# Model Order Reduction and Bayesian Optimization for MDO problems

ines.cardoso@onera.fr

Inês CARDOSO[a,b] – Gaspard BERTHELIN[a,b] – Sylvain DUBREUIL[a] – Michel SALAÜN[b] – Nathalie BARTOLI[a] – Christian GOGU[b]

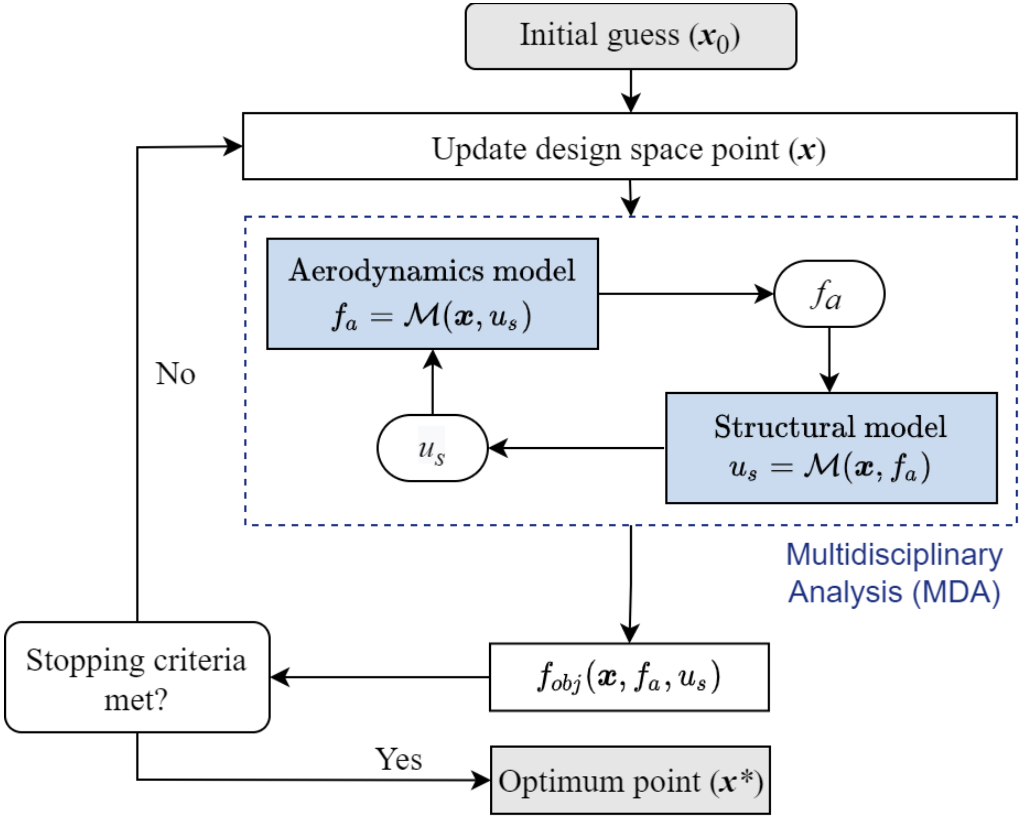
a. DTIS, ONERA, Université de Toulouse, Toulouse France

b. Université de Toulouse, CNRS, UPS, INSA, ISAE, Mines Albi, Institut Clément Ader (ICA), Toulouse, France

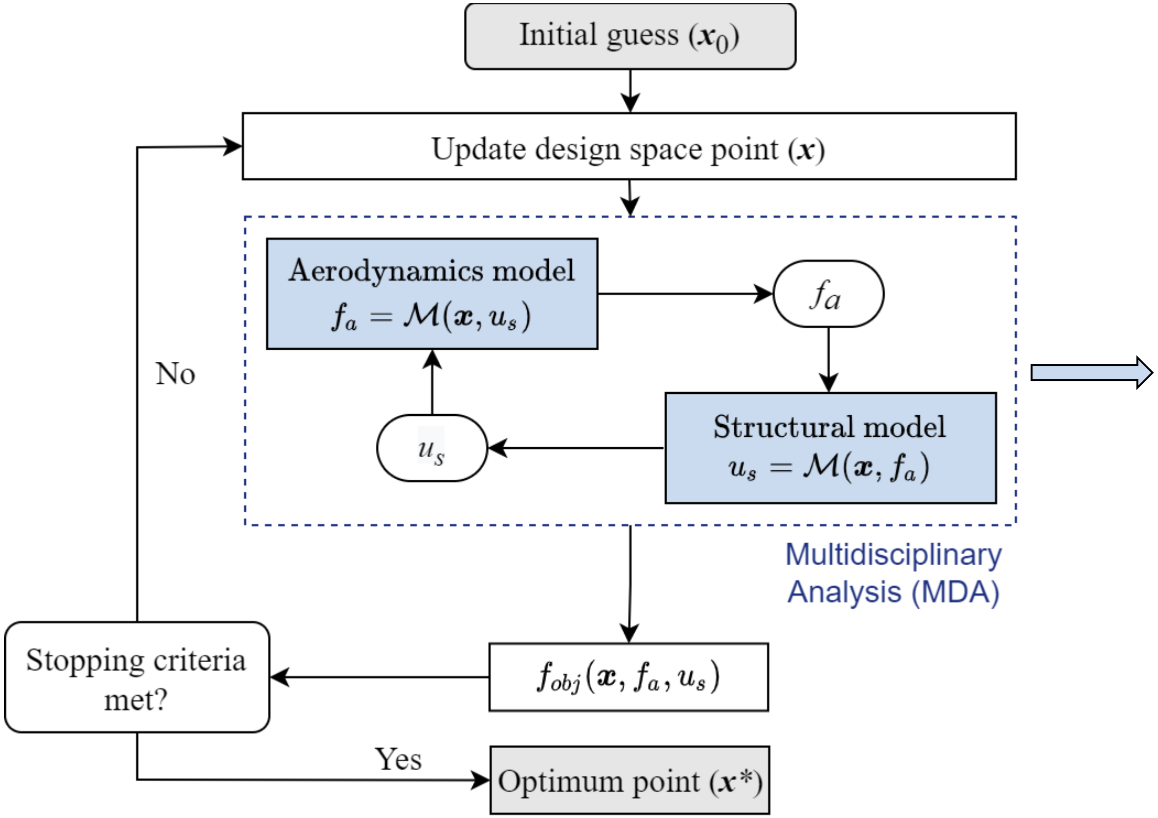
# Contents


I. Introduction
II. Application example
III. Reference framework
IV. Implemented frameworks
    I. DPOD+I & SLSQP
    II. DPOD+I & EGMDO
V. Conclusion

# Static aeroelastic optimization of an aircraft wing



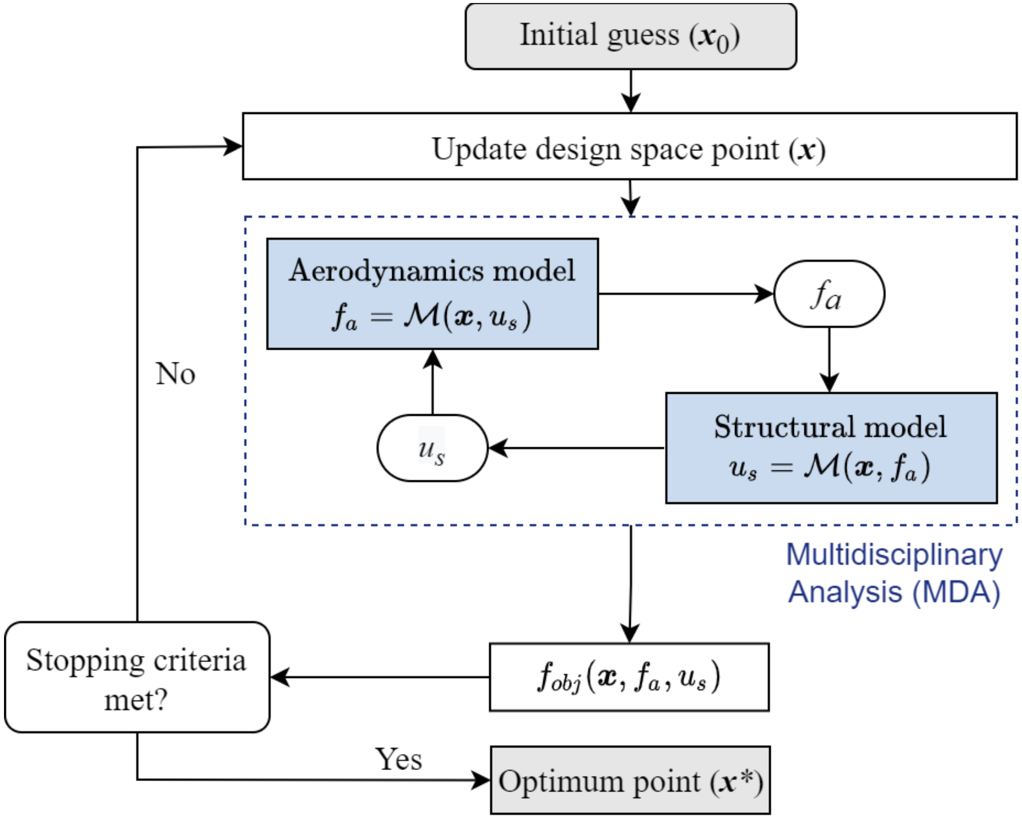
# Static aeroelastic optimization of an aircraft wing




The solution of the MDA is the **displacement field** ($u_s$) and the **vector of aerodynamic forces** ($f_a$).

Both disciplinary solvers depend on the **design variables** ($x$), handled by an **optimization algorithm**.

# Static aeroelastic optimization of an aircraft wing




The solution of the MDA is the **displacement field** ($u_s$) and the **vector of aerodynamic forces** ($f_a$).

Both disciplinary solvers depend on the **design variables** ($x$), handled by an **optimization algorithm**.

Assumptions:

- Partitioned approach: **non-intrusive coupling** between the disciplinary solvers.
- Multidisciplinary Feasible (MDF) approach: optimization problem and non-linear coupled problem **solved independently.**
- The coupling variables are **high dimensional** vectors.

# Problem statement

**Problem:**

- When using high fidelity solvers (e.g. FEM or CFD solvers), the **computational cost** may become intractable.

**Goal:**

- Reduce the computational cost, by **reducing the number of disciplinary solver calls** made.

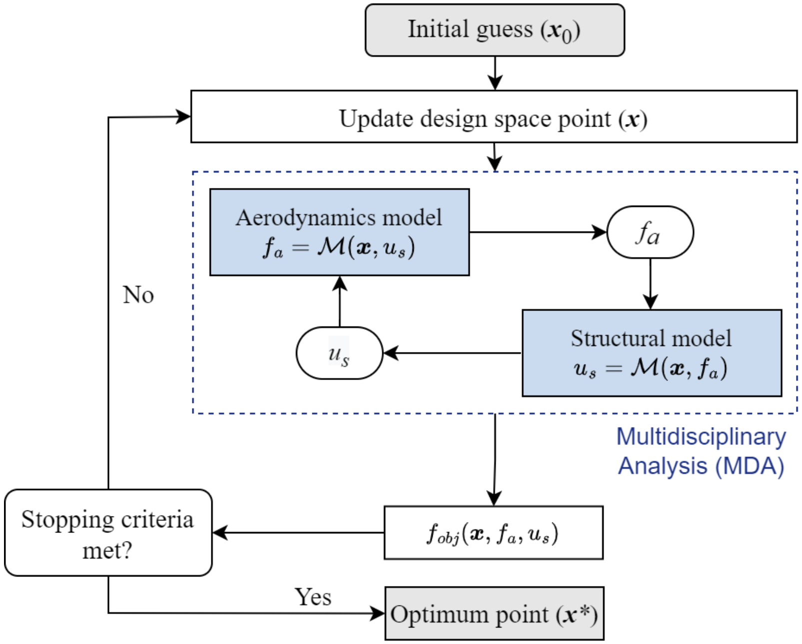
# Problem statement

**Problem:**

- When using high fidelity solvers (e.g. FEM or CFD solvers), the **computational cost** may become intractable.

**Goal:**

- Reduce the computational cost, by **reducing the number of disciplinary solver calls** made.



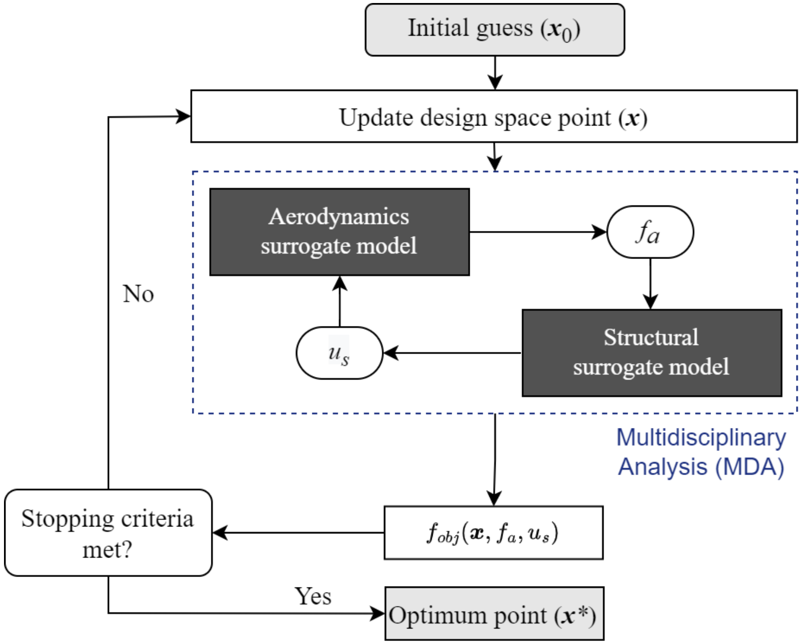
# Problem statement

**Problem:**

- When using high fidelity solvers (e.g. FEM or CFD solvers), the **computational cost** may become intractable.

**Goal:**

- Reduce the computational cost, by **reducing the number of disciplinary solver calls** made.



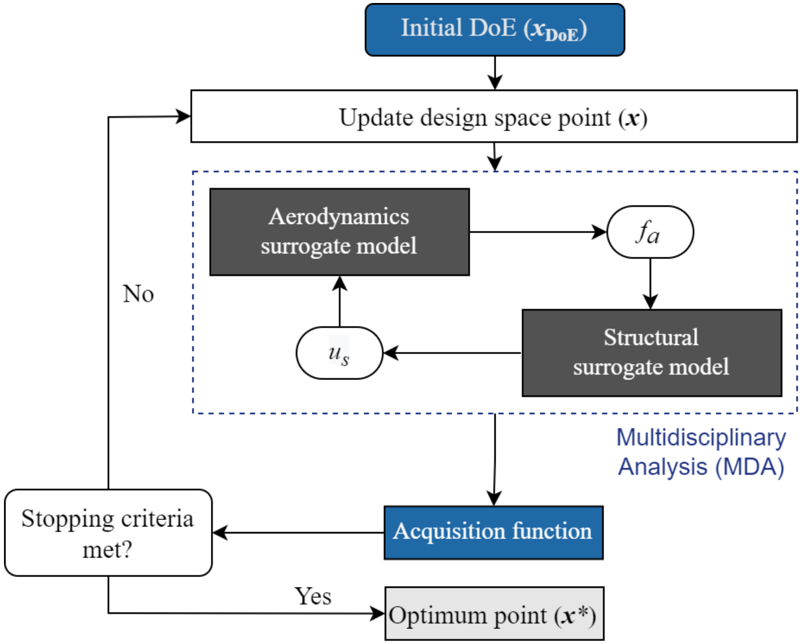
# Problem statement

**Problem:**

- When using high fidelity solvers (e.g. FEM or CFD solvers), the **computational cost** may become intractable.

**Goal:**

- Reduce the computational cost, by **reducing the number of disciplinary solver calls** made.

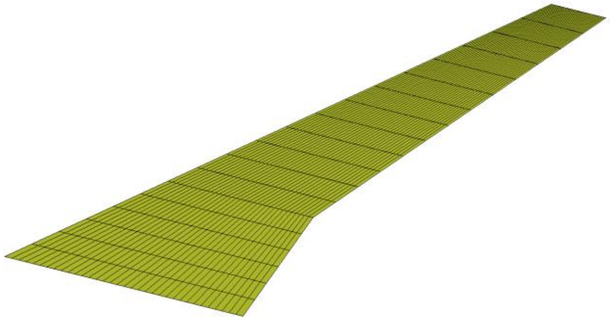


# Contents

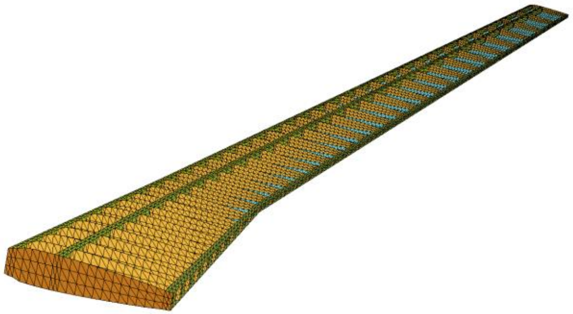

I. Introduction
II. Application example
III. Reference framework
IV. Implemented frameworks
    I. DPOD+I & SLSQP
    II. DPOD+I & EGMDO
V. Conclusion

# Aerodynamics and Structural solvers

- **Common Research Model (configuration uCRM-9)** [1]**:**



**Aerodynamics mesh** (VLM solver – 2100 degrees of freedom)

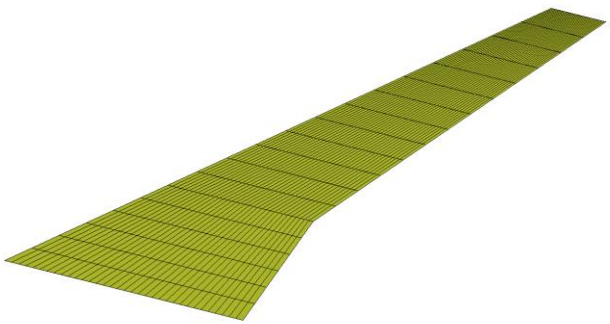


**Structural mesh** (FEM solver – 43416 degrees of freedom)

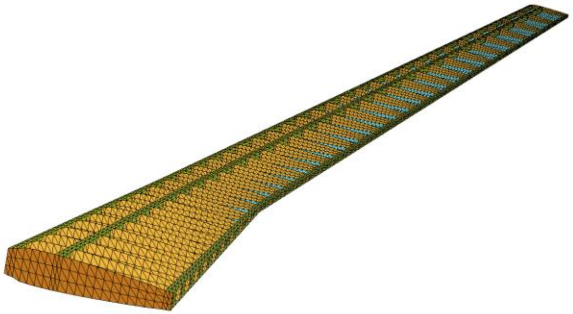
[1] T.R. Brooks, G.K. Kenway, and J.R.R.A. Martins. “Undeflected Common Research Model (uCRM): An Aerostructural Model for the Study of High Aspect Ratio Transport Aircraft Wings”. In: *35th AIAA Applied Aerodynamics Conference*. AIAA, 2017. doi: 10.2514/6.2017-4456.

# Aerodynamics and Structural solvers

- **Common Research Model (configuration uCRM-9)** [1]**:**



**Aerodynamics mesh** (VLM solver – 2100 degrees of freedom)



**Structural mesh** (FEM solver – 43416 degrees of freedom)

- Four considered **design variables** ($\boldsymbol{x} = \{\alpha, V_\infty, t_{sk}, t_{sp}\}$)**:**

| Variable | $\alpha[°]$ | $V_\infty[m/s]$ | $t_{sk}[m]$ | $t_{sp}[m]$ |
|---|---|---|---|---|
| Designation | Angle of attack | Air freestream velocity | Skin thickness | Spar thickness |
| Range of variation | $[1, 9]$ | $[220, 250]$ | $[0.003, 0.01]$ | $[0.01, 0.1]$ |

The design variables were scaled to take values in the range $[0,1]$.

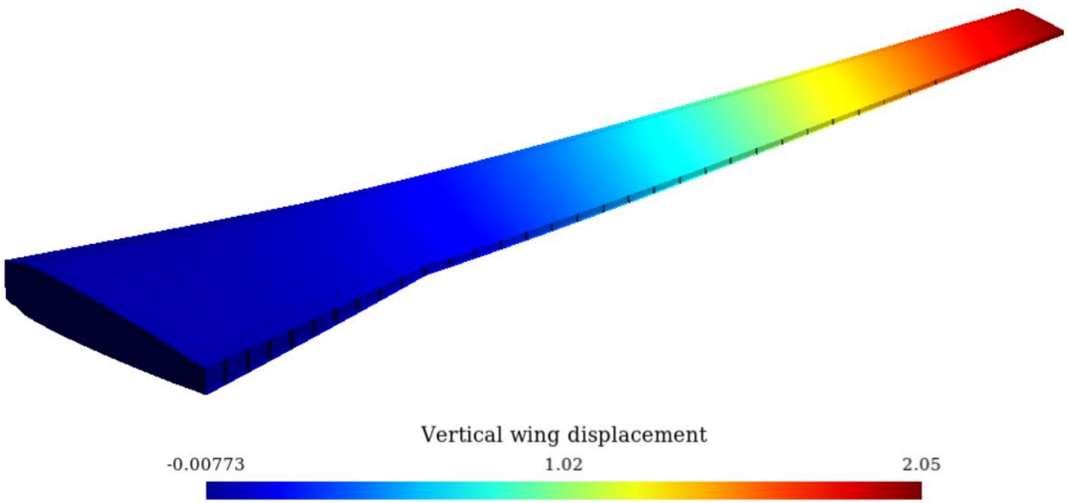
[1] T.R. Brooks, G.K. Kenway, and J.R.R.A. Martins. “Undeflected Common Research Model (uCRM): An Aerostructural Model for the Study of High Aspect Ratio Transport Aircraft Wings”. In: *35th AIAA Applied Aerodynamics Conference*. AIAA, 2017. doi: 10.2514/6.2017-4456.

# Objective function

- Objective function chosen as an **inverse problem:**

$$\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}\in\mathscr{X}} f_{obj}(\boldsymbol{x}) \quad \text{with} \quad f_{obj}(\boldsymbol{x}) = \frac{\|f_a(\boldsymbol{x}_{\text{ref}}) - f_a(\boldsymbol{x})\|_2}{\|f_a(\boldsymbol{x}_{\text{ref}})\|_2} + \frac{\|u_s(\boldsymbol{x}_{\text{ref}}) - u_s(\boldsymbol{x})\|_2}{\|u_s(\boldsymbol{x}_{\text{ref}})\|_2}$$

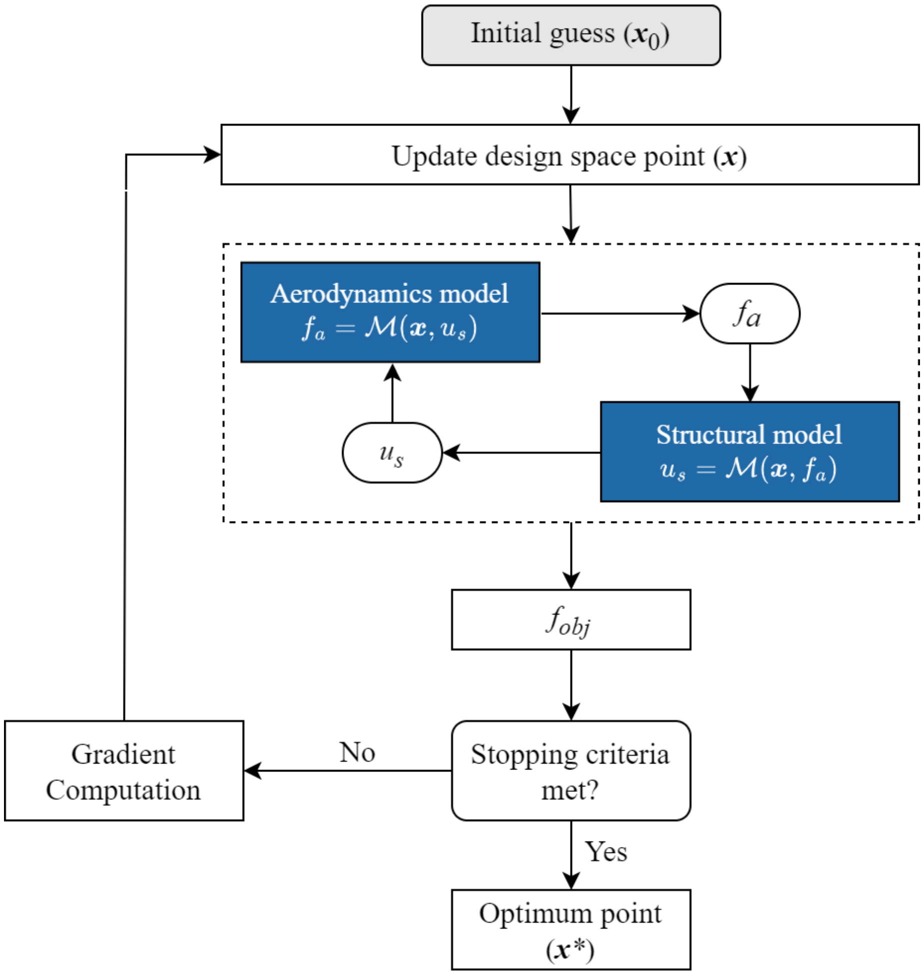
with $\boldsymbol{x}_{\text{ref}}$ the design space point that results in the **maximum wing tip displacement** $\boldsymbol{x}_{\text{ref}} = \{1,1,0,0\}$.



## Contents

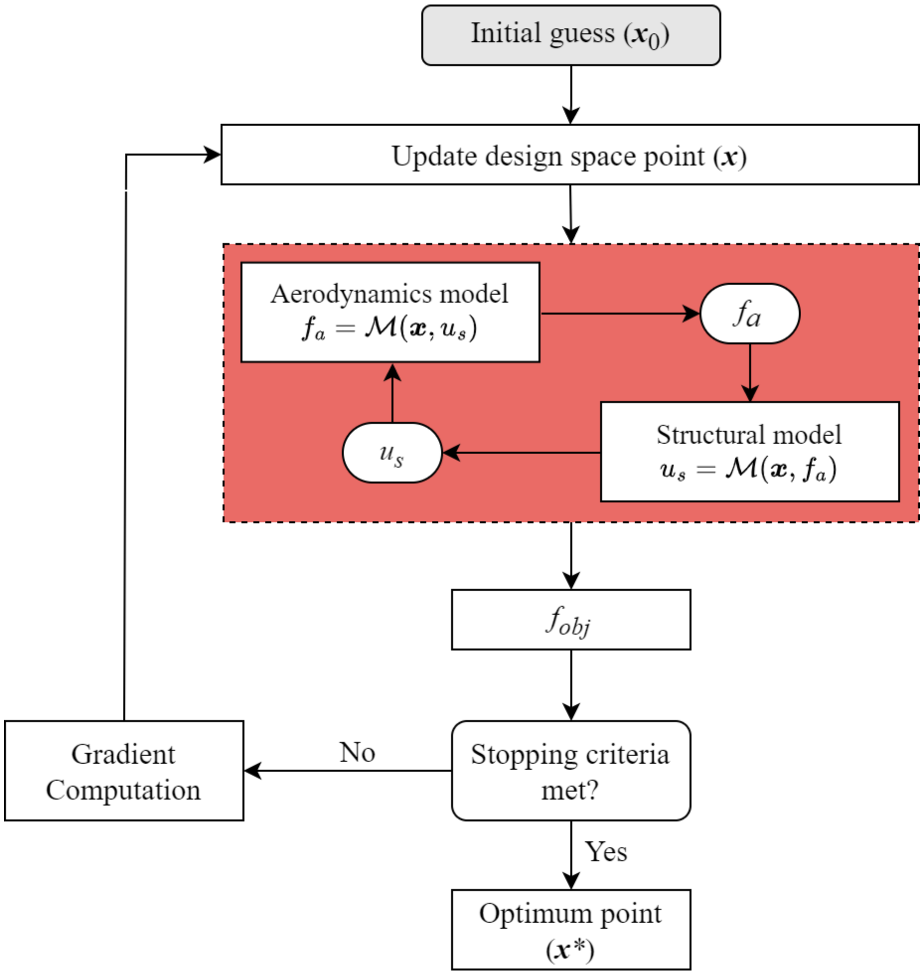

I. Introduction
II. Application example
III. Reference framework
IV. Implemented frameworks
  I. DPOD+I & SLSQP
  II. DPOD+I & EGMDO
V. Conclusion

# Reference optimization framework



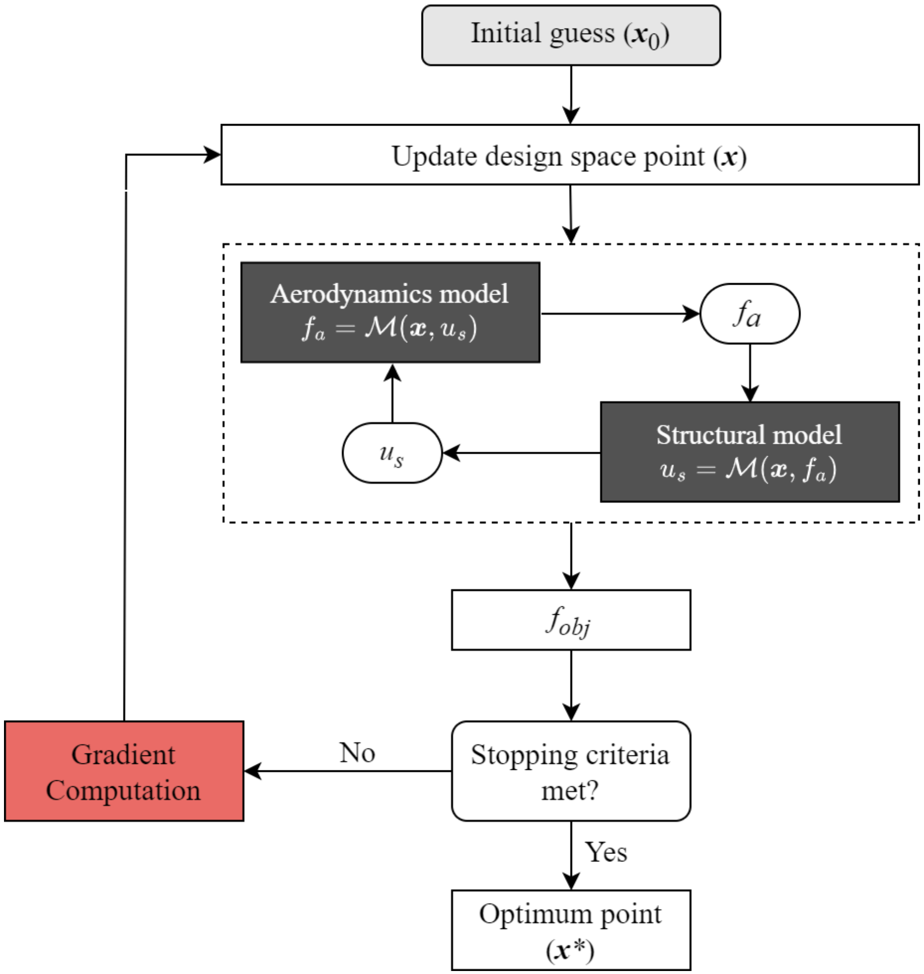

- The real solvers are used to model the disciplines.

# Reference optimization framework



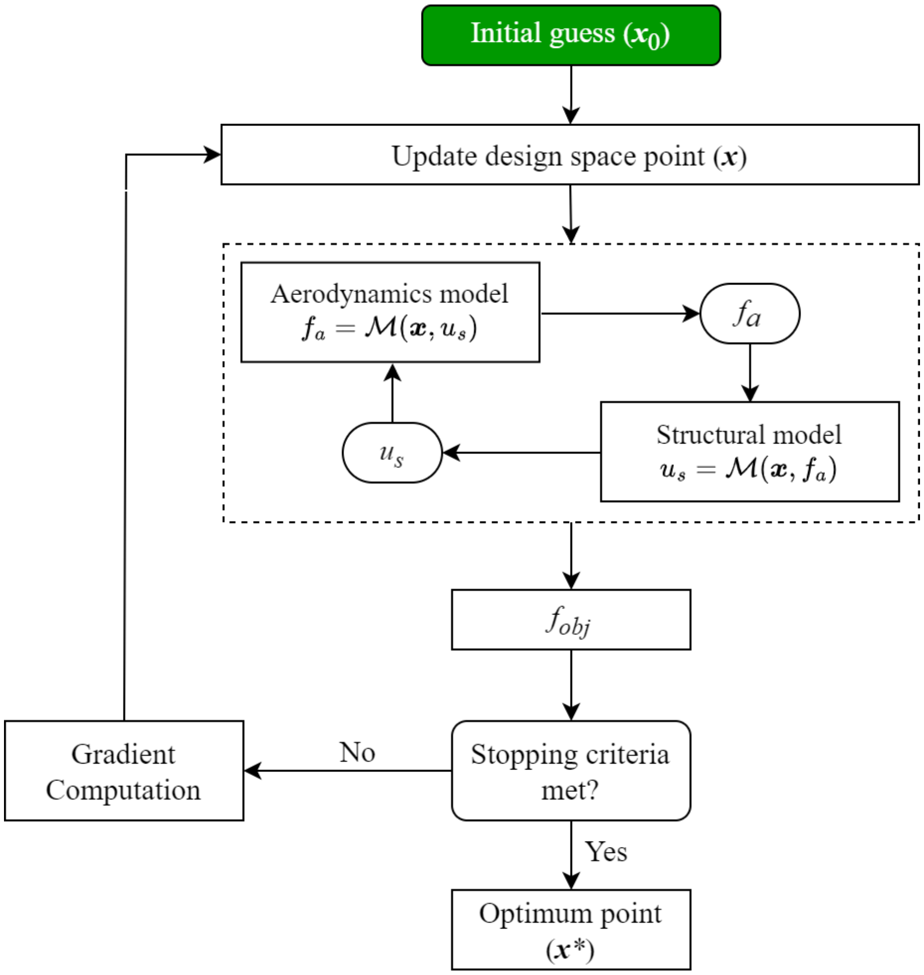

- The real solvers are used to model the disciplines.
- Due to the dimension of the coupling variables, the MDF approach was used.
  $\Rightarrow$ **MDA is solved at every iteration.**

# Reference optimization framework



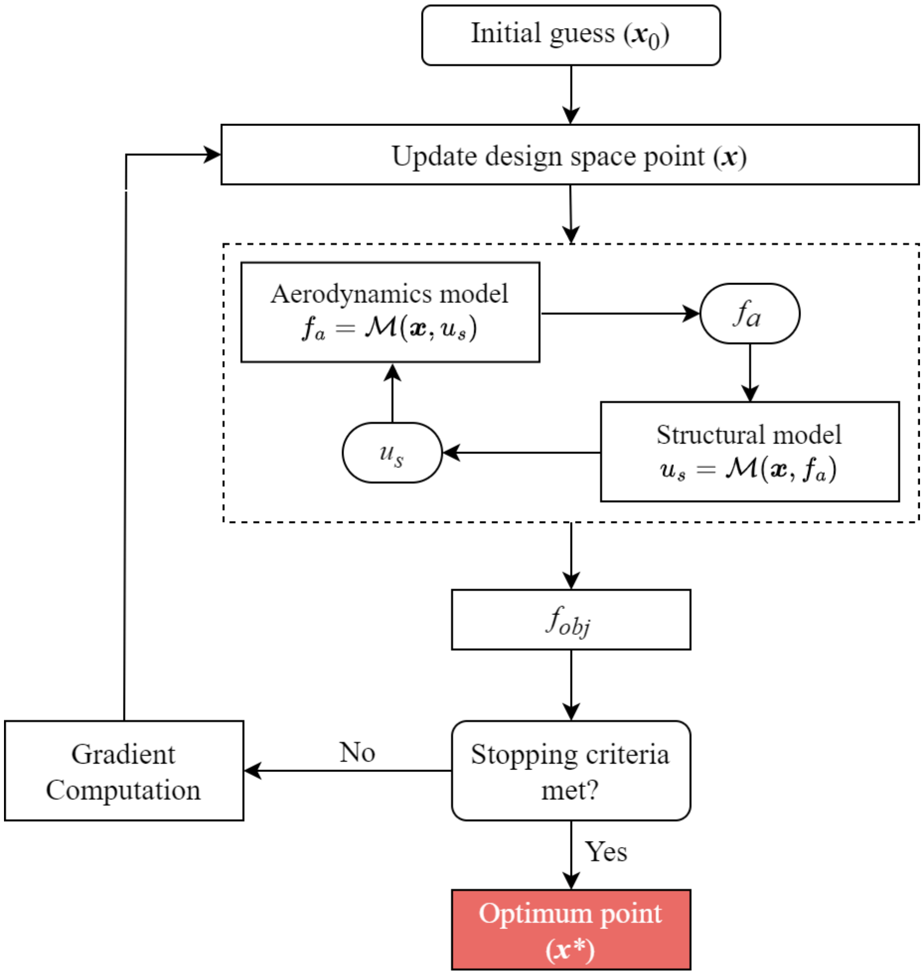

- The real solvers are used to model the disciplines.
- Due to the dimension of the coupling variables, the MDF approach was used. $\Rightarrow$ **MDA is solved at every iteration.**
- The MDA is solved via non-linear block Gauss-Seidel using Aitken acceleration.
- For the MDO, the gradient-based SLSQP solver is used. **Gradient calculation is made via finite differences.**

# Reference optimization framework




- The real solvers are used to model the disciplines.
- Due to the dimension of the coupling variables, the MDF approach was used. $\Rightarrow$ **MDA is solved at every iteration.**
- The MDA is solved via non-linear block Gauss-Seidel using Aitken acceleration.
- For the MDO, the gradient-based SLSQP solver is used. **Gradient calculation is made via finite differences.**
- Starting point chosen at the center of the design space $x_0 = \{0.5, 0.5, 0.5, 0.5\}$.

# Reference optimization framework



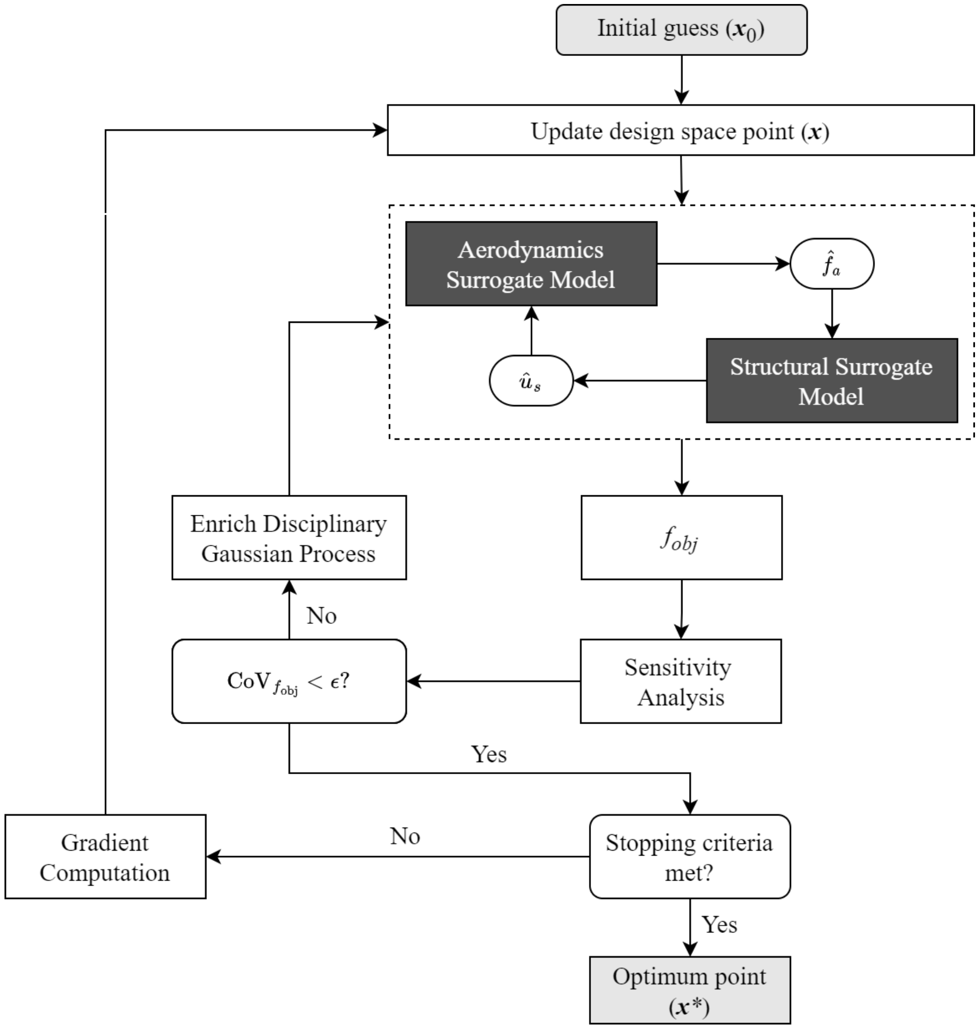

- The real solvers are used to model the disciplines.
- Due to the dimension of the coupling variables, the MDF approach was used. $\Rightarrow$ **MDA is solved at every iteration.**
- The MDA is solved via non-linear block Gauss-Seidel using Aitken acceleration.
- For the MDO, the gradient-based SLSQP solver is used. **Gradient calculation is made via finite differences.**
- Starting point chosen at the center of the design space $x_0 = \{0.5, 0.5, 0.5, 0.5\}$.

$\Rightarrow$ The algorithm needed 17 iterations and **286 calls** to each disciplinary solver in order to find the reference point.

# Contents


I. Introduction
II. Application example
III. Reference framework
IV. Implemented frameworks
    I. DPOD+I & SLSQP
    II. DPOD+I & EGMDO
V. Conclusion

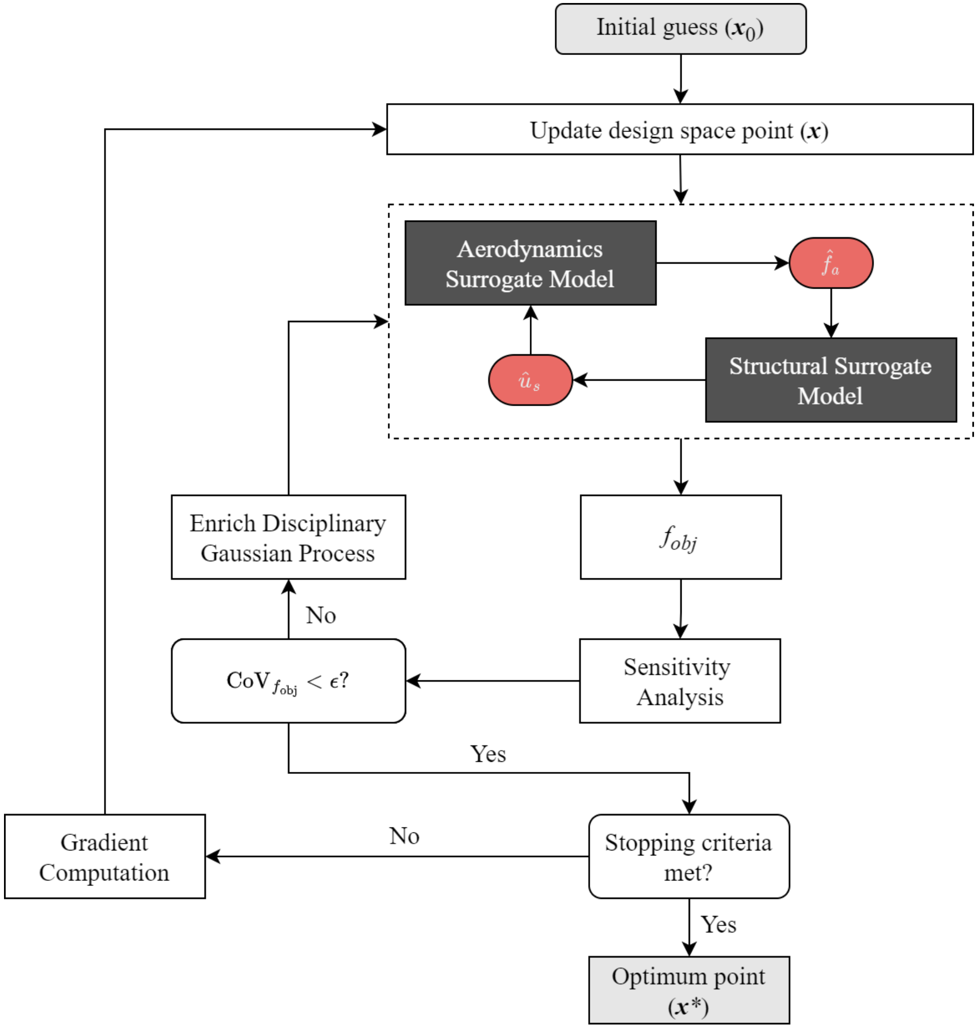
# DPOD+I & SLSQP framework




- Replacement of the disciplinary solvers by **Disciplinary Proper Orthogonal Decomposition + Interpolation (DPOD+I) surrogate models [2]**.

[2] G. Berthelin, S. Dubreuil, M. Salaün, N. Bartoli, and C. Gogu. "Disciplinary Proper Orthogonal Decomposition and Interpolation for the resolution of parametrized Multidisciplinary Analysis". In: International Journal for Numerical Methods in Engineering (2022).

# DPOD+I & SLSQP framework




- Replacement of the disciplinary solvers by **Disciplinary Proper Orthogonal Decomposition + Interpolation (DPOD+I) surrogate models [2]**.

  $\Rightarrow$ Model order reduction by **Disciplinary Proper Orthogonal Decomposition (DPOD):**

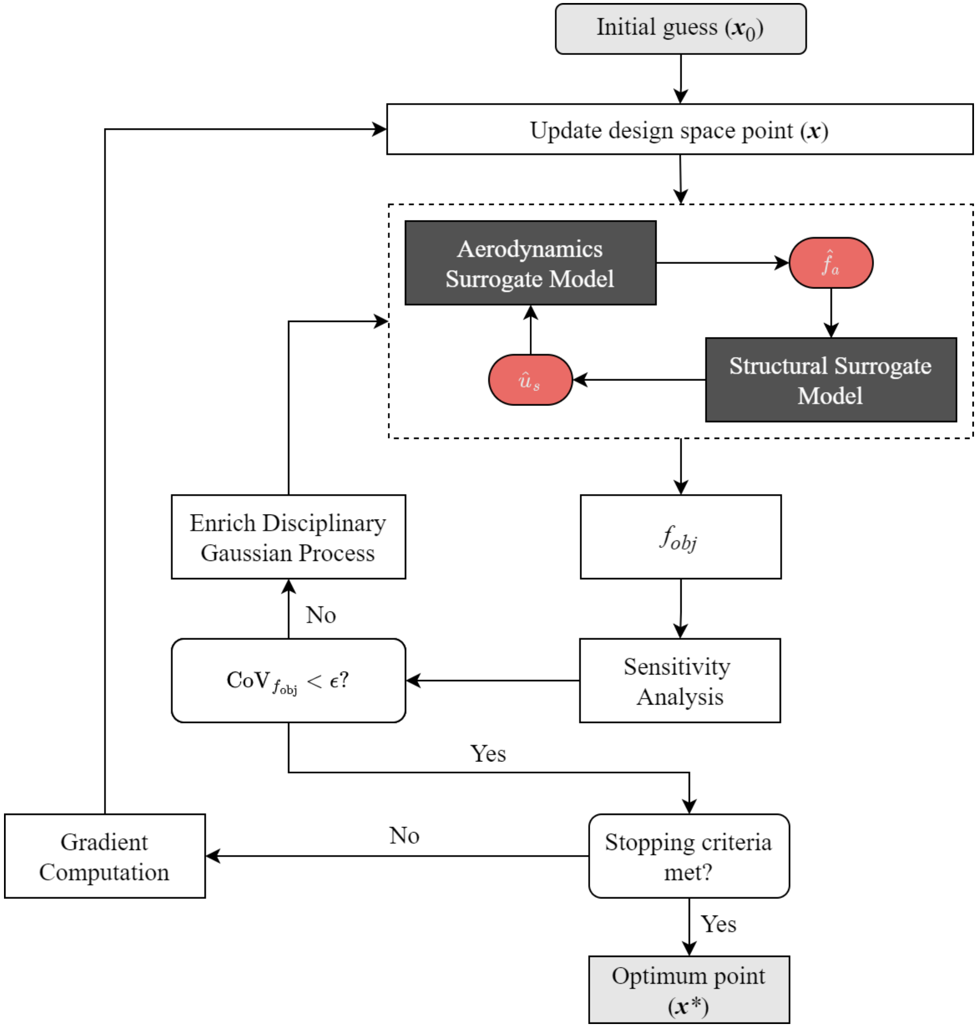
$$\hat{f}_a \approx \phi_0^a + \sum_{i=1}^{n_a} \alpha_i^a(\boldsymbol{x}, u_s)\phi_i^a$$

$$\hat{u}_s \approx \phi_0^s + \sum_{i=1}^{n_s} \alpha_i^s(\boldsymbol{x}, f_a)\phi_i^s$$

where $\phi_0$ is a constant vector, $\phi_i$ are the POD basis vectors, $\alpha_i$ are the POD coefficients, and $n_a$ and $n_s$ give the number of terms retained in the POD approximations.

[2] G. Berthelin, S. Dubreuil, M. Salaün, N. Bartoli, and C. Gogu. “Disciplinary Proper Orthogonal Decomposition and Interpolation for the resolution of parametrized Multidisciplinary Analysis”. In: International Journal for Numerical Methods in Engineering (2022).

# DPOD+I & SLSQP framework




- Replacement of the disciplinary solvers by **Disciplinary Proper Orthogonal Decomposition + Interpolation (DPOD+I) surrogate models [2]**.

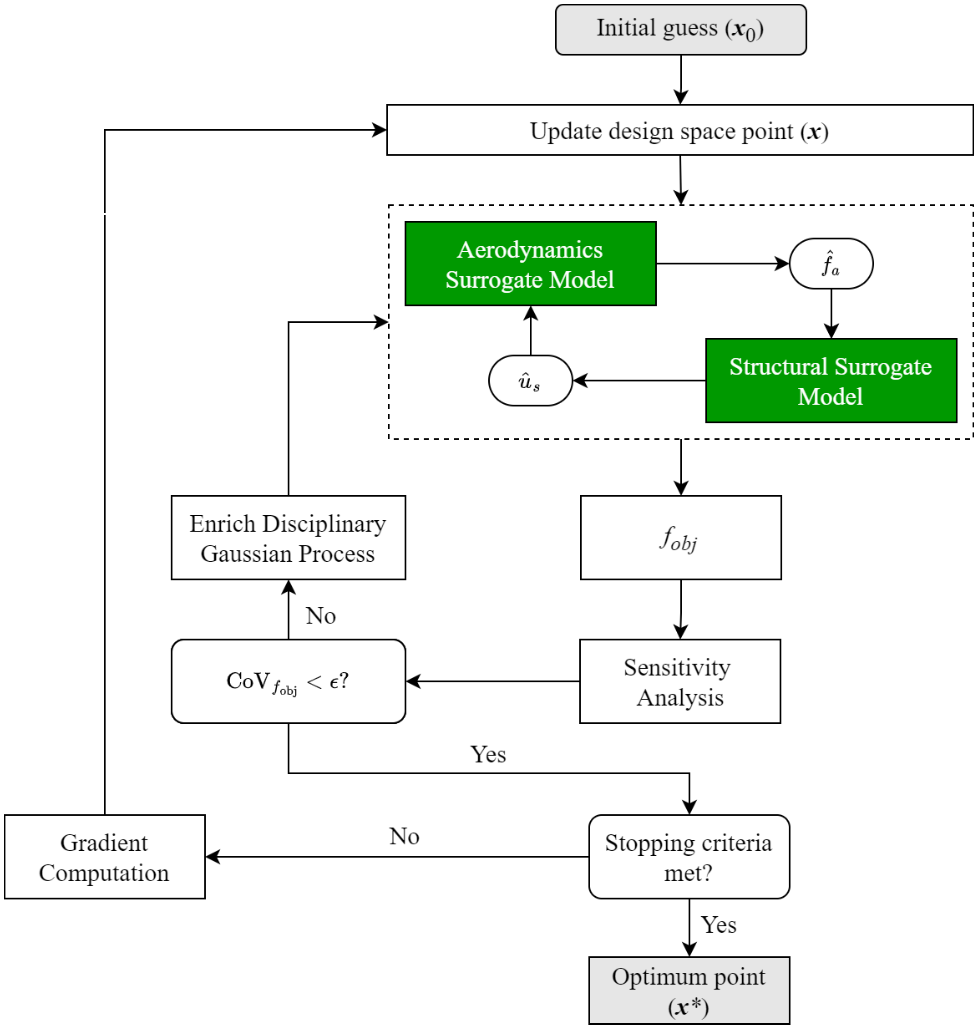
  $\Rightarrow$ Model order reduction by **Disciplinary Proper Orthogonal Decomposition (DPOD)** followed by the interpolation of each coefficient by **Gaussian Processes (GP)**:

$$\hat{\alpha}_i^a \sim \mathrm{GP}|_{\mathrm{DoE}_a}(\mu|_{\mathrm{DoE}_a}, k|_{\mathrm{DoE}_a})$$
$$\hat{\alpha}^s \sim \mathrm{GP}|_{\mathrm{DoE}_s}(\mu|_{\mathrm{DoE}_s}, k|_{\mathrm{DoE}_s})$$

where the GP approximation of the coefficients is denoted by $\hat{\alpha}_i$ and is characterized by a mean value $\mu$ and a covariance kernel $k$.

[2] G. Berthelin, S. Dubreuil, M. Salaün, N. Bartoli, and C. Gogu. "Disciplinary Proper Orthogonal Decomposition and Interpolation for the resolution of parametrized Multidisciplinary Analysis". In: International Journal for Numerical Methods in Engineering (2022).

# DPOD+I & SLSQP framework

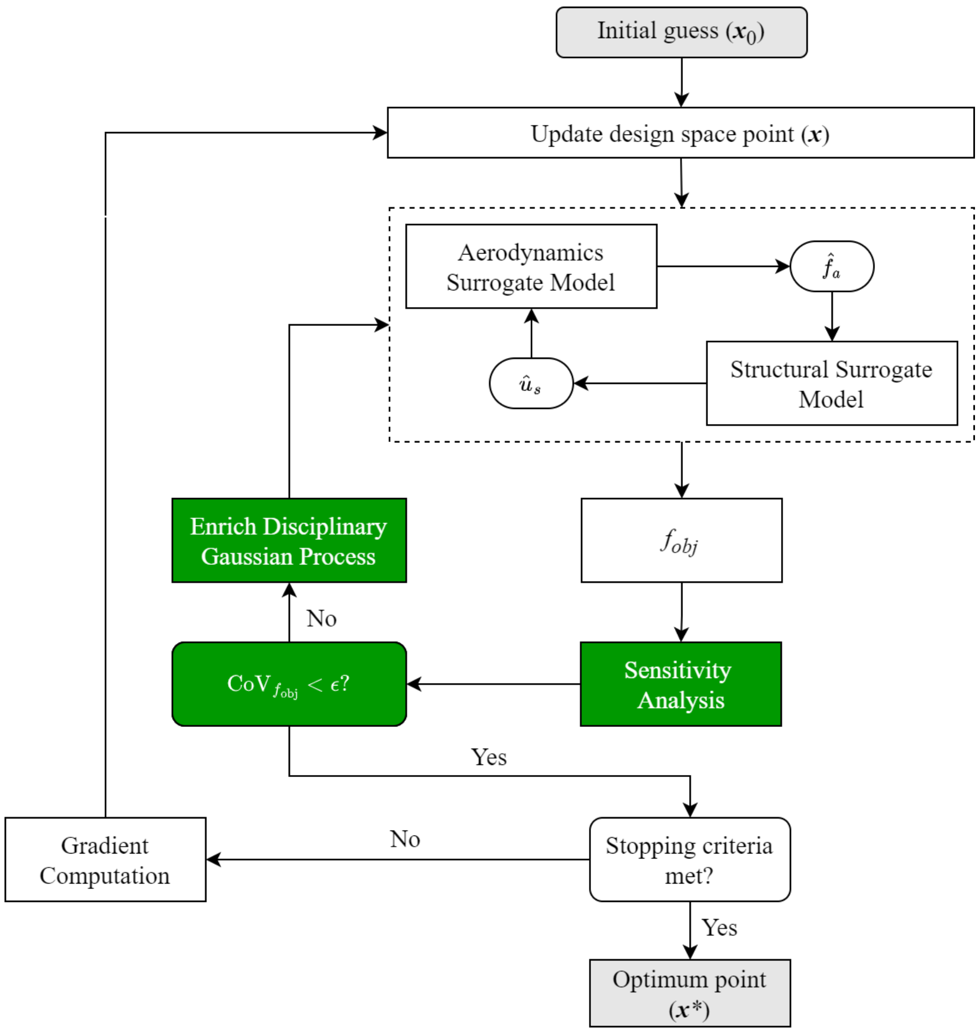



- Replacement of the disciplinary solvers by **Disciplinary Proper Orthogonal Decomposition + Interpolation (DPOD+I) surrogate models [2]**.

✓ The disciplinary surrogates are trained **independently** from different Designs of Experiments (DoE).

[2] G. Berthelin, S. Dubreuil, M. Salaün, N. Bartoli, and C. Gogu. "Disciplinary Proper Orthogonal Decomposition and Interpolation for the resolution of parametrized Multidisciplinary Analysis". In: International Journal for Numerical Methods in Engineering (2022).

# DPOD+I & SLSQP framework



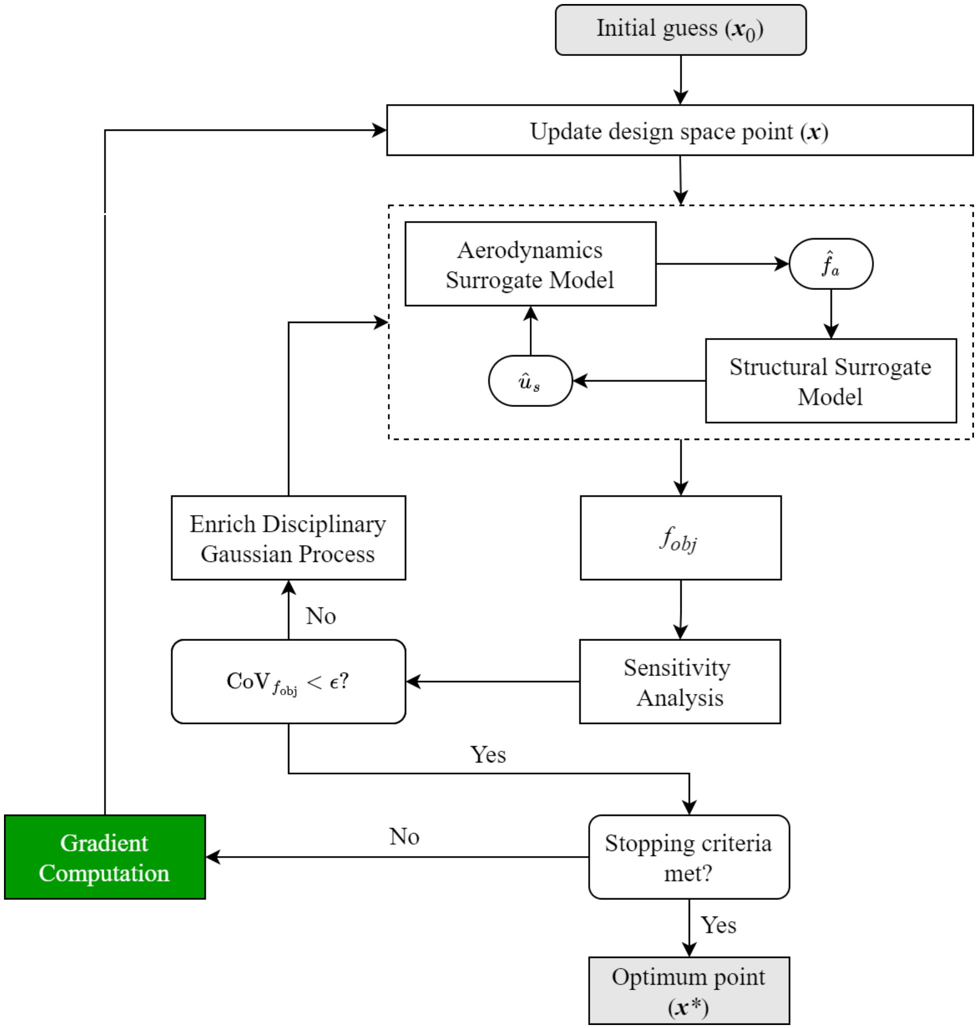

- Replacement of the disciplinary solvers by **Disciplinary Proper Orthogonal Decomposition + Interpolation (DPOD+I) surrogate models [2]**.

✓ The disciplinary surrogates are trained **independently** from different Designs of Experiments (DoE).

✓ It is possible to **enrich the disciplinary surrogates** throughout the MDA resolution until a given accuracy is achieved.

[2] G. Berthelin, S. Dubreuil, M. Salaün, N. Bartoli, and C. Gogu. "Disciplinary Proper Orthogonal Decomposition and Interpolation for the resolution of parametrized Multidisciplinary Analysis". In: International Journal for Numerical Methods in Engineering (2022).

# DPOD+I & SLSQP framework




- Replacement of the disciplinary solvers by **Disciplinary Proper Orthogonal Decomposition + Interpolation (DPOD+I) surrogate models [2]**.

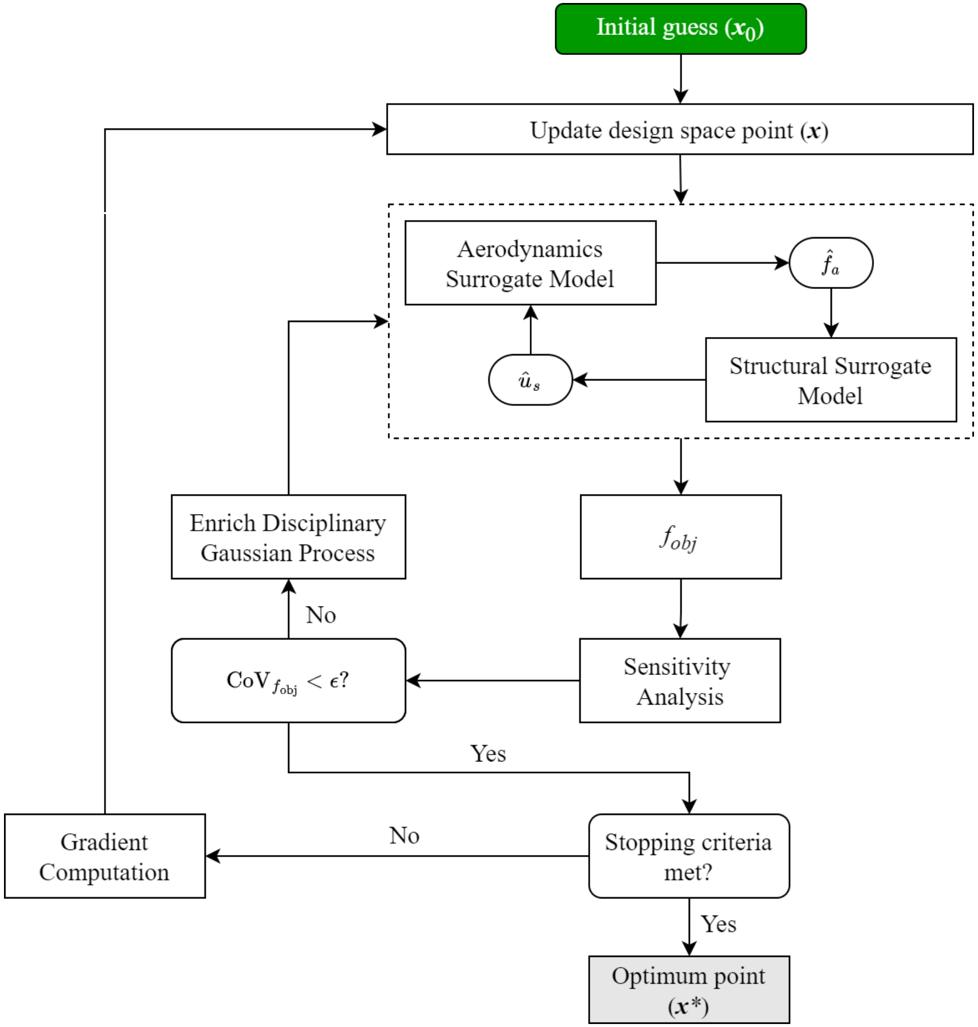
✓ The disciplinary surrogates are trained **independently** from different Designs of Experiments (DoE).

✓ It is possible to **enrich the disciplinary surrogates** throughout the MDA resolution until a given accuracy is achieved.

- The MDO is solved by a gradient-based optimizer (SLSQP). **Gradient computation uses the Gaussian Process derivatives.**

[2] G. Berthelin, S. Dubreuil, M. Salaün, N. Bartoli, and C. Gogu. "Disciplinary Proper Orthogonal Decomposition and Interpolation for the resolution of parametrized Multidisciplinary Analysis". In: International Journal for Numerical Methods in Engineering (2022).

# DPOD+I & SLSQP framework




- Replacement of the disciplinary solvers by **Disciplinary Proper Orthogonal Decomposition + Interpolation (DPOD+I) surrogate models [2]**.
- ✓ The disciplinary surrogates are trained **independently** from different Designs of Experiments (DoE).
- ✓ It is possible to **enrich the disciplinary surrogates** throughout the MDA resolution until a given accuracy is achieved.
- The MDO is solved by a gradient-based optimizer (SLSQP). **Gradient computation uses the Gaussian Process derivatives.**
- Starting point chosen at the center of the design space $\boldsymbol{x_0} = \{0.5, 0.5, 0.5, 0.5\}$.

[2] G. Berthelin, S. Dubreuil, M. Salaün, N. Bartoli, and C. Gogu. "Disciplinary Proper Orthogonal Decomposition and Interpolation for the resolution of parametrized Multidisciplinary Analysis". In: International Journal for Numerical Methods in Engineering (2022).

# DPOD+I & SLSQP results

- The GP surrogates are built upon random initial disciplinary DoEs $\Rightarrow$ **10 runs are performed.**

# DPOD+I & SLSQP results

- The GP surrogates are built upon random initial disciplinary DoEs $\Rightarrow$ **10 runs are performed.**
- Structural POD basis composed of an average of 6 coefficients and aerodynamics POD basis composed of an average of 5 coefficients.

$\Rightarrow$ **Initial DoE:** average of 42 points for the structural discipline and 52 for the aerodynamics discipline.

# DPOD+I & SLSQP results

- The GP surrogates are built upon random initial disciplinary DoEs $\Rightarrow$ **10 runs are performed.**
- Structural POD basis composed of an average of 6 coefficients and aerodynamics POD basis composed of an average of 5 coefficients.

$\Rightarrow$ **Initial DoE:** average of 42 points for the structural discipline and 52 for the aerodynamics discipline.

**Comparison between DPOD+I & SLSQP framework and reference framework**

| | | $\alpha^*$ | $V_\infty^*$ | $t_{sk}^*$ | $t_{sp}^*$ | $f(\boldsymbol{x}^*)$ | $n^a$ | $n^s$ |
|---|---|---|---|---|---|---|---|---|
| Reference | $\mathbb{E}$ | 1.0 | 1.0 | 0.0 | 0.0 | 0.0 | 286 | 286 |
| | CoV | — | — | — | — | — | — | — |
| DPOD+I & SLSQP | $\mathbb{E}$ | 1.0 | 1.0 | $3.4 \times 10^{-4}$ | $3.1 \times 10^{-4}$ | 0.0585 | 60.7 | 51.5 |
| | CoV | $\leq 10^{-12}$ | $\leq 10^{-12}$ | 3.0 | 2.6346 | 0.1523 | 0.1904 | 0.1784 |

where $n^a$ and $n^s$ are, respectively, the number of aerodynamics solver calls and the number of structural solver calls.

# DPOD+I & SLSQP results

- The GP surrogates are built upon random initial disciplinary DoEs ⇒ **10 runs are performed.**
- Structural POD basis composed of an average of 6 coefficients and aerodynamics POD basis composed of an average of 5 coefficients.

⇒ **Initial DoE:** average of 42 points for the structural discipline and 52 for the aerodynamics discipline.

**Comparison between DPOD+I & SLSQP framework and reference framework**

| | | $\alpha^*$ | $V_\infty^*$ | $t_{sk}^*$ | $t_{sp}^*$ | $f(\boldsymbol{x}^*)$ | $n^a$ | $n^s$ |
|---|---|---|---|---|---|---|---|---|
| Reference | $\mathbb{E}$ | 1.0 | 1.0 | 0.0 | 0.0 | 0.0 | 286 | 286 |
| | CoV | – | – | – | – | – | – | – |
| DPOD+I & SLSQP | $\mathbb{E}$ | 1.0 | 1.0 | $3.4 \times 10^{-4}$ | $3.1 \times 10^{-4}$ | 0.0585 | 60.7 | 51.5 |
| | CoV | $\leq 10^{-12}$ | $\leq 10^{-12}$ | 3.0 | 2.6346 | 0.1523 | 0.1904 | 0.1784 |

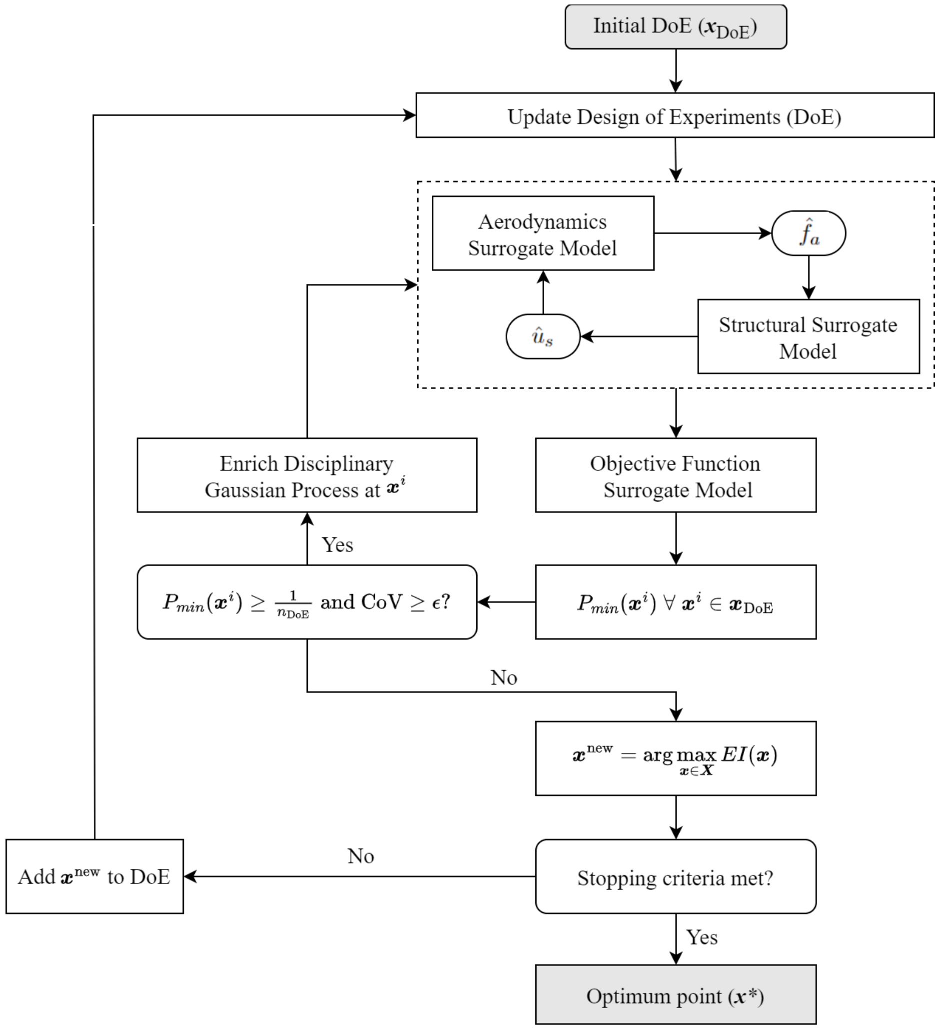
where $n^a$ and $n^s$ are, respectively, the number of aerodynamics solver calls and the number of structural solver calls.

- **Reduction by a factor of 5** in the number of necessary disciplinary solver calls.
- An average of **only 10 calls** was made to each disciplinary solver during the optimization process.

# Contents

I. Introduction
II. Application example
III. Reference framework
IV. Implemented frameworks
  I. DPOD+I & SLSQP
  II. DPOD+I & EGMDO
V. Conclusion

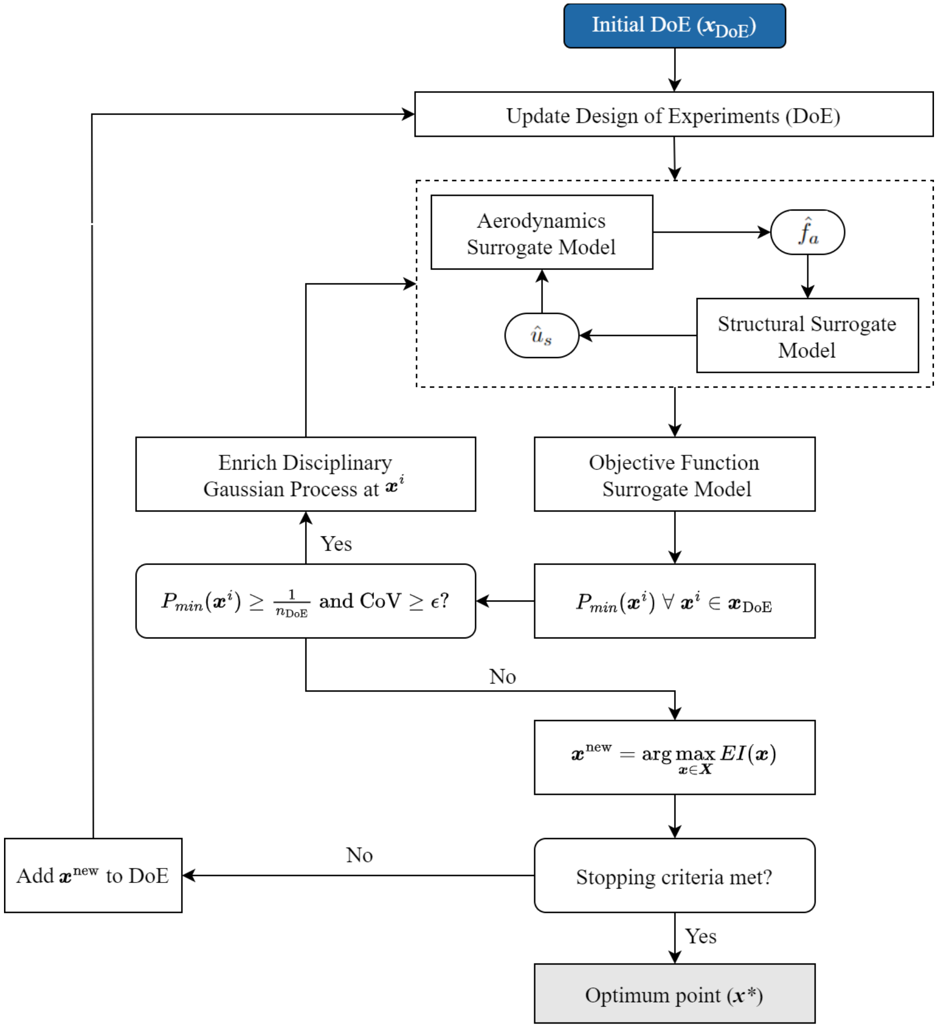
# DPOD+I & EGMDO framework




- The MDA remains the same as in the previous framework.
- Replacement of the optimizer by the **EGMDO (Efficient Global Multidisciplinary Optimization)** algorithm [3].

[3] S. Dubreuil, N. Bartoli, C. Gogu, and T. Lefebvre. "Towards an efficient global multidisciplinary design optimization algorithm". In: Structural and Multidisciplinary Optimization 62 (2020), pp. 1–27.

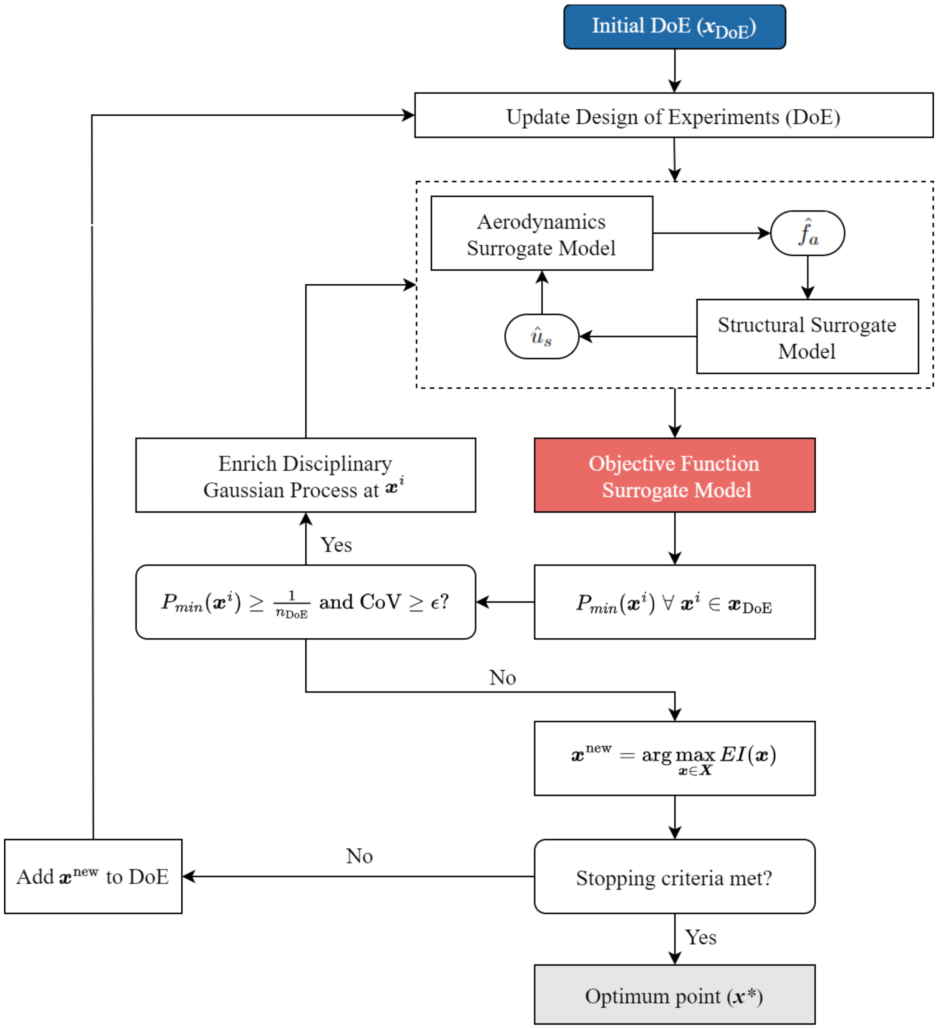
# DPOD+I & EGMDO framework




- The MDA remains the same as in the previous framework.
- Replacement of the optimizer by a global optimization algorithm: **EGMDO** [3].

[3] S. Dubreuil, N. Bartoli, C. Gogu, and T. Lefebvre. "Towards an efficient global multidisciplinary design optimization algorithm". In: Structural and Multidisciplinary Optimization 62 (2020), pp. 1–27.

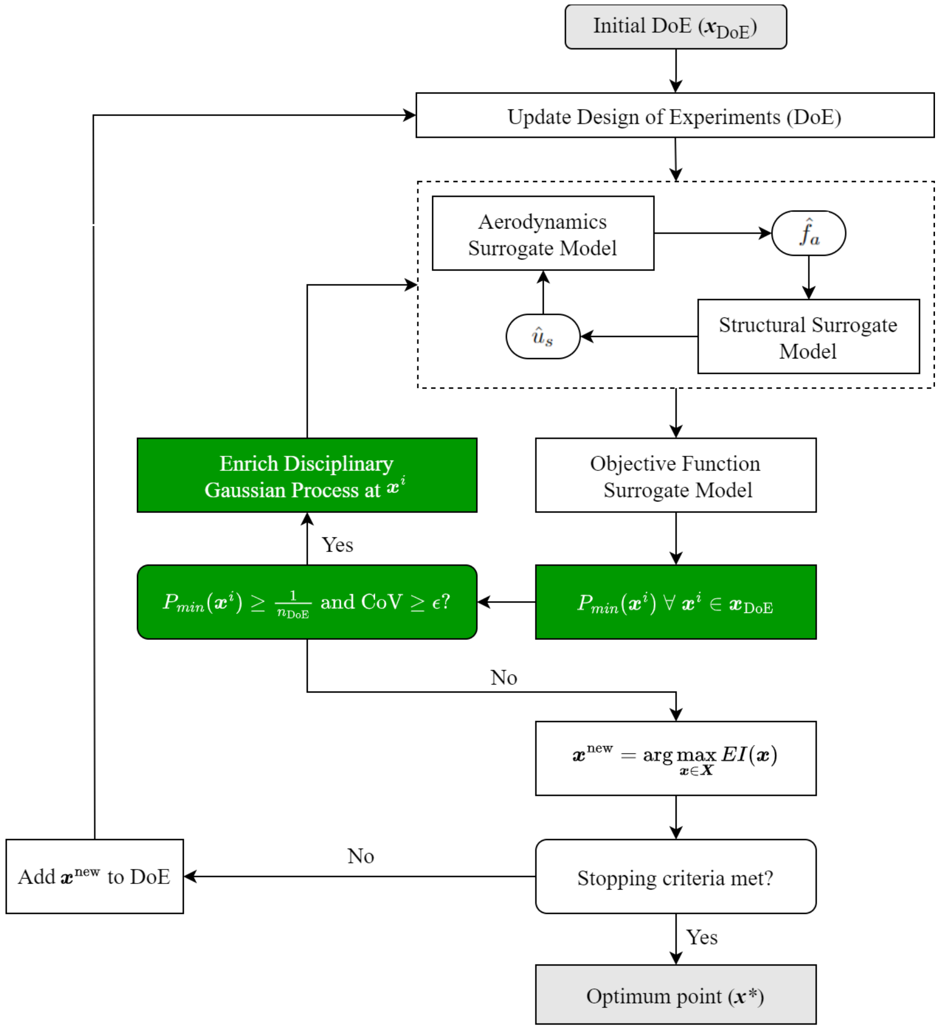
# DPOD+I & EGMDO framework




- The MDA remains the same as in the previous framework.
- Replacement of the optimizer by a global optimization algorithm: **EGMDO** [3].
- Non-linear MDA solved using disciplinary GPs leads to a **non-Gaussian objective function.**

[3] S. Dubreuil, N. Bartoli, C. Gogu, and T. Lefebvre. “Towards an efficient global multidisciplinary design optimization algorithm”. In: Structural and Multidisciplinary Optimization 62 (2020), pp. 1–27.

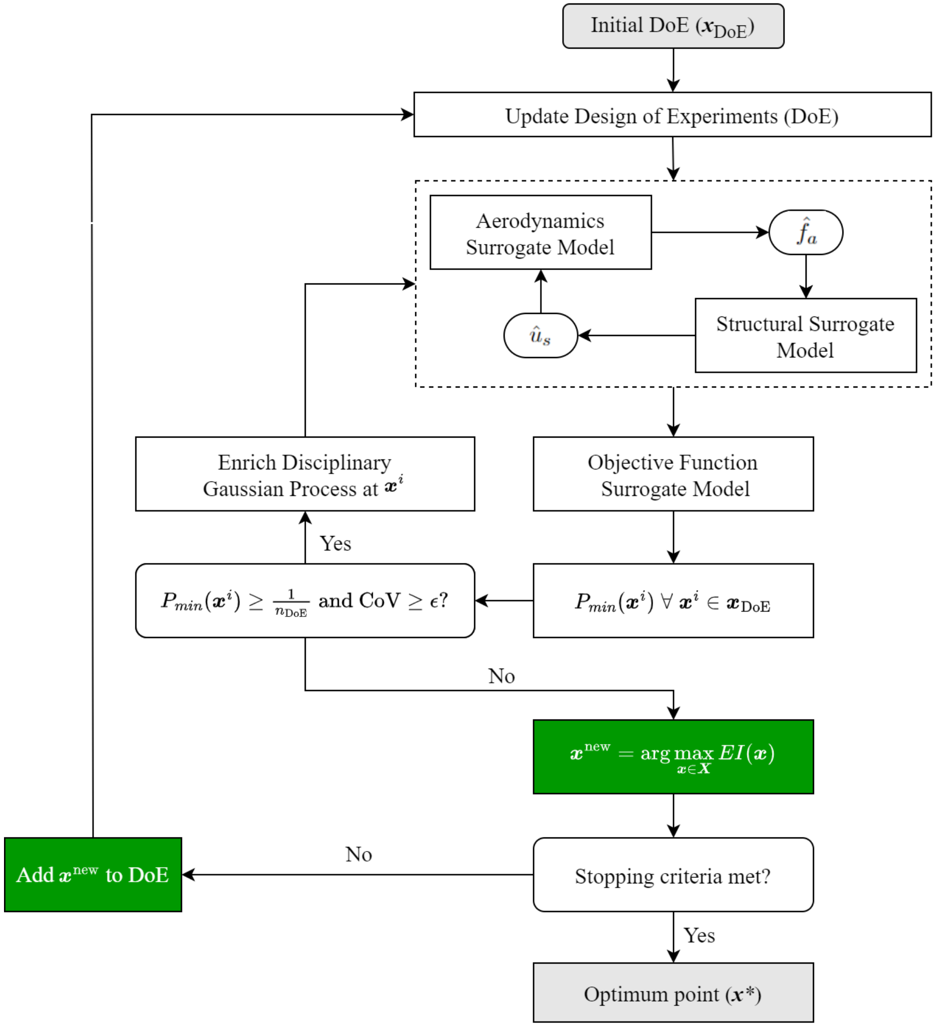
# DPOD+I & EGMDO framework




- The MDA remains the same as in the previous framework.
- Replacement of the optimizer by a global optimization algorithm: **EGMDO** [3].
- Non-linear MDA solved using disciplinary GPs leads to a **non-Gaussian objective function.**
- It remains possible to enrich the disciplinary surrogate models at DoE points.
- ✓ The point must have some **likelihood of being the minimum** to be enriched.

[3] S. Dubreuil, N. Bartoli, C. Gogu, and T. Lefebvre. "Towards an efficient global multidisciplinary design optimization algorithm". In: Structural and Multidisciplinary Optimization 62 (2020), pp. 1–27.

# DPOD+I & EGMDO framework




- The MDA remains the same as in the previous framework.
- Replacement of the optimizer by a global optimization algorithm: **EGMDO** [3].
- Non-linear MDA solved using disciplinary GPs leads to a **non-Gaussian objective function.**
- It remains possible to enrich the disciplinary surrogate models at DoE points.

✓ The point must have some **likelihood of being the minimum** to be enriched.

- Possible to add new points to the DoE.

✓ A modified **Expected Improvement (EI)** criterion is used.

$\implies$ Due to the non-Gaussian nature of $f_{obj}$ the **EI is estimated via Monte Carlo Simulation.**

[3] S. Dubreuil, N. Bartoli, C. Gogu, and T. Lefebvre. "Towards an efficient global multidisciplinary design optimization algorithm". In: Structural and Multidisciplinary Optimization 62 (2020), pp. 1–27.

# DPOD+I & EGMDO results

- Random initial disciplinary DoEs and random initial optimizer DoE $\Rightarrow$ **10 runs are performed.**
- **Initial DoE:** average of 49 points for the structural discipline and 59 for the aerodynamics discipline.

# DPOD+I & EGMDO results

- Random initial disciplinary DoEs and random initial optimizer DoE $\Rightarrow$ **10 runs are performed.**
- **Initial DoE:** average of 49 points for the structural discipline and 59 for the aerodynamics discipline.

**Comparison between DPOD+I & EGMDO framework and reference framework**

| | | $\alpha^*$ | $V_\infty^*$ | $t_{sk}^*$ | $t_{sp}^*$ | $f(\boldsymbol{x}^*)$ | $n^a$ | $n^s$ |
|---|---|---|---|---|---|---|---|---|
| Reference | $\mathbb{E}$ | 1.0 | 1.0 | 0.0 | 0.0 | 0.0 | 286 | 286 |
| | CoV | – | – | – | – | – | – | – |
| DPOD+I & SLSQP | $\mathbb{E}$ | 1.0 | 1.0 | $3.4 \times 10^{-4}$ | $3.1 \times 10^{-4}$ | 0.0585 | 60.7 | 51.5 |
| | CoV | $\leq 10^{-12}$ | $\leq 10^{-12}$ | 3.0 | 2.6346 | 0.1523 | 0.1904 | 0.1784 |
| DPOD+I & EGMDO | $\mathbb{E}$ | 0.999 | 0.987 | 0.006 | $3 \times 10^{-7}$ | 0.044 | 61 | 51 |
| | CoV | 0.003 | 0.02 | 0.9 | 3.16 | 0.44 | 0.1117 | 0.1225 |

where $n^a$ and $n^s$ are, respectively, the number of aerodynamics solver calls and the number of structural solver calls.

# DPOD+I & EGMDO results

- Random initial disciplinary DoEs and random initial optimizer DoE $\Rightarrow$ **10 runs are performed.**
- **Initial DoE:** average of 49 points for the structural discipline and 59 for the aerodynamics discipline.

**Comparison between DPOD+I & EGMDO framework and reference framework**

| | | $\alpha^*$ | $V_\infty^*$ | $t_{sk}^*$ | $t_{sp}^*$ | $f(\boldsymbol{x}^*)$ | $n^a$ | $n^s$ |
|---|---|---|---|---|---|---|---|---|
| Reference | $\mathbb{E}$ | 1.0 | 1.0 | 0.0 | 0.0 | 0.0 | 286 | 286 |
| | CoV | – | – | – | – | – | – | – |
| DPOD+I & SLSQP | $\mathbb{E}$ | 1.0 | 1.0 | $3.4 \times 10^{-4}$ | $3.1 \times 10^{-4}$ | 0.0585 | 60.7 | 51.5 |
| | CoV | $\leq 10^{-12}$ | $\leq 10^{-12}$ | 3.0 | 2.6346 | 0.1523 | 0.1904 | 0.1784 |
| DPOD+I & EGMDO | $\mathbb{E}$ | 0.999 | 0.987 | 0.006 | $3 \times 10^{-7}$ | 0.044 | 61 | 51 |
| | CoV | 0.003 | 0.02 | 0.9 | 3.16 | 0.44 | 0.1117 | 0.1225 |

where $n^a$ and $n^s$ are, respectively, the number of aerodynamics solver calls and the number of structural solver calls.

➢ **Reduction by a factor of 5** on the number of necessary disciplinary solver calls compared to the reference framework.

# Contents


I. Introduction
II. Problem statement
III. Reference framework
IV. Implemented frameworks
  I. DPOD+I & SLSQP
  II. DPOD+I & EGMDO
V. Conclusion

# Conclusion and perspectives

- The optimization of a two-disciplinary problem requires many disciplinary solver calls if the real solvers are used for the MDA.

# Conclusion and perspectives

- The optimization of a two-disciplinary problem requires many disciplinary solver calls if the real solvers are used for the MDA.
- DPOD+I surrogates allow us to **perform multi-disciplinary optimization using high-fidelity solvers**, at a reduced computational cost.

# Conclusion and perspectives

- The optimization of a two-disciplinary problem requires many disciplinary solver calls if the real solvers are used for the MDA.
- DPOD+I surrogates allow us to **perform multi-disciplinary optimization using high-fidelity solvers**, at a reduced computational cost.
- The EGMDO algorithm allows to **perform global optimization when the disciplinary solvers are replaced by disciplinary Gaussian Processes**, reducing the disciplinary solver calls during the optimization process.

# Conclusion and perspectives

- The optimization of a two-disciplinary problem requires many disciplinary solver calls if the real solvers are used for the MDA.
- DPOD+I surrogates allow us to **perform multi-disciplinary optimization using high-fidelity solvers**, at a reduced computational cost.
- The EGMDO algorithm allows to **perform global optimization when the disciplinary solvers are replaced by disciplinary Gaussian Processes**, reducing the disciplinary solver calls during the optimization process.
- Some perspectives to the proposed framework include the implementation of **other dimension reduction techniques**, for instance via local POD basis or non-linear model order reduction, to account for more complex disciplinary models. **Other approximation models**, such as the Kriging with Partial Least Squares model [4] could allow the construction of GPs for a greater number of design variables.

**Bibliography**:

[1] T.R. Brooks, G.K. Kenway, and J.R.R.A. Martins. “Undeflected Common Research Model (uCRM): An Aerostructural Model for the Study of High Aspect Ratio Transport Aircraft Wings”. In: *35th AIAA Applied Aerodynamics Conference*. AIAA, 2017. doi: 10.2514/6.2017-4456.

[2] G. Berthelin, S. Dubreuil, M. Salaün, N. Bartoli, and C. Gogu. “Disciplinary Proper Orthogonal Decomposition and Interpolation for the resolution of parametrized Multidisciplinary Analysis”. In: *International Journal for Numerical Methods in Engineering* (2022). Accepted Author Manuscript. doi: 10.1002/nme.6981.

[3] S. Dubreuil, N. Bartoli, C. Gogu, and T. Lefebvre. “Towards an efficient global multidisciplinary design optimization algorithm”. In: *Structural and Multidisciplinary Optimization* 62 (2020), pp. 1–27. doi: 10.1007/s00158-020-02514-6.

[4] M. Bouhlel, N. Bartoli, J. Morlier, and A. Otsmane. “Improving kriging surrogates of high-dimensional design models by Partial Least Squares dimension reduction”. In: Structural and Multidisciplinary Optimization 53 (5) (2016), pp. 935–952. doi: 10.1007/s00158-015-1395-9.