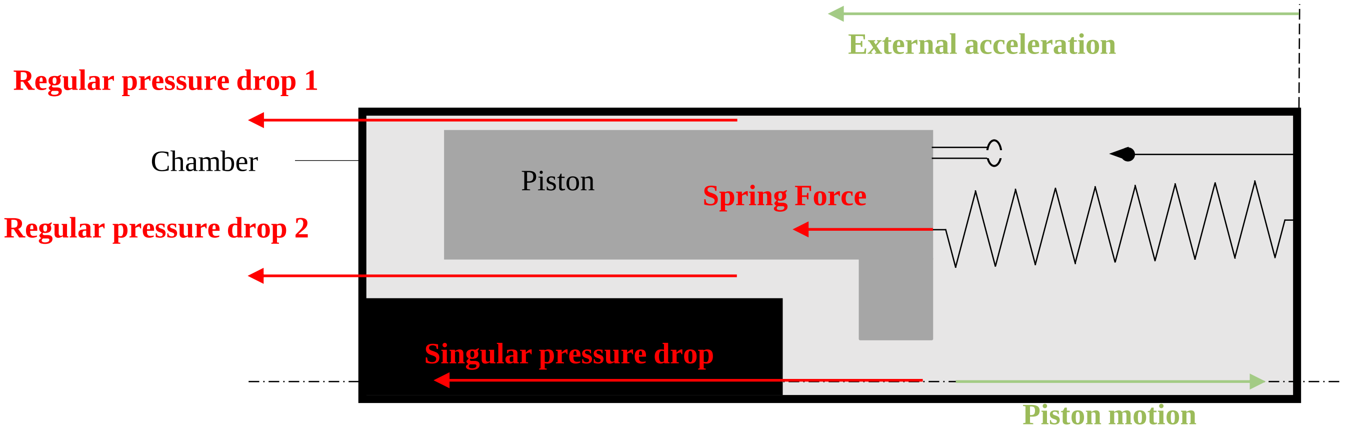
- Difficulties in conducting experiments (more complex experiences);
- Increasing demand for computer resources;
- Emergence of metamodeling strategies.

Thesis objectives

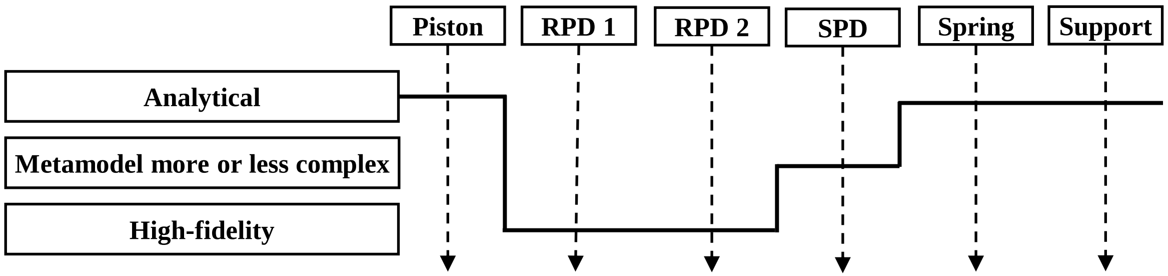
To develop Multi-fidelity modelling strategy based on a system approach including NURBS metamodels to solve engineer system model.

- To reduce computing time;
- To do quick preliminary design;
- To apply computing resources just where it is necessary.

Typical application: airbag switching system



- Idea: **to subdivide the system into elementary blocks** and to determine the **critical components** (pressure drop components).
- Friction coefficient (oil/piston): regular pressure drop → several level of physical description are available (Bernoulli / Metamodel / Finite Elements);
- Our goal: to evaluate different **robust** and **adaptative precision metamodeling** strategies.



The thesis

The means (work in progress)

- System divide in **elementary blocks:**
  - **Modelica**
- To our typical problems, sensitivity analysis could be useful
  - **Sobol indices**
- Inclusion (FMI norm) of **external code or metamodel**:
  - NURBS, CATIA, Abaqus…
- Determination of the sufficient level of description:
  - L^2 **norm convergence** (still under reflexion)

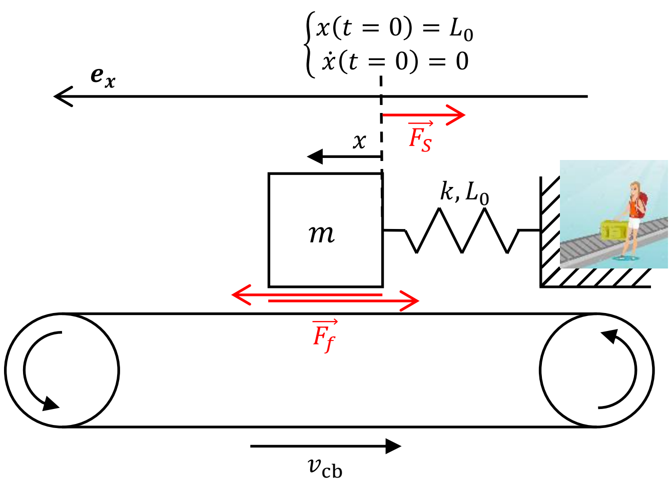
What's new ?

- To do multi-fidelity model with a system approach;
- To use Modelica coupled with other codes;
- To evaluate NURBS / PGD as a metamodel strategy.

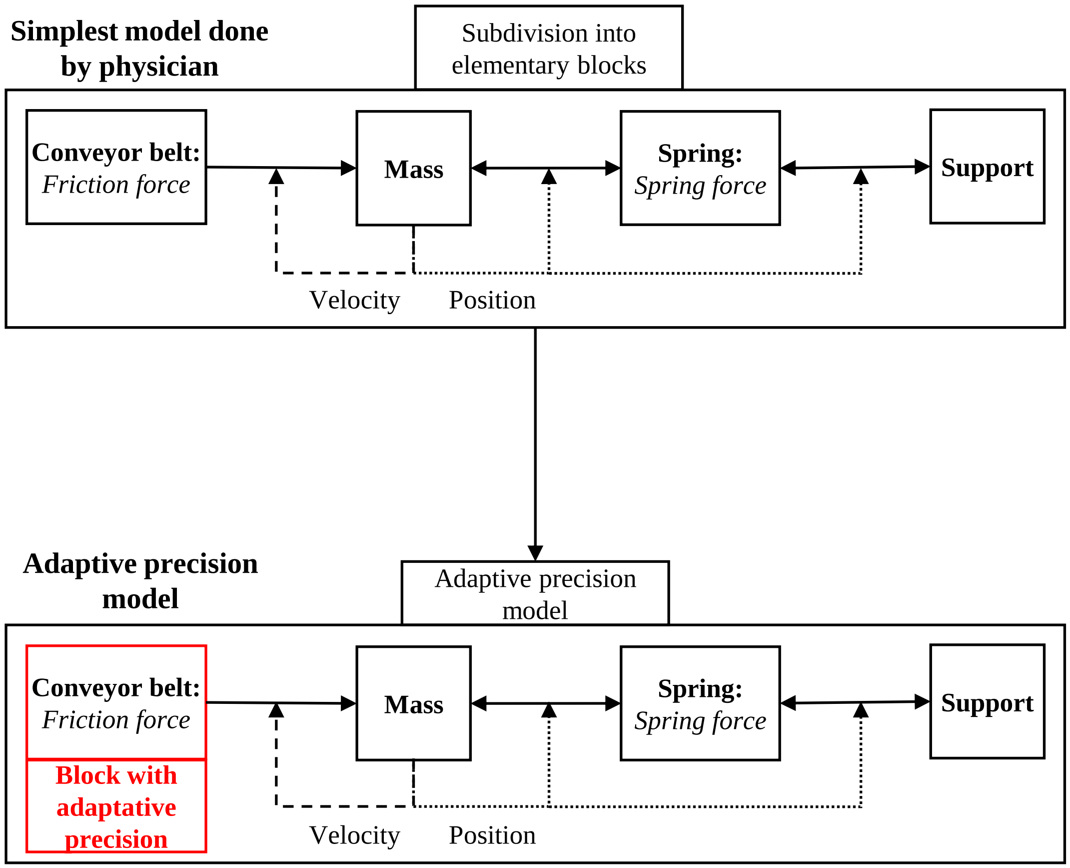
Methodology illustration

Methodology illustration

[2], [3], [4], [5]

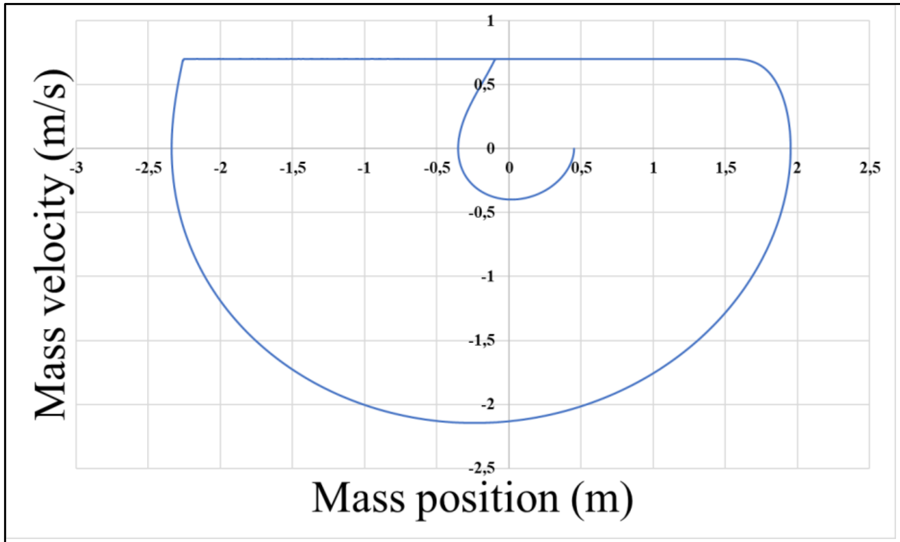


Scheme

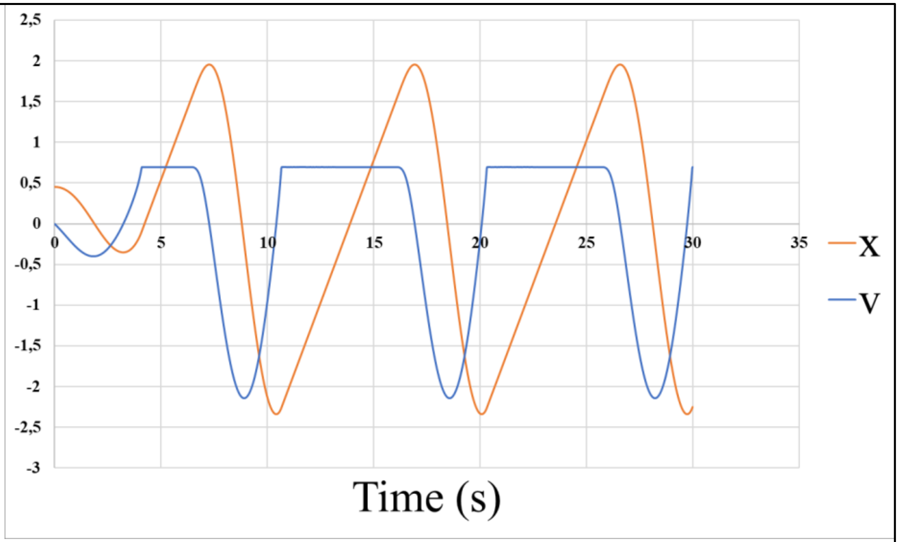


Functional scheme

Selected solution for sensitivity analysis



Phase diagram



Velocity and position vs time

To be sure to get the expected solution, a physician advice is required because the problem is highly non linear (cf. bibliography).

For sensibility analysis of the friction force block, we will consider the stable solution (velocity plateau > 0.3 s)

[14], [15], [16]

The Sobol's indices

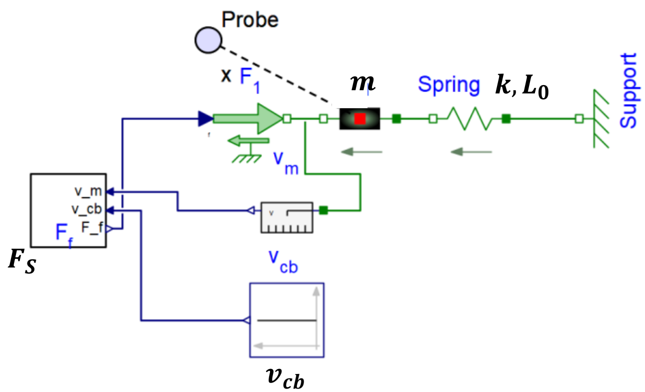
- Elementary indices translate the influence of one parameter on the observed output:

$$S_{E_i} = \frac{Var\left(\mathbb{E}(Y|X_i)\right)}{Var(Y)}$$

- Total indices translate the influence of one parameter and its interactions with other parameters:

$$S_{T_i} = S_{E_i} + S_{(i,j)} + S_{(i,j,k)} + \cdots$$

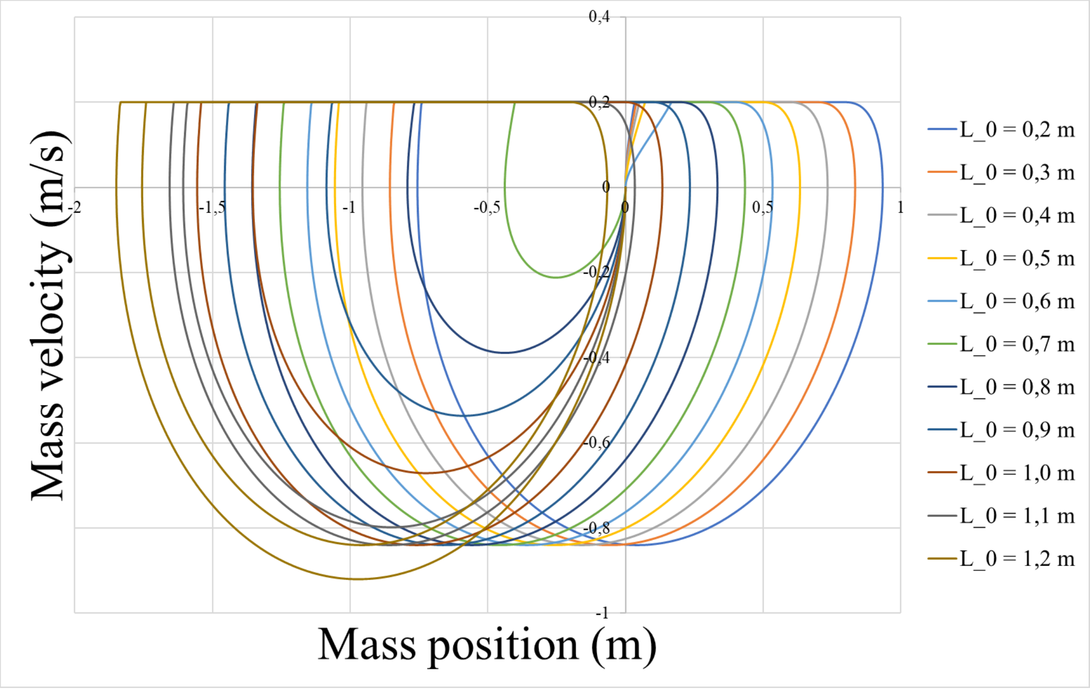
Example : motion stability (see previous solution)



Studied parameters	Range variation
k	$\mathcal{U}([0.5; 5.0])\ \mathrm{N.m^{-1}}$
L_0	$\mathcal{U}([0.2; 1.2])\ \mathrm{m}$
m	$\mathcal{U}([0.5;\ 2.0])\ kg$
v_{cb}	$\mathcal{U}([0.2; 5.0])\ m.s^{-1}$
F_S	$\mathcal{U}([1.0; 5.0])\ \mathrm{N}$

Sensitivity analysis

Studied parameters	Elementary indices	Total indices
k	12.69 %	44.25
L_0	0.00 %	2.08
m	0.00 %	5.67
v_{cb}	34.05 %	74.69
F_S	11.47 %	44.19
Interactions	41.79 %	-

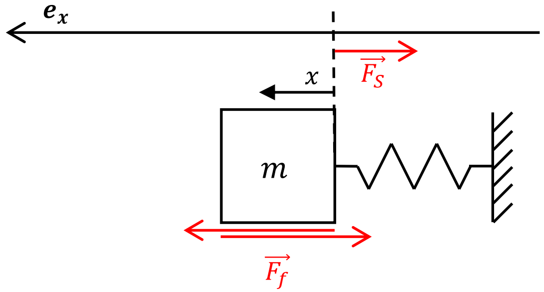


- Conveyor belt velocity is the most influential parameter.
- The unstretched spring length L_0 is the less influential parameter.
- We have to describe more precisely the elementary block which contains v_{cb} and F_S (the frictional force block).

The means

[11], [12], [13]

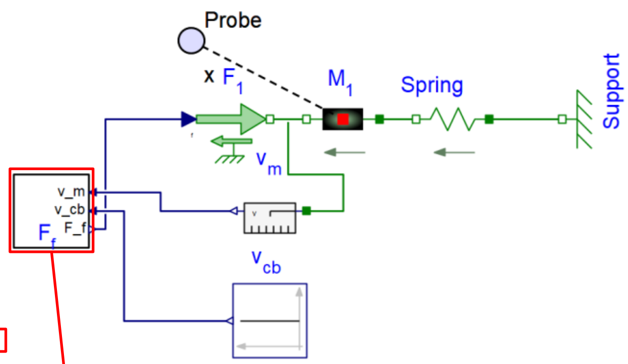
The computer language: system approach and acausal resolution



Real model

```
model StickSlip
    import SI = Modelica.SIunits;
    parameter SI.SpringConstant k = 1;
    parameter SI.Lenght L0 = 0.45;
    parameter SI.Force F_S = 2;
    parameter SI.Mass m=1;
    parameter SI.Velocity v_cb = 0.2;
    parameter Real delta = 3;
    SI.Position x;
    SI.Velocity v, v_rel;
    SI.Acceleration a;
    SI.Force F_Spring, F_f;
equation
    v = der(x);
    a = der(v);
    F_Spring = k*(x-L0);
    F_f = -(F_S*sign(v_rel))/(1 + delta * abs(v_rel));
    v_rel = v - v_cb;
    m*a = F_Spring + F_f;
end StickSlip;
```

Computed model



Scheme block

Metamodeling strategy

Modelica vs. classical language:

In Modelica: write just the constraint equations

Classical language: write in an explicit way the final Differential Algebraic Equation system

Identification of some physical problems of interest: thermo-mechanical system, latches system, thermal battery

Metamodeling (work in progress)

Choice of metamodeling strategy

- The importance of the *off-line* part;
- The difficulty to use it;
- The difficulty to integrate it;
- Their precision;
- How it take in account the uncertainty.

What I will evaluate in my thesis

- NURBS
  - Approximation strategy, improved by the I2M team (genetic algorithm), especially during the thesis of Yohann AUDOUX.
- PGD
  - Model reduction method (contact with Francisco CHINESTA [10]).

[6], [8]

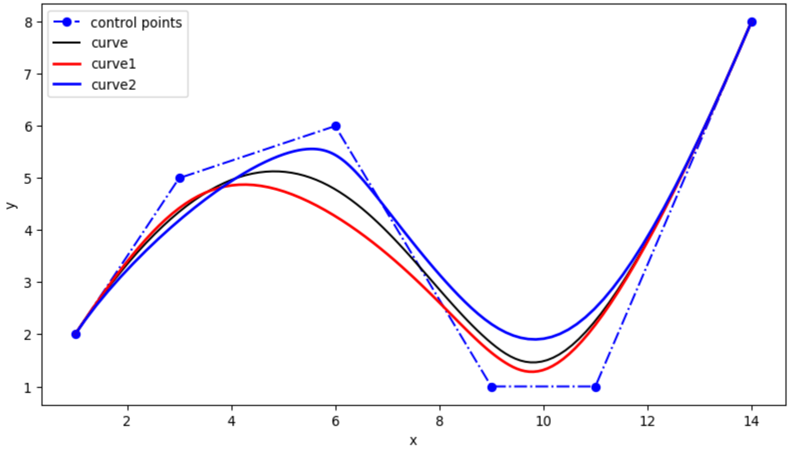
NURBS curves (Non Uniform Rational Basis Spline)

$$\mathbf{C}(t) = \frac{\sum_{i=0}^{p-1} N_i^n(t)\omega_i \mathbf{P}_i}{\sum_{i=0}^{p-1} N_i^n(t)\omega_i}$$

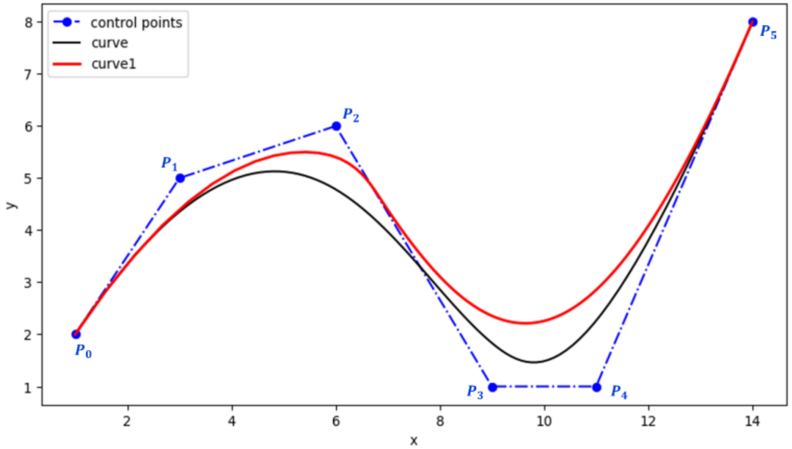
- p : the number of control points $\mathbf{P_i}$ (forming a control polygon);
- ω_i : the weights associated to each control points $\mathbf{P_i}$;
- t : node of the node vector $\boldsymbol{t} = [t_0, \dots, t_m]$, with $m = \mathrm{n} + \mathrm{p} + 1$;
- $N(t)$: parametrical functions of degree n, defined on $\mathrm{t} \in [t_k; t_{k+1}]$ with $k \in [0; m-1]$ and expressed recursively as follows:

$$\begin{cases} N_{i,0}(t) = \begin{cases} 1, & \text{if } t_i < t < t_{i+1}, \\ 0, & \text{else,} \end{cases} \\ N_{i,\tau}(t) = \dfrac{t - t_i}{t_{i+\tau} - t_i} N_{i,\tau-1}(t) + \dfrac{t_{i+\tau+1} - t}{t_{i+\tau+1} - t_{i+1}}, \end{cases} \quad \text{with } i \in [0; p-1] \text{ and } \tau \in [0; n].$$

Metamodeling strategy – NURBS



In black: simple B-Spline;
In red: high weight on node 3;
In blue: low weight on node 3.



In black: simple B-Spline with uniform node vector $\boldsymbol{t} = [0; 0; 0; 0; \frac{1}{3}; \frac{2}{3}; 1; 1; 1; 1]$;
In red: NURBS with non-uniform node vector $\boldsymbol{t} = [0; 0; 0; 0; 0.1; 0.2; 1; 1; 1; 1]$.

Metamodeling strategy – NURBS

[7], [9]

Use in metamodeling strategy:

- Pioneer in this field: Turner
- Algorithm developed and generalised at I2M
- Formalised by Yohann Audoux during his thesis
  - Use of a NURBS algorithm coupled with a genetic optimisation algorithm to compute the node vector and point weight values.
- **Can be included in Modelica as all functions with several parameters**

Conclusion and perspectives

Conclusion

What has been done (*1st PhD year*)

- Identification of some systems application;
- First sensitivity analysis;
- Physical / engineering analysis (by the user) + Modelica;
- Bibliography: Modelica and its uses/viability, metamodeling, NURBS, PGD.

Future work (*2nd PhD year)*

- To evaluate NURBS and PGD programs;
- To compare others metamodel strategies with NURBS and PGD;
- To take into account: uncertainty, multi-fidelity (to determine the good level of description) [1]

Thank for your attention

Supervisors : Marco MONTEMURRO,
Enrico PANETTIERI,
Ludovic HALLO

PhD student : Bruno VUILLOD

16 September, 2021

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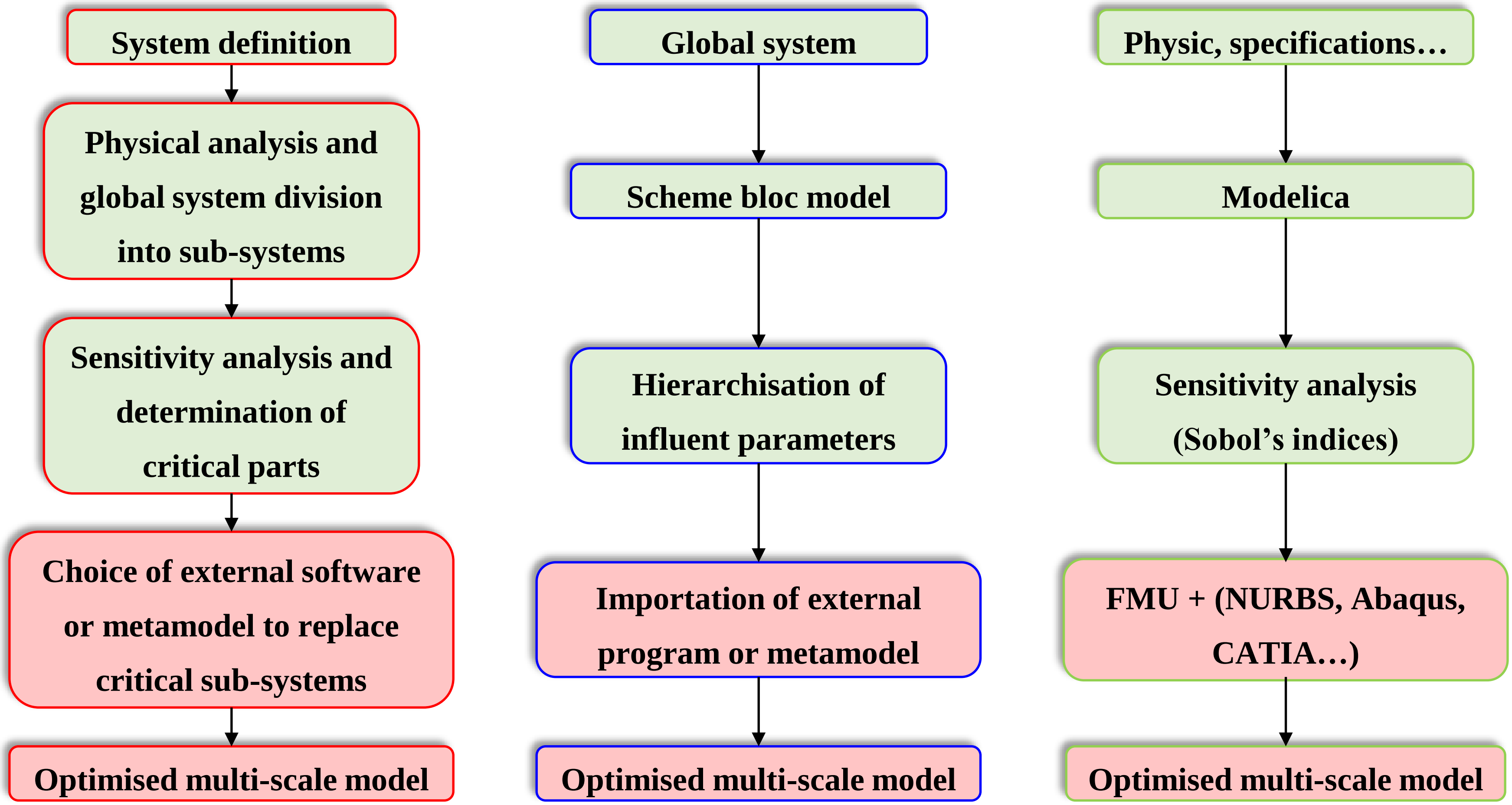
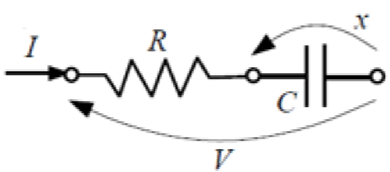
Summary of the methodology developed through my thesis



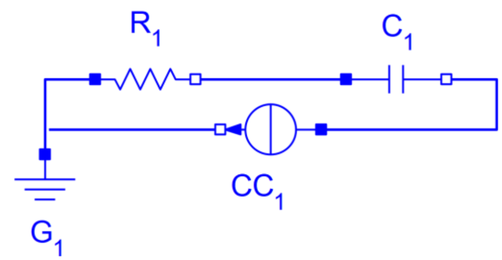
The computer language



Real model

```
model CircuitRC
    import SI = Modelica.SIunits;
    parameter SI.Current I=1;
    parameter SI.Resistance R=1;
    parameter SI.Capacitance C=1;
    SI.Voltage V, x(start = 0);
equation
    x + R*I = V;
    C * der(x) = I;
end CircuitRC;
```

Computed model



Bloc scheme

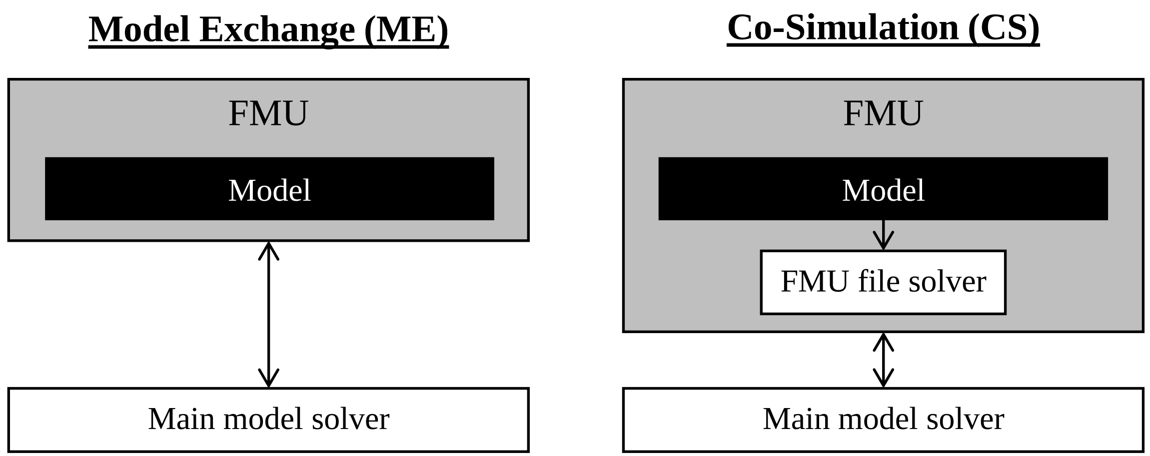
$$\begin{cases} x + RI = V \\ C\dot{x} = I \end{cases}$$

Initial system

$$C = \frac{I}{\dot{x}} \text{ ou } R = \frac{V-x}{I}$$

Equations which could be taken into account

The Functional Mock-up Interface norm (FMI)



A FMU file is like a ZIP file which contains:

- An XML document with the description of the mdoel
- A C/C++ code of the model
- Other documents that the model need to be run (Excel table point for example)

The elementary Sobol's indices

- It corresponds to the influence of a parameter translated by **its variance**, **normalized** on the **global model variance**.
- Explanation :
  - The parameter ξ_i variance is determined by studying the output variance Y by fixing ξ_i : $Var(Y|\xi_i = x_i) \xrightarrow[global\ method]{} E\big(Var(Y|\xi_i)\big)$. This corresponds to a **conditional variance** or **conditional expectation**.
  - If $E\big(Var(Y|\xi_i)\big)$ is small, fixing ξ_i reduces the variability of Y, so ξ_i is influential.
  - By using the total variance theorem : $Var(Y) = E\big(Var(Y|\xi_i)\big) + Var\big(E(Y|\xi_i)\big)$, it can be deduced that if ξ_i is influential, $Var\big(E(Y|\xi_i)\big)$ is high.
  - Normalizing by $Var(Y)$, we obtain the influence of the parameter ξ_i on the output Y in percent, which is much more exploitable. This is **the elementary Sobol index** S_{E_i}.

$$S_{E_i} = \frac{Var\big(E(Y|\xi_i)\big)}{Var(Y)} = \frac{E\left(\big(E(Y|\xi_i) - E(Y)\big)^2\right)}{Var(Y)}$$

The total Sobol's indices

- It corresponds to the influence of a parameter alone and with its interactions with other parameters translated by **its variance** and **coupled variances**, **normalized** on the **global model variance**.
- Explanation:
  - A parameter can have a self influence on the system and cross-influence with another parameter.
  - To take it in case, we use the **total Sobol's index** which correspond to the sum of elementary Sobol's index, second order Sobol's index, third order…
  - A Sobol's index of order greater than one, is determined by fixing the value of more than one parameter. Mathematically, this is expressed has follows:

$$S_{E_{(i,j)}} = \frac{1}{Var(Y)}\left[Var\left(E\left(Y|\xi_i,\xi_j\right)\right) - Var\left(E(Y|\xi_i)\right) - Var\left(E\left(Y|\xi_j\right)\right)\right]$$

$$S_{E_{(i,j,k)}} = \frac{1}{Var(Y)}\left[Var\left(E\left(Y|\xi_i,\xi_j,\xi_k\right)\right) - Var\left(E(Y|\xi_i)\right) - Var\left(E\left(Y|\xi_j\right)\right) - Var\left(E(Y|\xi_k)\right)\right]$$

- Finally, by summing all the Sobol's index of order one and more, we deduce the **total Sobol's index** :

$$S_{T_i} = S_{E_i} + S_{E_{(i,j)}} + S_{E_{(i,j,k)}} + \cdots$$

Sobol's index property

- By considering a model of d parameters, we have the following low:

$$\sum_{i=1}^{d} S_{E_i} + \sum_{\substack{i=1,j=1 \\ i\neq j}}^{d} S_{E_{(i,j)}} + \sum_{\substack{i=1,j=1,k=1 \\ i\neq j\neq k}}^{d} S_{E_{(i,j,k)}} + \cdots = 1$$

Caution when using this indicator

It's important to well make the difference with elementary and total Sobol's index. Indeed, a parameter can not have influence alone but can have great with interactions. So, it's necessary to first study total Sobol's index and after elementary.

Model definition

Input parameters $\boldsymbol{X}$
- Physical characteristics
- Chemical characteristics
- …

Studied system
- Mathematical system
- Digital simulation
- …

Ouput variables $\boldsymbol{f}(\boldsymbol{X})$
- Movements
- Stability
- …

A metamodeling strategy will be to determine a function $\hat{f}(\boldsymbol{X})$ which will approximate the initial function $f(\boldsymbol{X})$ using fewer resources and with the lowest possible approximation error $\boldsymbol{\varepsilon}$.

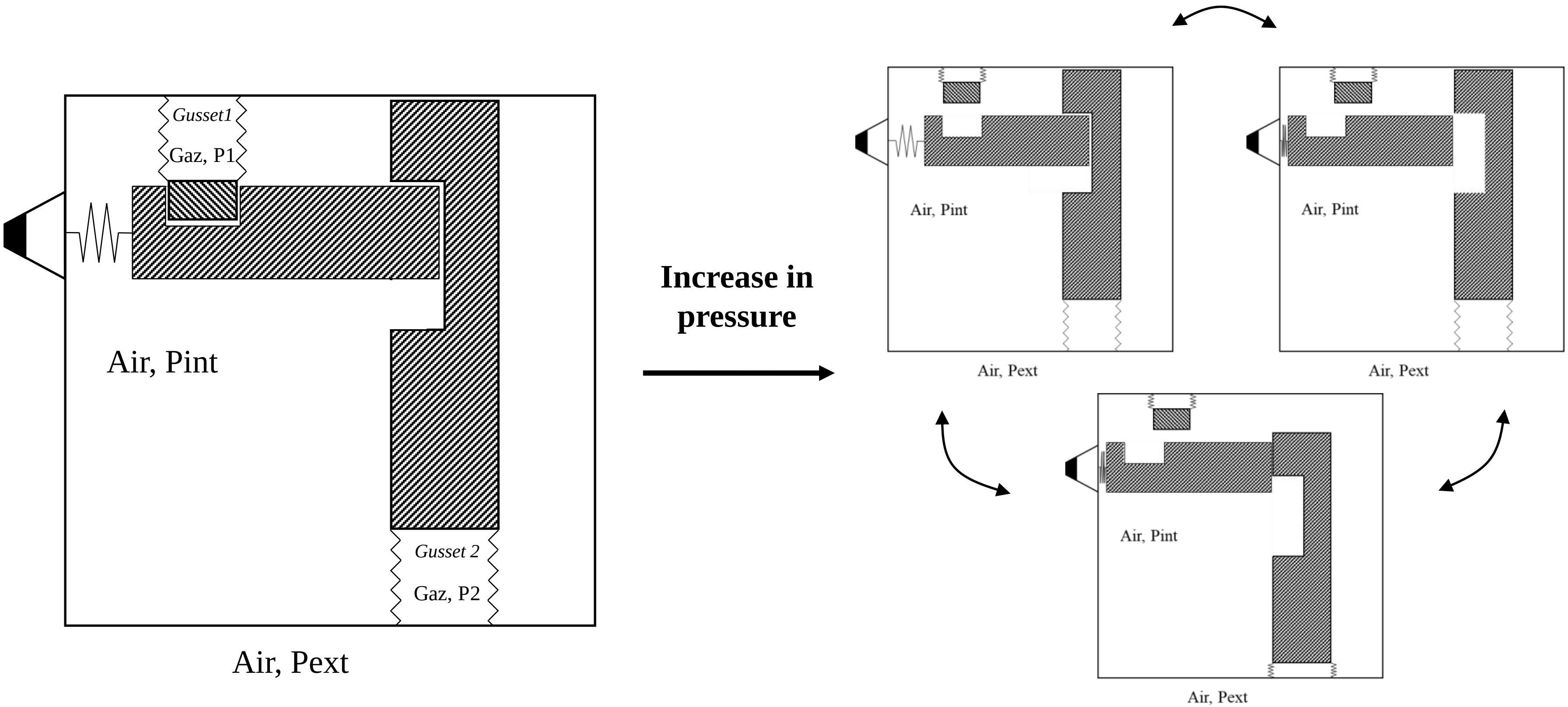
There are 2 parts in metamodeling

***Off-line* part**
- Calibration
- Learning
- Choice of metamodel parameters

→

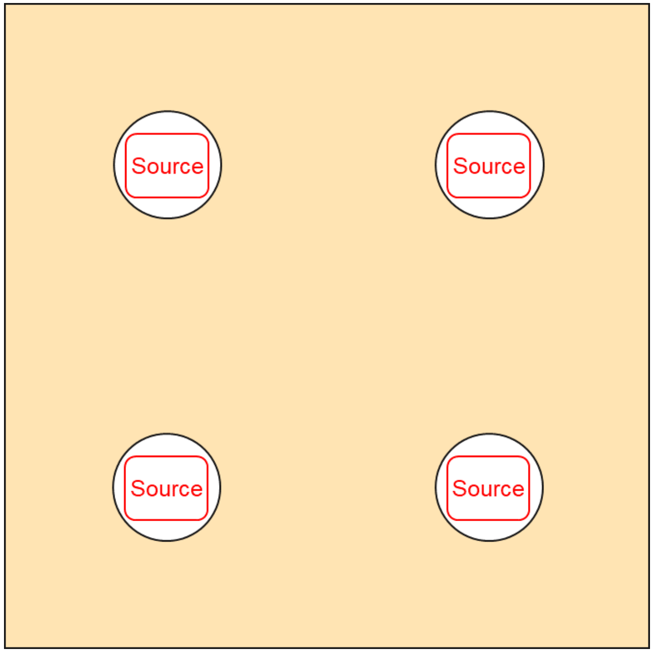
***On-line* part**
- Use of the metamodel
- Optimisation process
- Integration into a general model

Latches systems



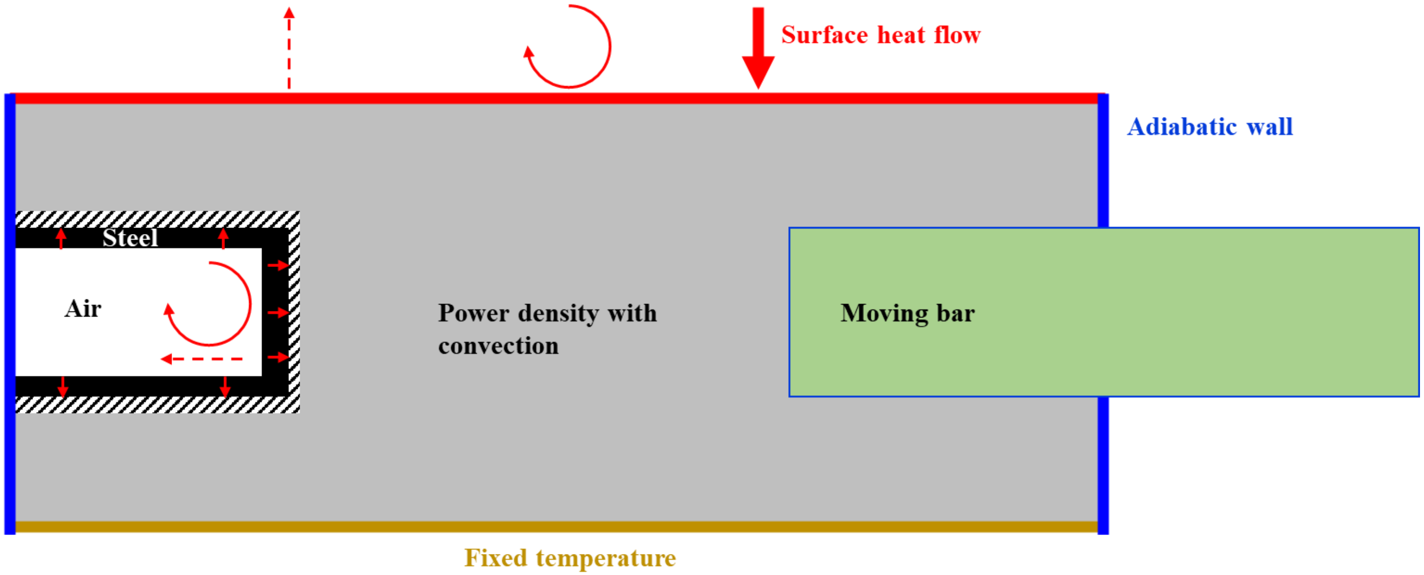
Criteria	Comments
Physic(s)	Hydromechanical
Metamodel – microscopical	Pressure law
Analytical model – macroscopical	Latches motions

Thermal battery



Criteria	Comments
Physic(s)	Thermal
Metamodel – microscopical	Phase change material
Analytical model – macroscopical	Heat exchanges

Thermo-mechanical system



Criteria	Comments
Physic(s)	Thermomechanical
Metamodel – microscopical	Material with volume power (thermal) Air cavity (thermal)
Analytical model – macroscopical	Heat transfer Movement of the bar