Using Generative Adversarial Networks to constrain inverse problems resolution.

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ETICS | October 2022

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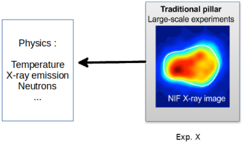
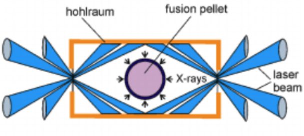
Summary

Inertial Confinement Fusion



Inertial confinement fusion



X-ray emission calibration

- Context : LMJ installation → to reproduce the ICF
- Objective: Find physical characteristics (e.g a mesh) from an image measurement of the pellet.

This is an inverse problem.

Summary

Inverse problem formalization

Given Ω the set of $N_{pix} \times N_{pix}$ sized images showing a specifical physic characteristic.

$$\mathcal{P} : \begin{array}{l} \Omega \subset \mathbb{R}^{N_{pix} \times N_{pix}} \to \mathbb{R}^{N_{pix_2} \times N_{pix_2}} \\ x \mapsto \mathcal{P}(x) = y \end{array}$$

where:

- x is an image or a vector (e.g. the physical mesh),
- y is an image (the measurement),
- $\mathcal{P}$ is the simulation code's operator.

Objective: Solving the following inverse problem:

Knowing y determine $\hat{x}$ such that:

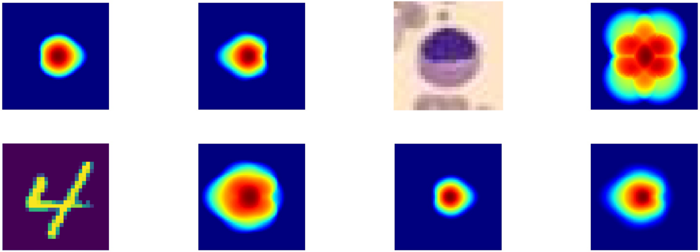
$$\hat{x} = \underset{x \in \mathbb{R}^{N_{pix} \times N_{pix}}}{\operatorname{argmin}} \parallel \mathcal{P}(x) - y \parallel^2 .$$

Issues

The inverse problem is an ill-posed problem.

Learning of Ω

- First objective: Describe Ω
- Issues: Very high dimension ($dim \simeq 10^3$)



Tools

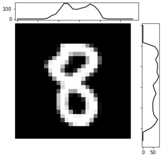
Deep Learning with Generative Adversarial Networks (GAN).

Operator $\mathcal{P}$: an example with the digits

Mathematically $\mathcal{P}$ is the following operator:

$$\mathcal{P}: \begin{array}{l} \mathbb{R}^{N_{pix} \times N_{pix}} \longrightarrow \mathbb{R}^{2N_{pix}} \\ x \longmapsto \begin{pmatrix} \sum_i x_{ij} \\ \sum_j x_{ij} \end{pmatrix} = y \end{array}$$

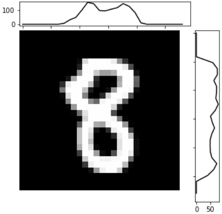
$x_{ref} \in \Omega \subset \mathbb{R}^{N_{pix} \times N_{pix}}$ is the targeted image.



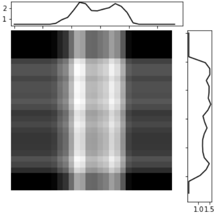
x_{ref}

We denote : $y_{ref} = \mathcal{P}(x_{ref})$.

Operator $\mathcal{P}$: an example with the digits - Results



x_{ref}



Generalized inverse

This is an ill-posed poblem.
The generalized inverse is not the solution expected visually.

Summary

Generative Adversarial Network

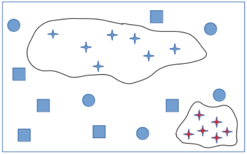
The GAN is a Neural network whose function is to generate fake images looking like examples.

Example of GAN created faces

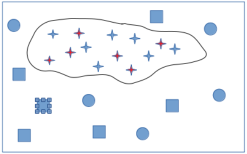
GAN

This image generating unsupervised deep learning tool has been introduced by Ian Goodfellow in 2014.

Introduction of the distributions



Initial set of the images from $\mathbb{R}^{N_{pix} \times N_{pix}}$



Final set of the images from $\mathbb{R}^{N_{pix} \times N_{pix}}$ with an optimal generator

- The discriminator **D** aims to distinguish the two distributions.
- The generator **G** aims to get the two distributions closer.

Advantage

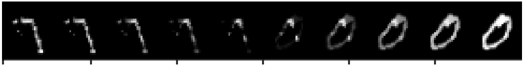
The GAN generates images and therefore gives access to a generic distribution describing a specific image nature that is now characterizable.

Computational settings

Characteristics

- The GAN is a Convolutionnal network.
- It is a Wasserstein GAN (based on a Wasserstein Distance metric).
- The Optimizer is ADAM (a stochastic gradient descent variant).
- The latent space used has a dimension $d = 100$.

Advantages



Elements of a line in $\mathbb{R}^{N_{pix} \times N_{pix}}$ between two elements of Ω

Elements of that line between these same elements made in $\mathcal{Z}$

Advantage

The use of the GAN and its latent space on which it is trained brings the notion of *manifold*.

GAN for the inverse problem

Now, let's focus on the generator $\mathbf{G} : \mathcal{Z} \subset \mathbb{R}^d \mapsto \Omega$.
From here, $\mathbf{G}$ is already trained.

Objective : Resolution of the following inverse problem:

Knowing y determine $\hat{x}$ such that:

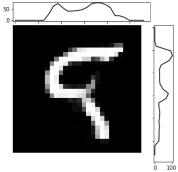
$$\hat{x} = \underset{\substack{x \in \mathbb{R}^{N_{pix} \times N_{pix}} \\ x = \mathbf{G}(z), z \in \mathcal{Z}}}{\operatorname{argmin}} \| \mathcal{P}(x) - y \|^2 .$$

Conclusion

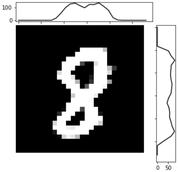
The introduction of G in the problem resolution allows to restrain the domain of search of x.

Results

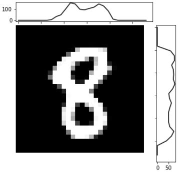
- This time the minimization is computed on $z \in \mathcal{Z}$ that has been clipped.



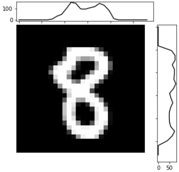
Initial $\mathbf{G}(z)$



$\mathbf{G}(z)$ after 2000 steps



Final $\mathbf{G}(z)$



x_{ref}

Perspectives

Perspectives

- Focus on higher dimension data.
- Keep the physical characteristic of the images generated.
- Bring in a physic validation for generated images.
- Constrain the distribution of z instead of clipping the latent vector.

Thank you for your attention.
Any question ?