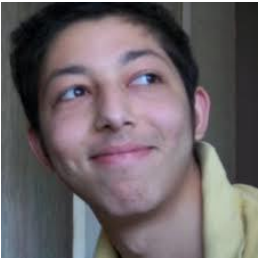


2 APPLICATIONS OF THE STEPWISE UNCERTAINTY REDUCTION STRATEGY FOR EXCURSION SET ESTIMATION

Delphine Sinoquet¹

Reda El Amri¹, Celine Helbert², Morgane Menz¹, Miguel Munoz Zuniga¹, Clémentine Prieur³

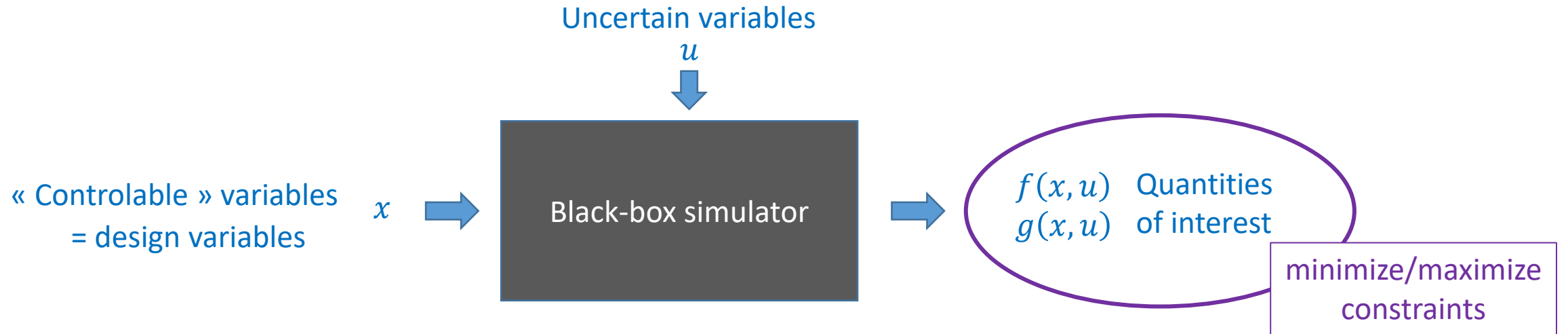


¹ IFP Énergies Nouvelles

² École Centrale de Lyon

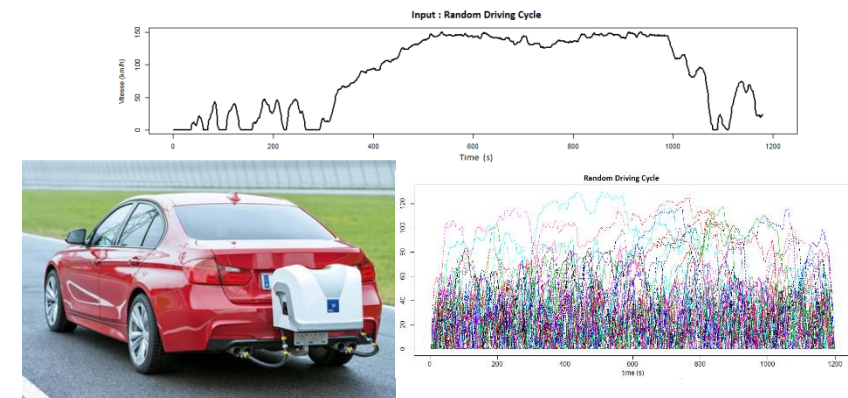
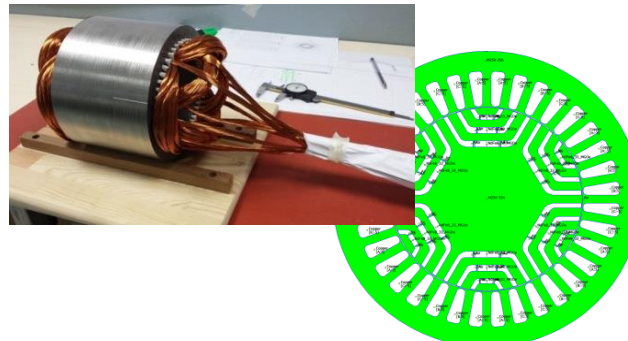
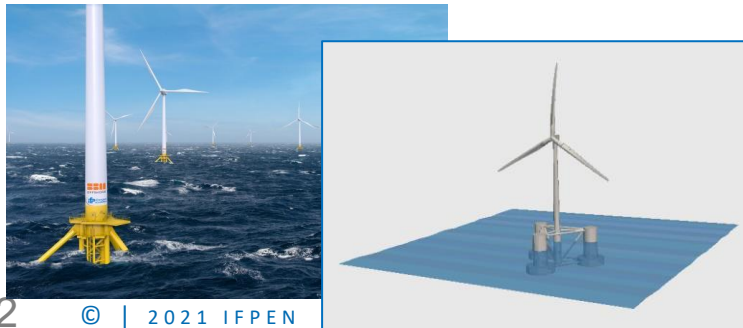
³ Université Grenoble Alpes, INRIA

CONTEXT : ROBUST/RELIABLE CONCEPTION



Applications for optimized design

- Reliability w.r.t environmental conditions (e.g. wind, wave)
- Robustness to dispersions of design variables (manufacturing), to component characteristics (e.g. magnetic properties of magnets), ...

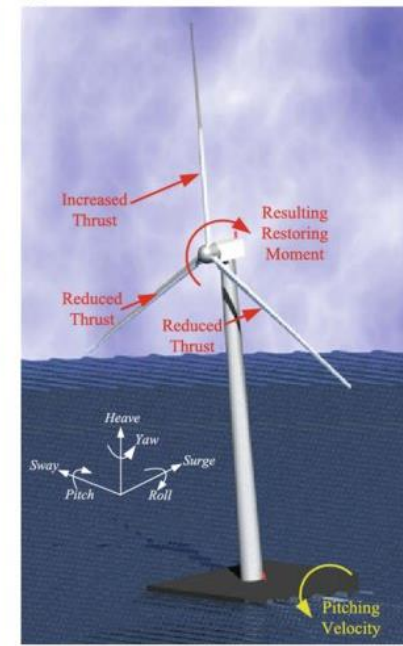
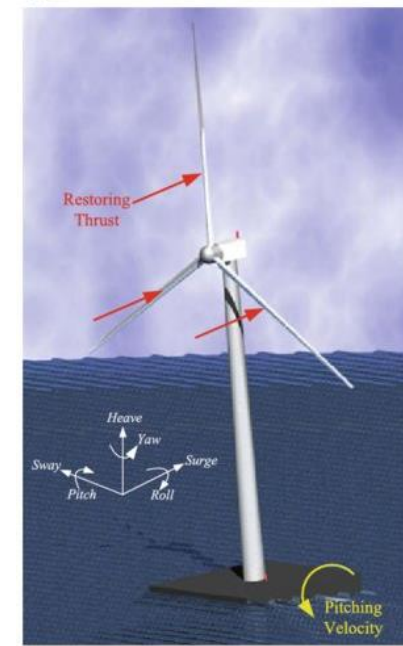


APPLICATIONS OF EXCURSION SET ESTIMATION

Feasible designs for complex optimal design problems

- **Pre-calibration** (*PhDs C. Duhamel, A. Hirvoas*) for wind turbine find a set of structure parameters x , such that, for environmental conditions \mathbf{U}

$$\Gamma^* := \{x \text{ s.t. } \mathbb{P}_{\mathbf{U}}(\|D^{sim}(\mathbf{U}, x) - D^{meas}(\mathbf{U})\| < \text{seuil}) > 1 - \alpha\}$$



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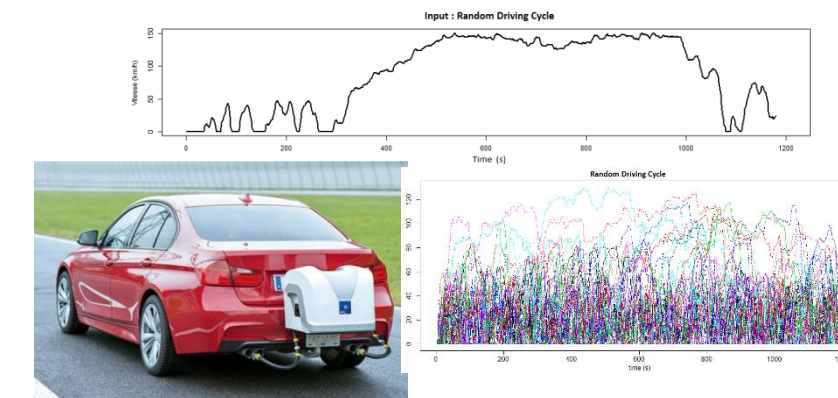
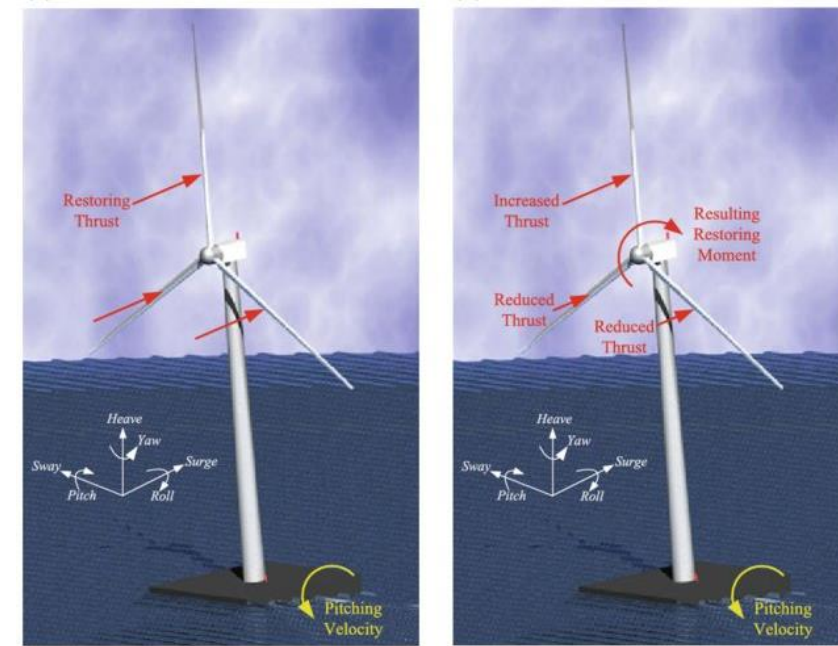
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- **Robust tuning of control systems**

- for offshore wind turbines: avoid an unstable float-push system while staying above a minimum production level (*ANR Samourai*)
- for the de-pollution system of a vehicle (*PhD. R. El Amri*)

$$\Gamma^* := \{x \text{ s.t. } \mathbb{E}_{\mathbf{U}}(g_1(x, U)) \leq s\}$$



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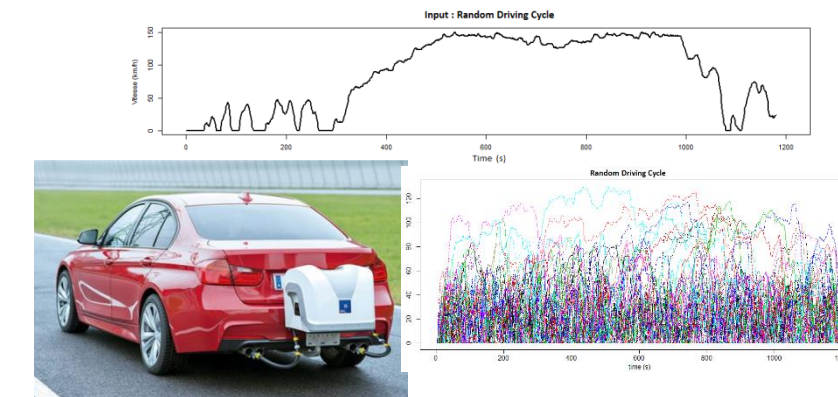
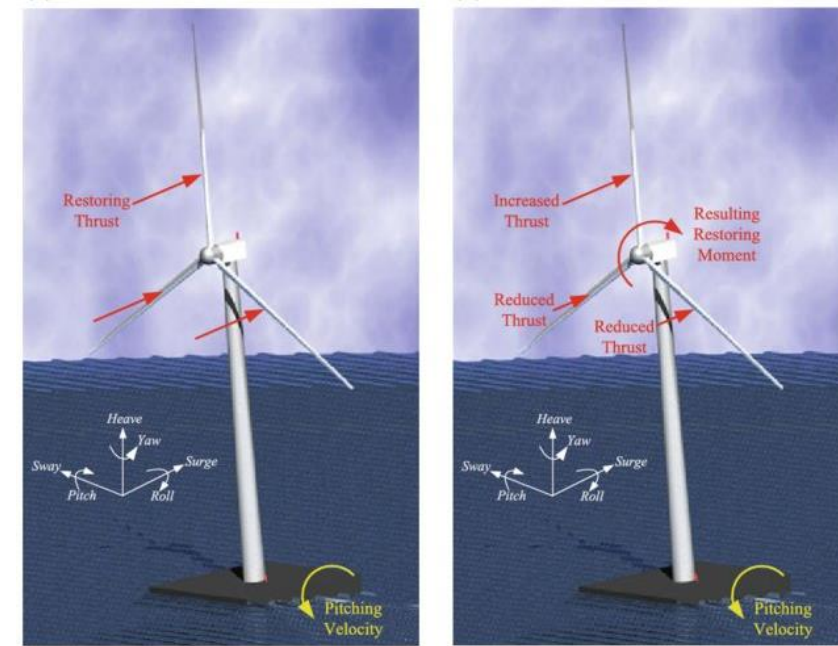
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Feasible input set for complex simulators

- **Hidden constraints leading to simulation crashes**

$$\Gamma^* = \{x \in \Omega: f(x) \neq \text{NAN}\} = \{x \in \Omega: \mathbb{1}_{f(x) \neq \text{NAN}} = 1\}$$

(*ANR Samourai*)



OUTLINE

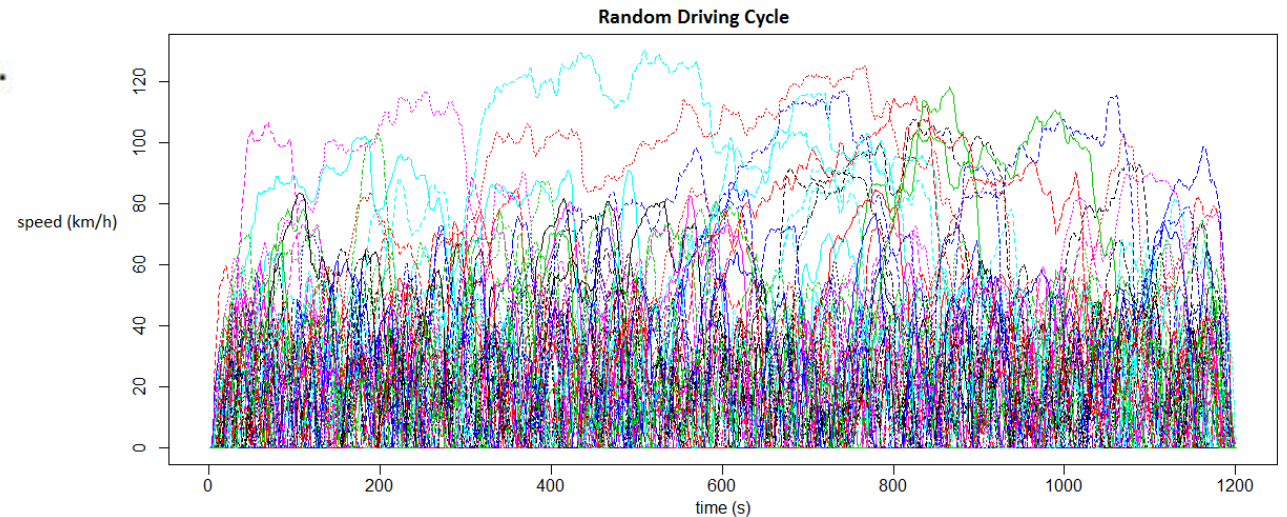
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for the de-pollution system of a vehicle (PhD. R. El Amri)
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hidden constraints leading to simulation crashes

ROBUST TUNING OF CONTROL SYSTEMS



IFPEN test case : control strategy for an automotive depollution system

- v^1, \dots, v^{100} random driving cycles.



Control parameters of the depollution system
(SCR: Selective Catalytic Reduction)

- NH_3^{\max} function of x_1 and x_2 , $D = [0, 0.6]^2$

$$\Gamma^* = \{ \mathbf{x} = (x_1, x_2) \in D : \mathbb{E}[\text{NH}_3^{\max}(\mathbf{x}, \mathbf{V})] \leq s \}.$$

Ammoniac spike must be smaller than **30 ppm**

The model is expensive to evaluate (chemical kinetic model).

ROBUST INVERSION WITH FUNCTIONAL INPUTS

$$\Gamma^* := \{ \mathbf{x} \in D \text{ s.t. } f(\mathbf{x}) = \mathbb{E}[g(\mathbf{x}, \mathbf{V})] \leq s \}$$

Objective:

estimate $\Gamma^* \subset \mathbb{R}^p$ from model evaluations.

Issues :

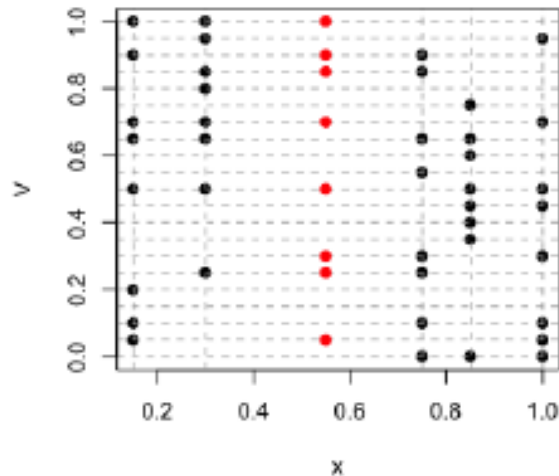
- each evaluation $f(\mathbf{x})$ requires the estimation of $\mathbb{E}[g(\mathbf{x}, \mathbf{V})]$,
- \mathbf{V} is known through κ realizations $\mathbf{v}^1, \dots, \mathbf{v}^\kappa$,
- each evaluation $g(\mathbf{x}, \mathbf{v})$ is costly.

TWO STRATEGIES

$$\Gamma^* := \{ \mathbf{x} \in D \text{ s.t. } f(\mathbf{x}) = \mathbb{E}[g(\mathbf{x}, \mathbf{V})] \leq s \}$$

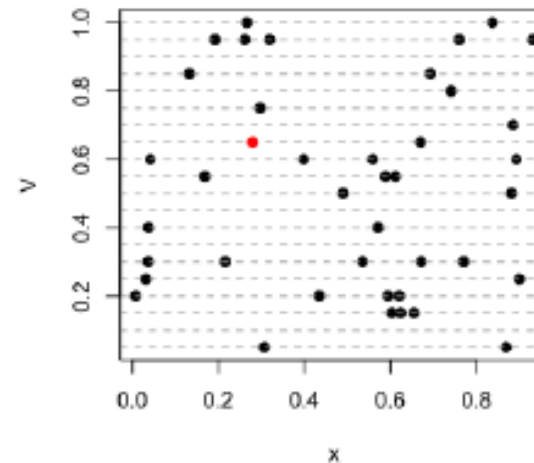
Strategy I

- Build a meta-model of $f(x)$
- Choose $x_{n+1} \in D$,
- Estimate $f(\mathbf{x}_{n+1}) = \mathbb{E}[g(\mathbf{x}_{n+1}, \mathbf{V})]$ with l evaluations of $g(\mathbf{x}_{n+1}, \cdot)$ selected by quantization in $(\mathbf{v}^1, \mathbf{v}^2, \dots, \mathbf{v}^k)$



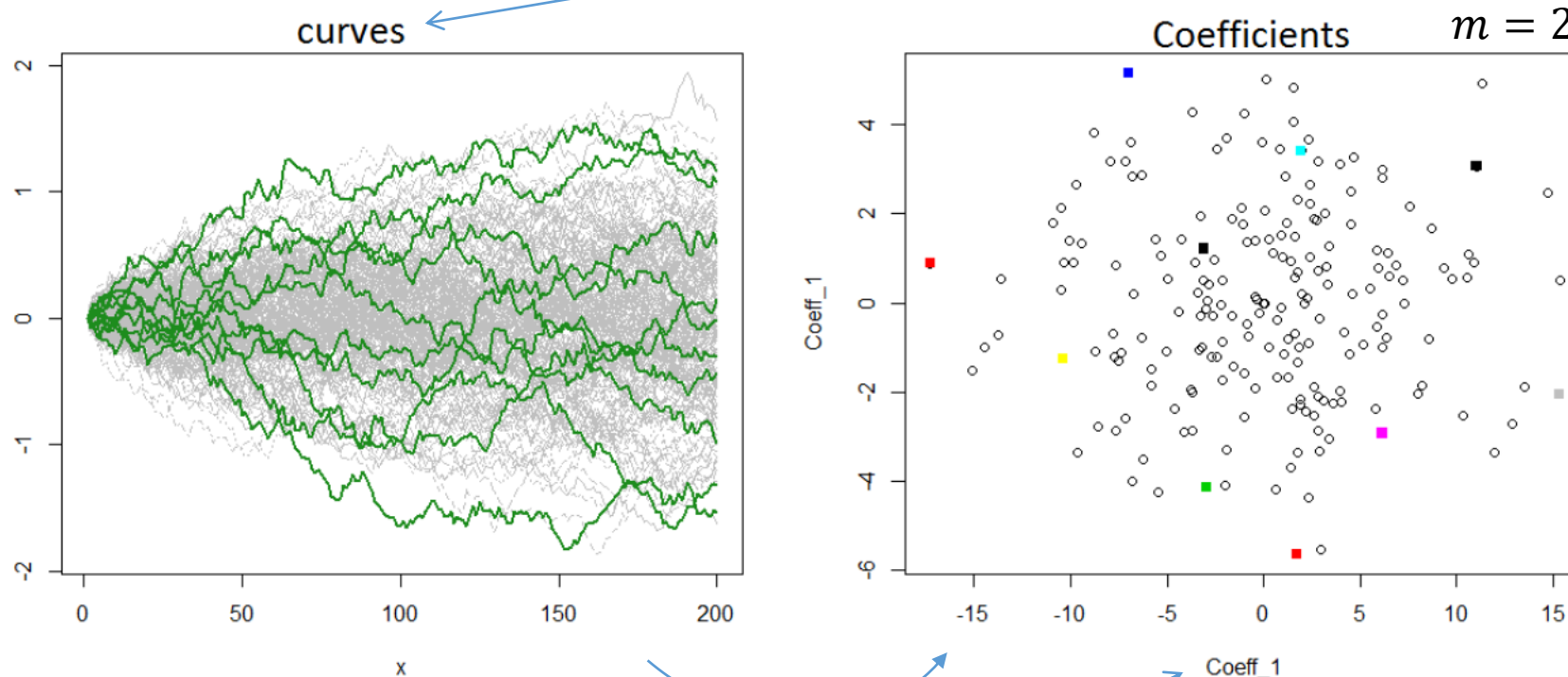
Strategy II

- Build a meta-model of $g(x, \mathbf{V})$
- Choose $(x_{n+1}, \mathbf{v}^{n+1})$,
- Evaluate $\mathbb{E}[\hat{g}(\mathbf{x}_{n+1}, \mathbf{V})]$ at this new point



DIMENSION REDUCTION OF FUNCTIONAL VARIABLES

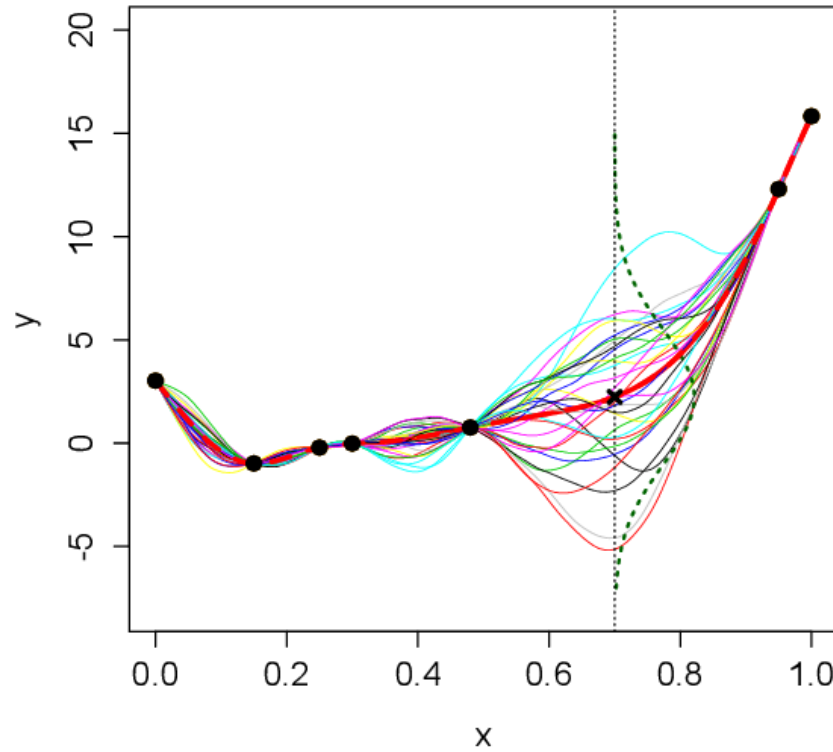
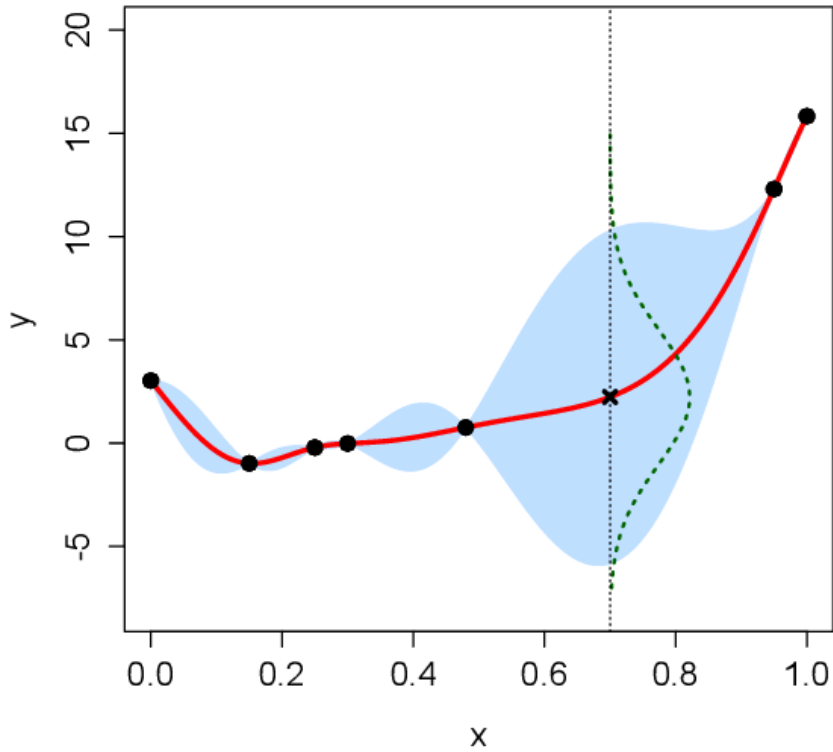
$$\Gamma^* := \{x \in D \text{ s.t. } f(x) = \mathbb{E}[g(x, V)] \leq s\}$$



Karhunen-Loève (KL) expansion

$$V(t) = \mu v(t) + \sum_{k=1}^m \sqrt{\lambda_k} \eta_k \phi_k(t).$$

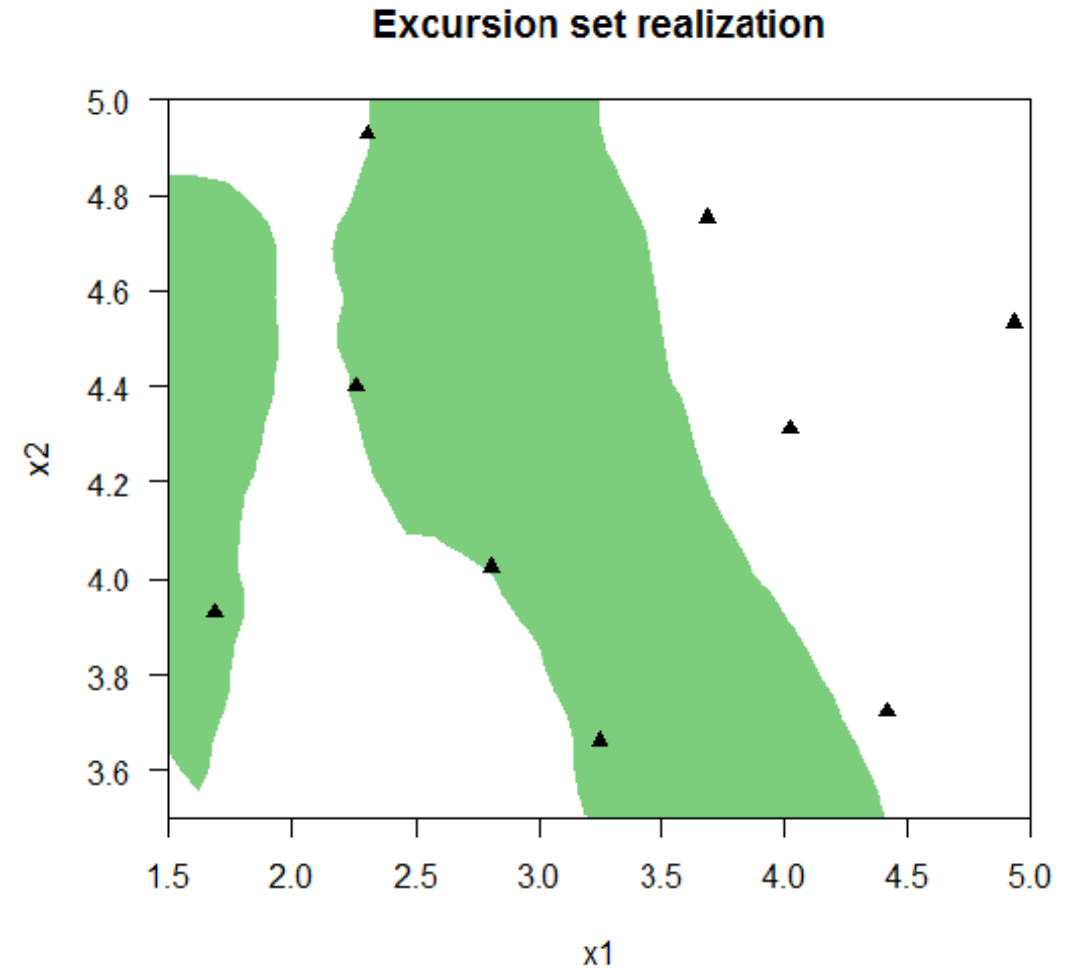
GAUSSIAN PROCESS METAMODELING



Meta-models based on **Gaussian Process** that provides **uncertainty estimation**

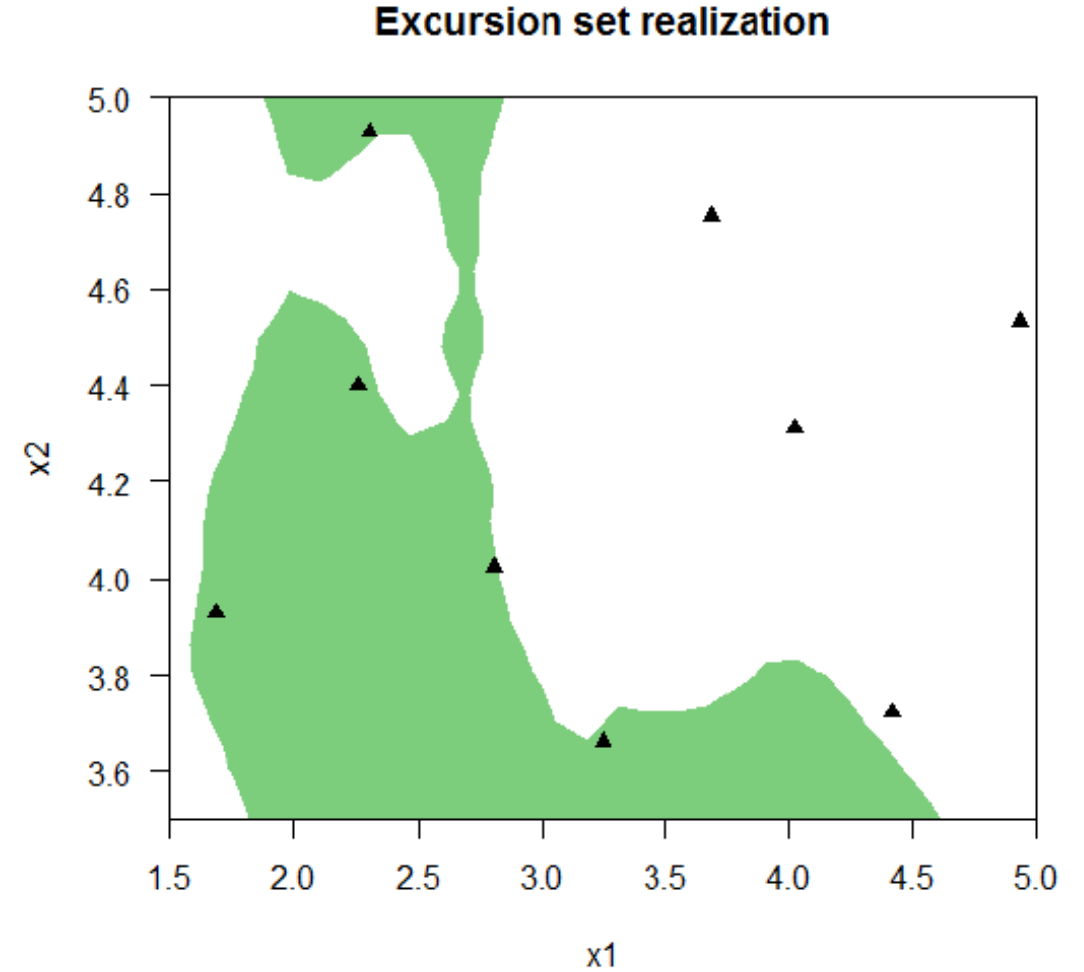
GP INDUCES RANDOM EXCURSION SET

$$\Gamma := \{x \in D: Z_x | Z_{x_n} \leq s\}$$



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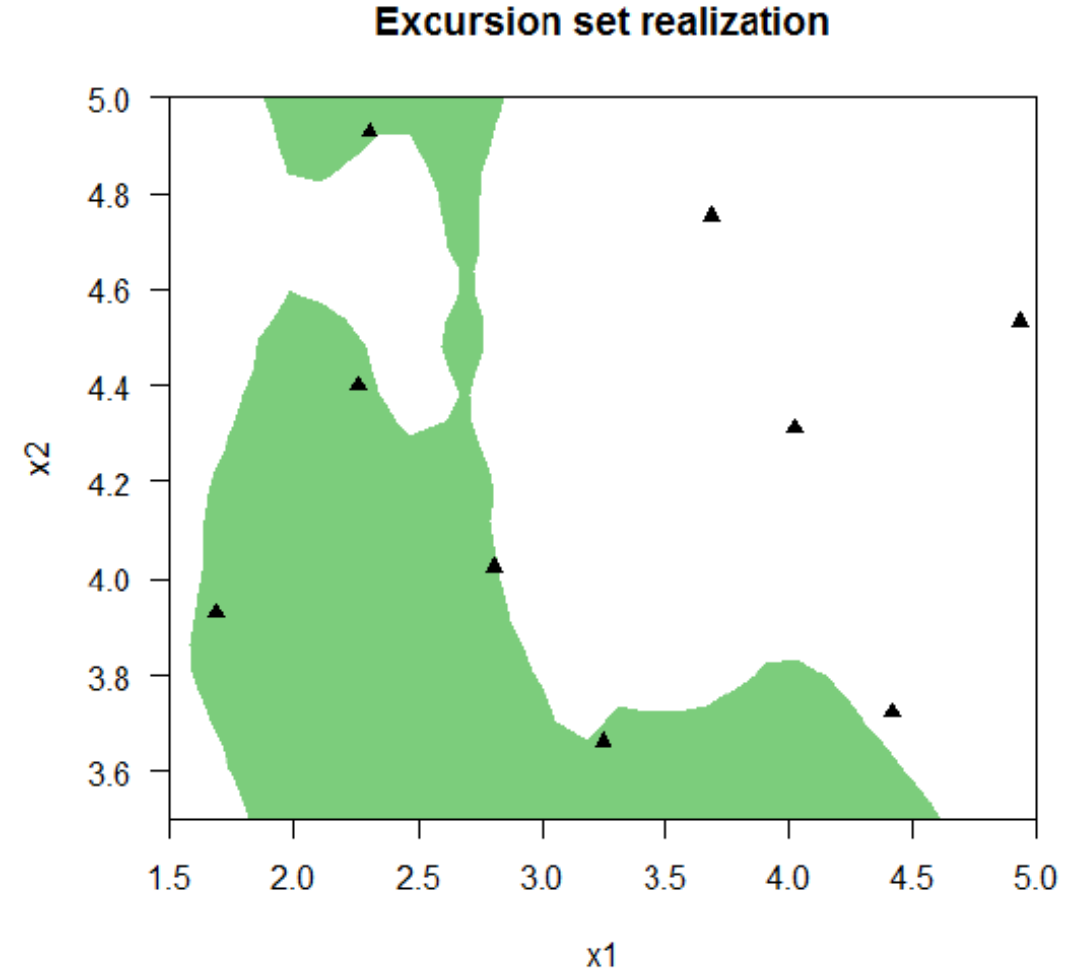
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GP INDUCES RANDOM EXCURSION SET

How to summarize the distribution on sets?

- **Estimate** Γ^* with Expectation of random closed sets
(Molchanov, 2006)

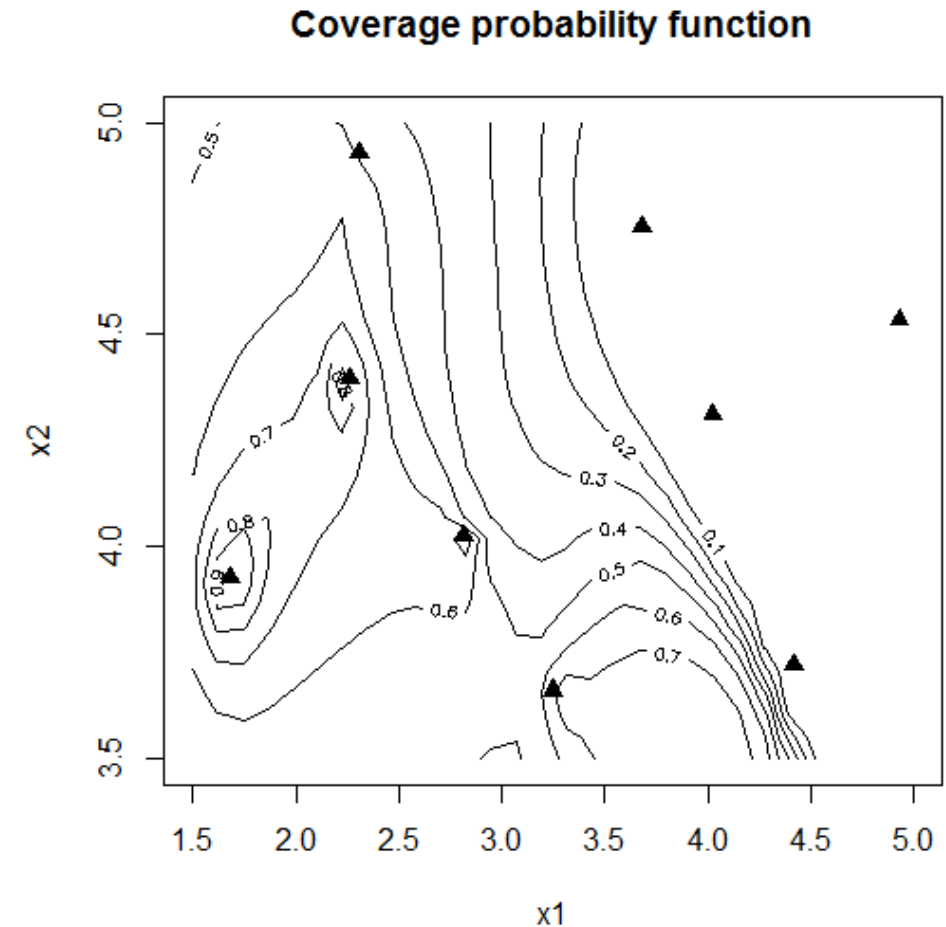


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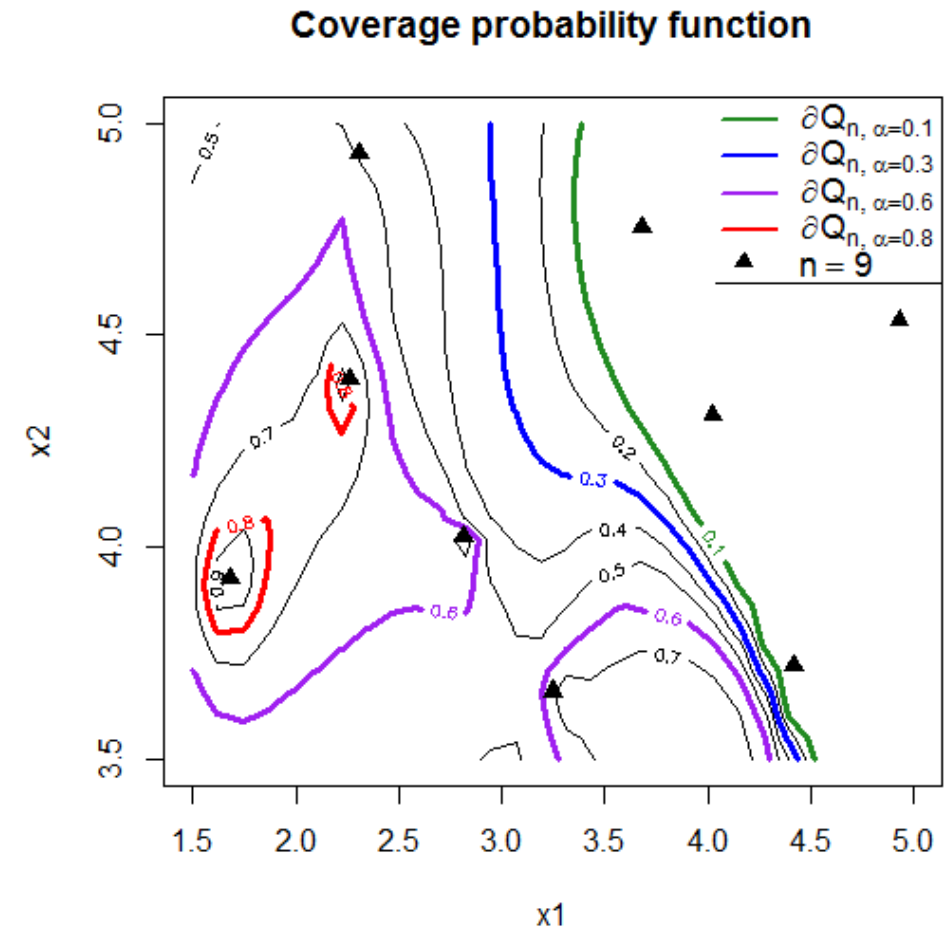
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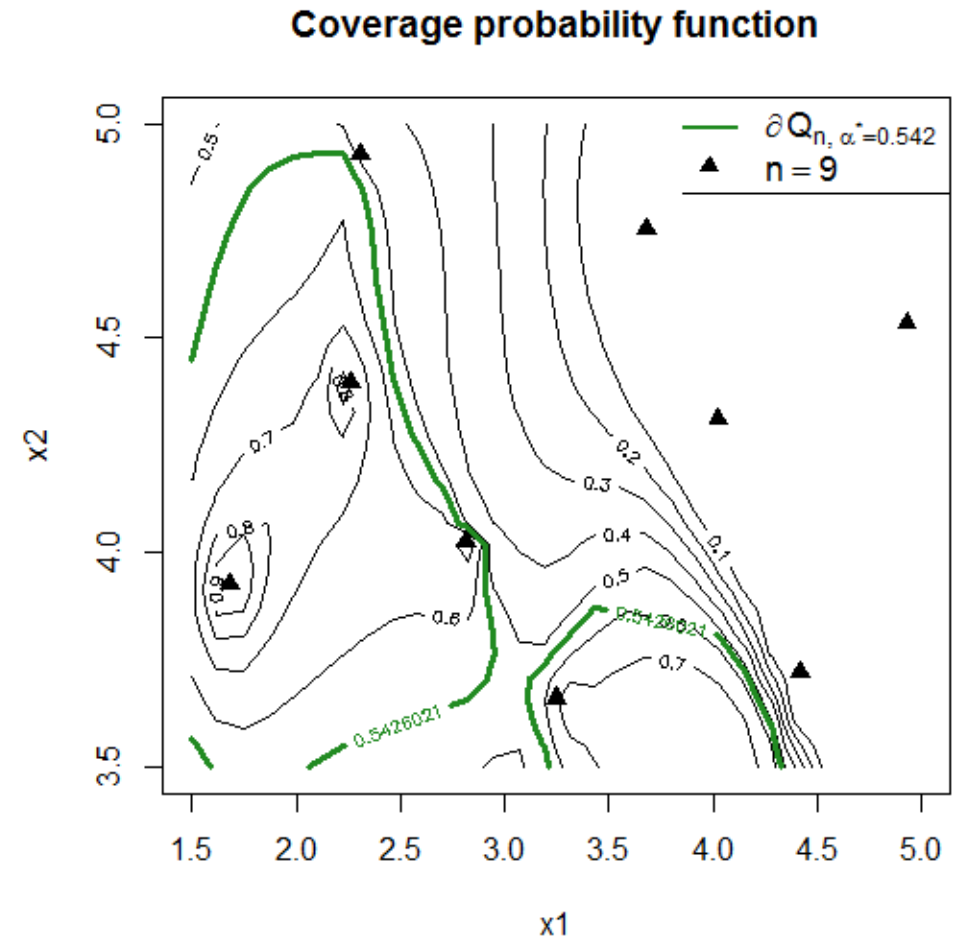
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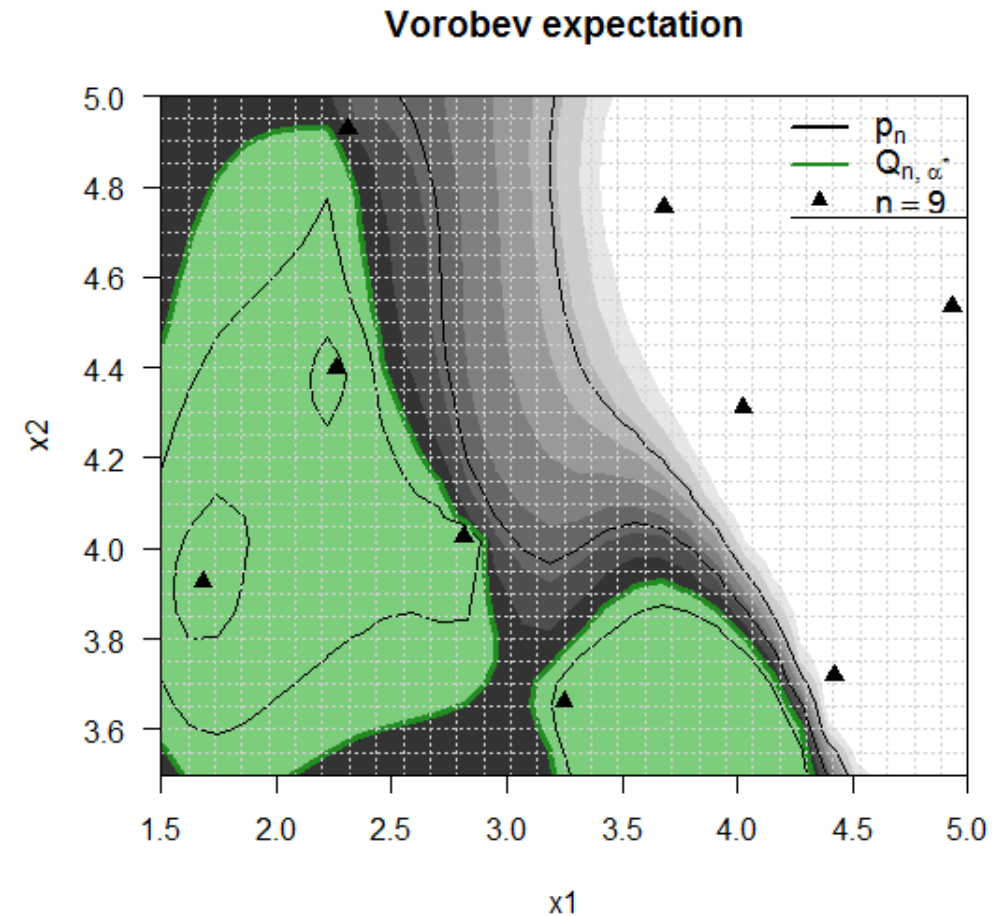
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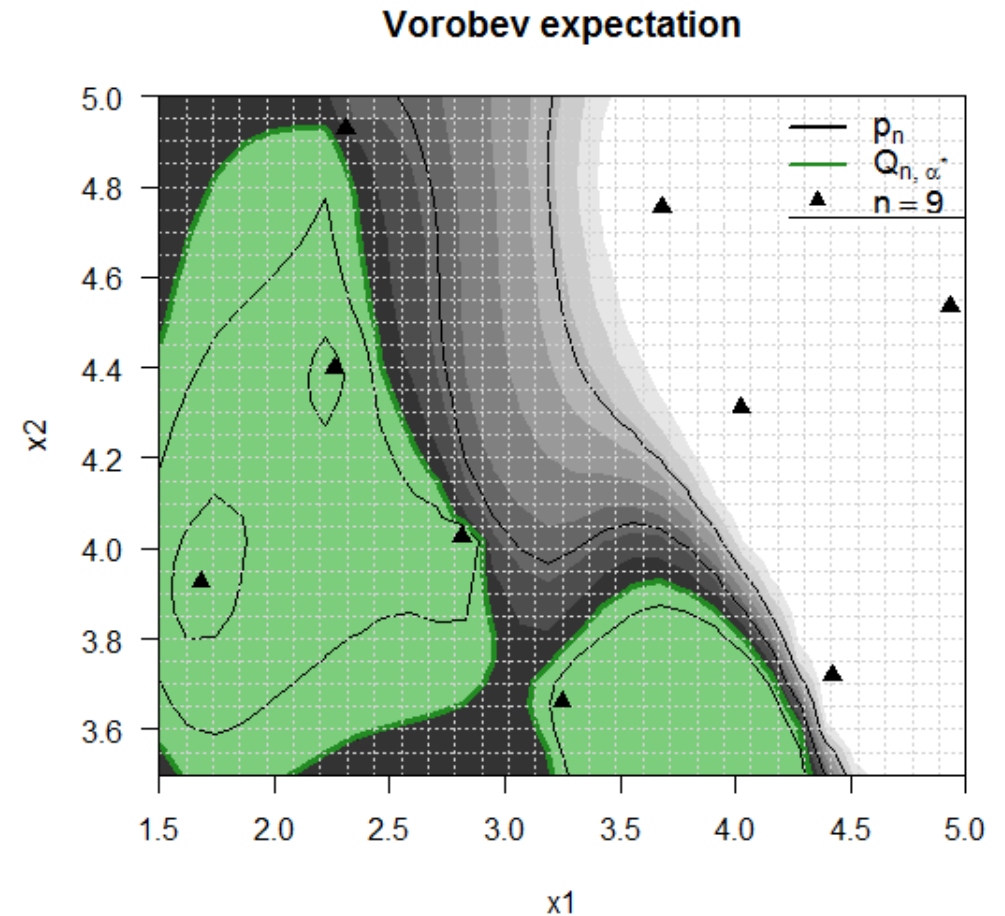
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- **Vorob'ev deviation** : $Var_n(\Gamma) = \mathbb{E}(\mu(Q_{n,\alpha^*} \Delta \Gamma))$



STEPWISE UNCERTAINTY REDUCTION BASED ON VOROB'EV THEORY

- Choose next point x_{n+1} to reduce expected uncertainty H_{n+1} on the future model:

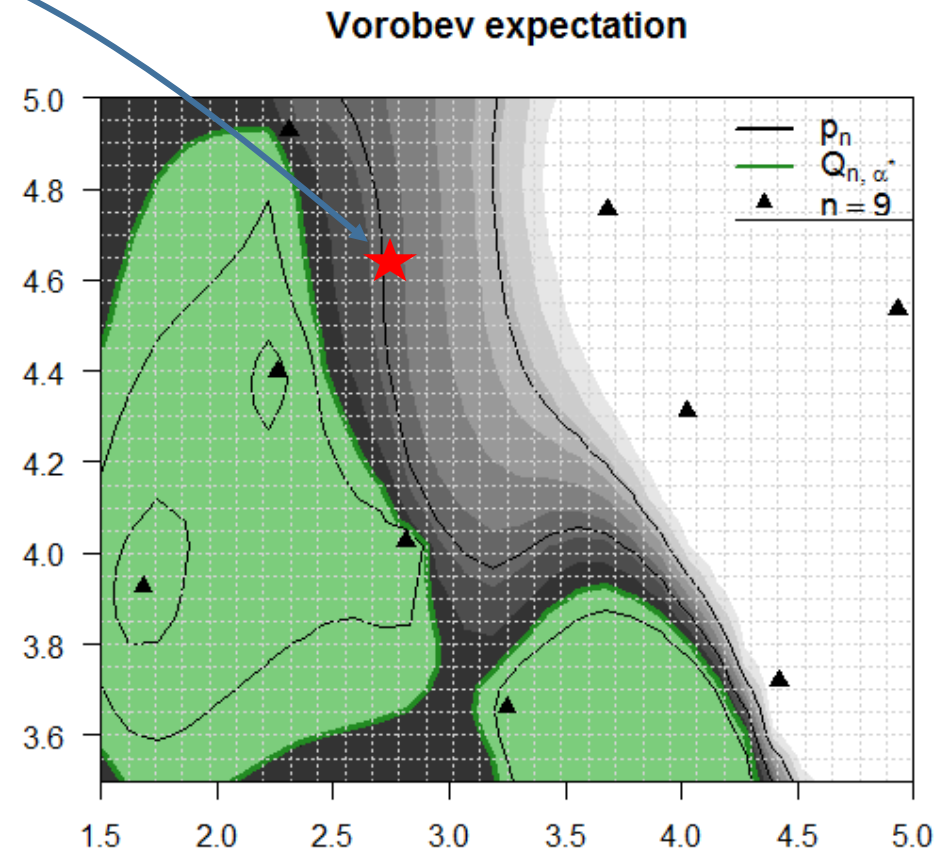
$$x_{n+1} = \underset{x \in D}{\operatorname{argmin}} \mathbb{E}_{n, Y_{n+1}}(x) [H_{n+1}]$$

Uncertainty function : Vorob'ev deviation

$$\mathcal{H}_n^{\text{uncert}} = \mathbb{E}_n[\mu(Q_{\alpha_n^*} \Delta \Gamma)]$$

- Choose next curve (Strategy II)

$$\mathbf{V}_{(n+1)} \leftarrow \underset{x}{\operatorname{argmin}} \operatorname{VAR}(Y_{x(n+1)}^{n+1})$$



APPLICATION ON AN ANALYTICAL EXAMPLE WITH \mathbf{V} A BROWNIAN MOTION

$$f : (\mathbf{x}, v) \mapsto \max_t v_t \cdot |0.1 \cos(x_1 \max_t v_t) \sin(x_2) \cdot (x_1 + x_2 \min_t v_t)^2| \cdot \int_0^T (30 + v_t)^{\frac{x_1 \cdot x_2}{20}} dt,$$

Excursion set to retrieve

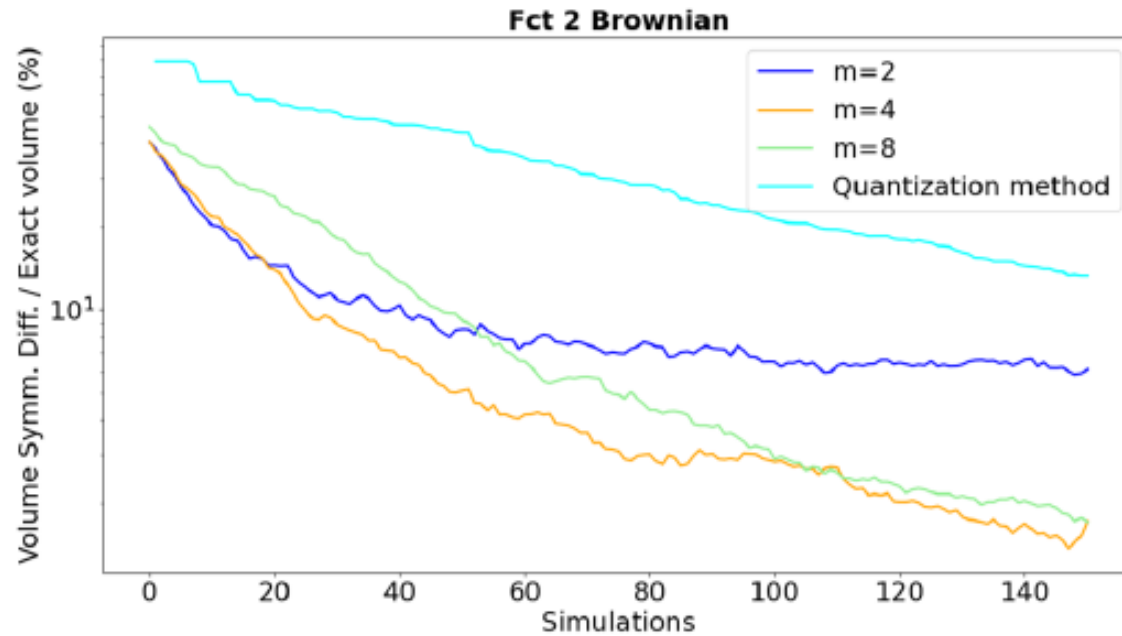
$$\Gamma^* := \{\mathbf{x} \in [1.5, 5] \times [3.5, 5] : f(\mathbf{x}) = \mathbb{E}[g(\mathbf{x}, \mathbf{V})] \leq c\}.$$

For the experiments we chose:

- a threshold $c = 1.2$,
- $\text{card}(\Xi) = 200$,
- a constant mean function m , a Matérn 5/2 covariance kernel,
- an initial DoE of size $n = 30$,
- a KL truncation argument $m_{KL} = 2, 4, 8$.

APPLICATION ON AN ANALYTICAL EXAMPLE WITH \mathbb{V} A BROWNIAN MOTION

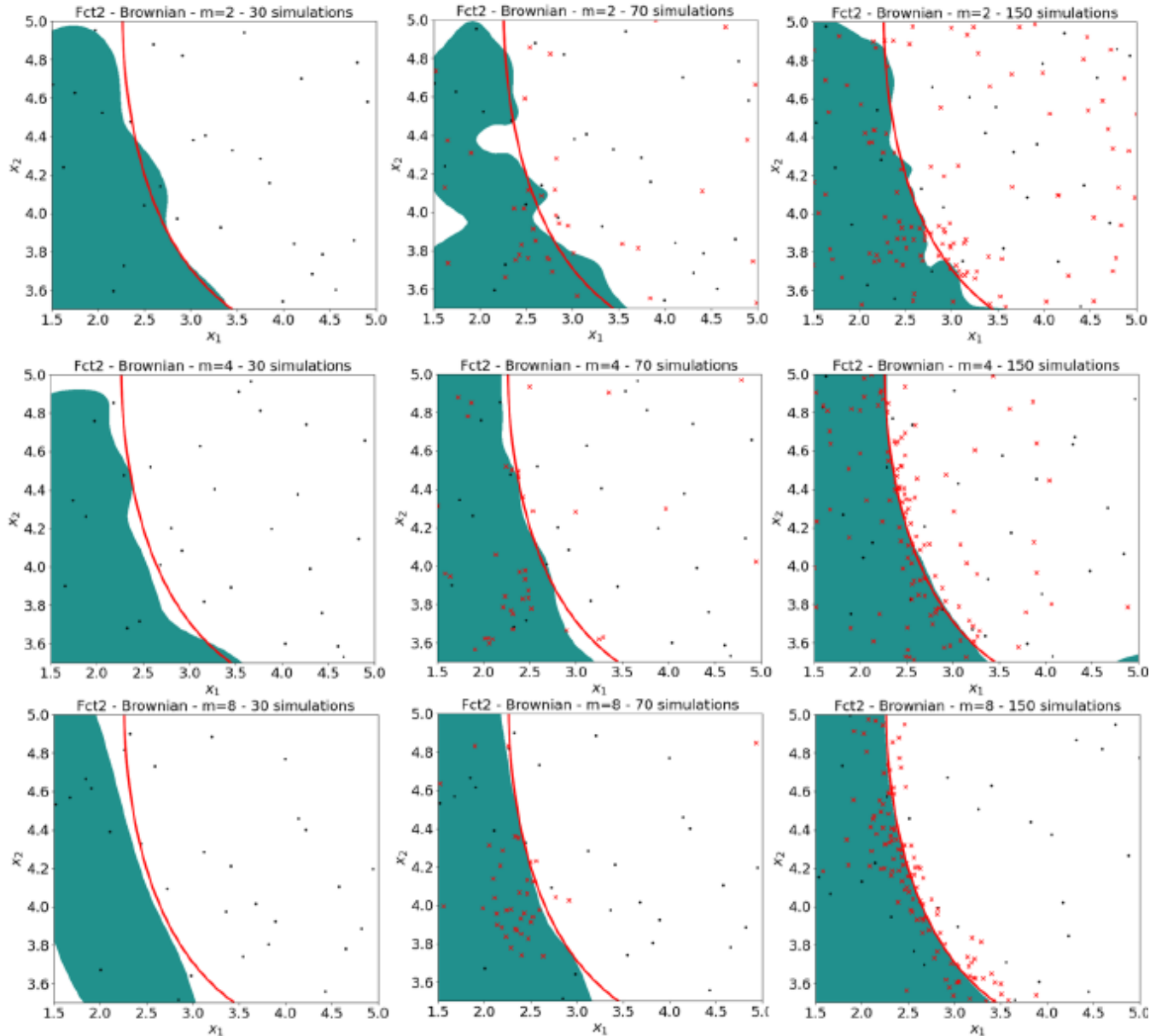
COMPARISON STRATEGY I (QUANTIZATION) AND STRATEGY II (MM IN JOINT SPACE)



For the approach based on quantization, we chose an initial DoE in \mathbb{X} of size 9. We have neglected the cost due to the quantization on this initial DoE. Indeed, around 20 points in average are required to estimate the expectation at each of these 9 points.

STRATEGY II (MM IN JOINT SPACE)

simulations \rightarrow



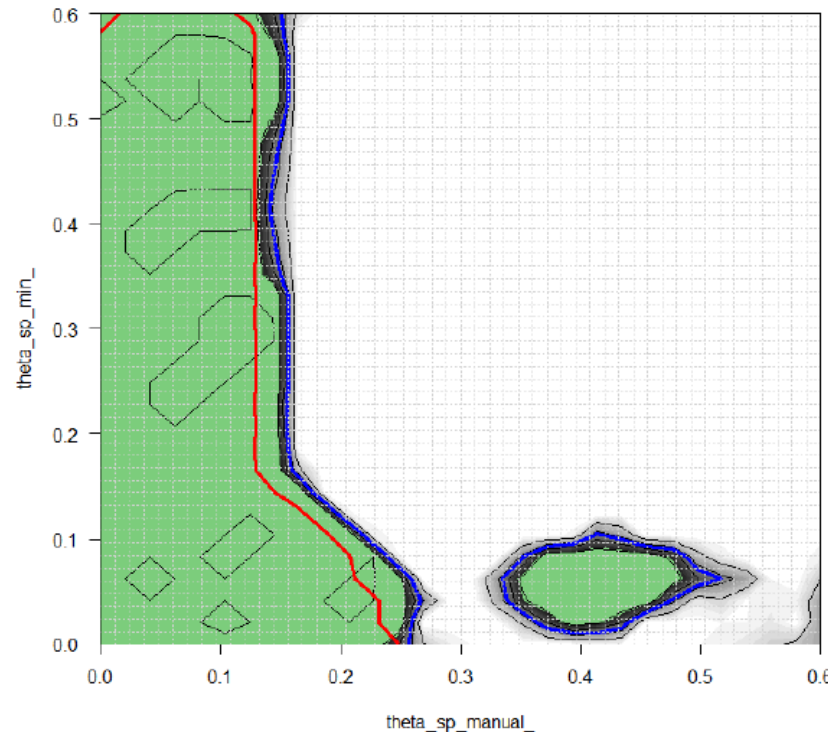
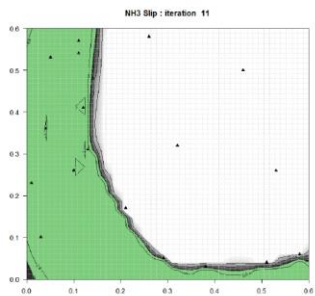
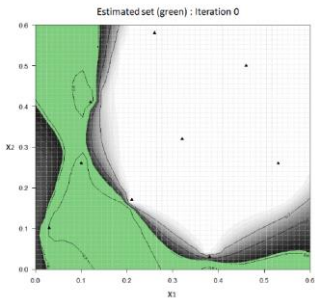
KL truncation
Argument m

NUMERICAL RESULTS

IFPEN test case: control strategy for an automotive NO_x depollution system

- NH₃^{out} function of '*theta.sp.min*' and '*theta.sp.manual*',
 $\mathbb{X} = [0, 0.6] \times [0, 0.6]$,
- $n = 510$ calls to f ,

The estimated set by **Algorithm I** (green set) and **Algorithm II** (red boundary)



- Small dimension
Algo II > Algo I
- Larger dimension
Algo I > Algo II

CONCLUSIONS FOR ROBUST INVERSION PART

- Two strategies for robust inversion strategy based on GP regression
 - MM in control space with expectation estimated by quantization (El Amri et al, 2020)
 - **MM in joint space of control and uncertain variables** (El Amri et al, 2021)
- Application to a depollution control system

Perspectives : PhD thesis of Clément Duhamel (INRIA AIRSEA team, IFPEN)

- Extend strategies to correlated responses (multiple objectives)
- Deal with other measures of robustness : quantiles ...
- Other applications : offshore wind turbines

OUTLINE

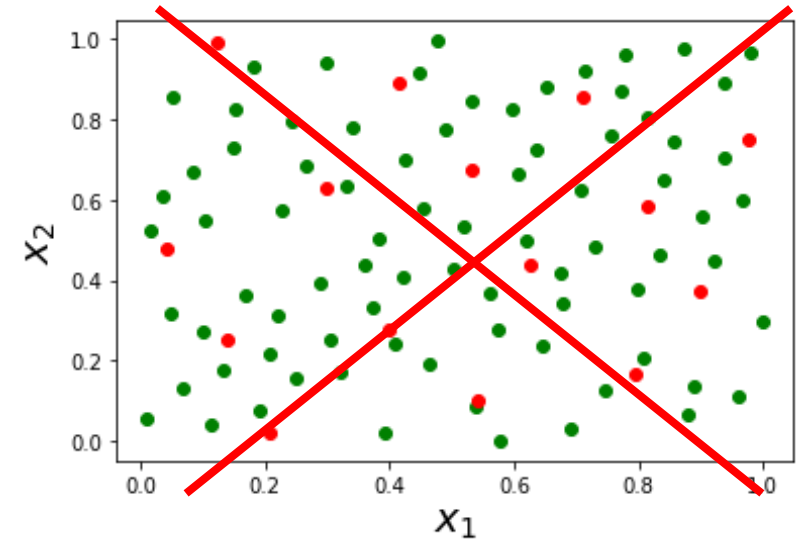
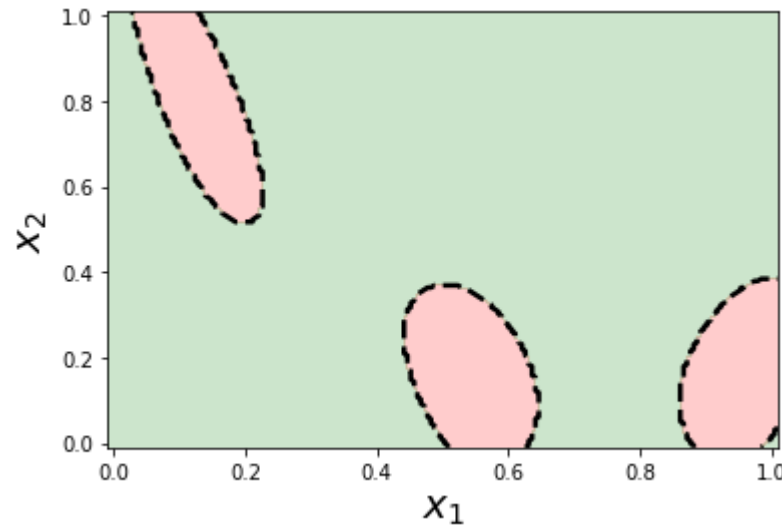
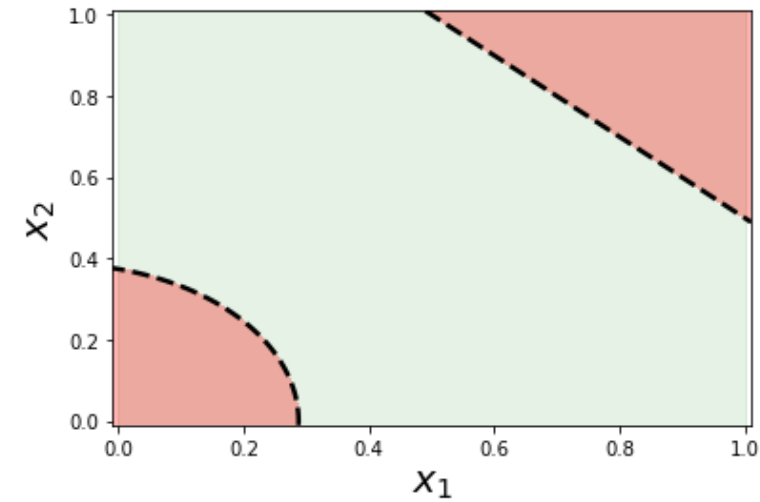
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SECOND PART: LEARNING HIDDEN CONSTRAINTS

Context

● crash of expensive simulator

➔ Learn hidden constraints with limited number of evaluations



PROBLEM STATEMENT

- f : a simulator with inputs $x \in \Omega \subset \mathbb{R}^m$ with simulation failures on Ω
- Our objective: determine the feasible set

$$\Gamma^* = \{x \in \Omega : f(x) \neq \text{NAN}\} = \{x \in \Omega : \mathbb{1}_{f(x) \neq \text{NAN}} = 1\}$$

GAUSSIAN PROCESS BASED CLASSIFICATION

Learning hidden constraint is a binary classification problem

● We have binary observations $(\mathcal{X}, \mathcal{Y}) = (x_j, y_j)_{j=1, \dots, n}$

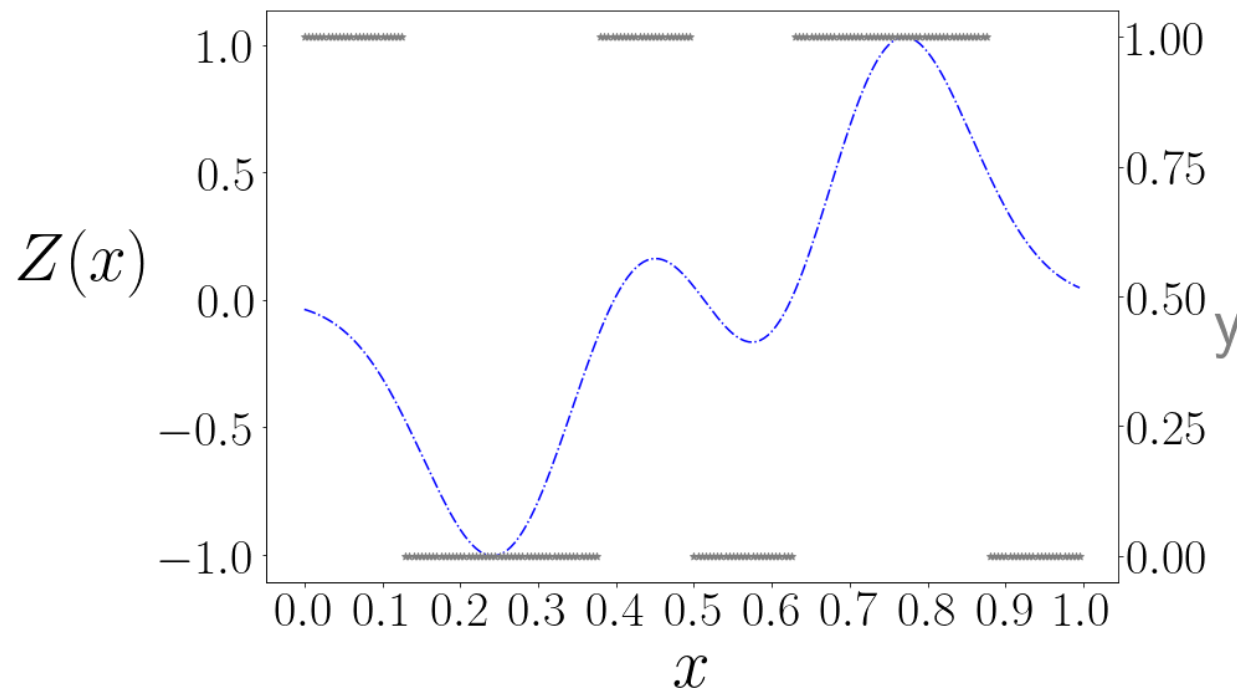
with $y_j = \mathbb{1}_{f(x_j) \neq \text{NAN}}$

● Objective: predict the probability of belonging to the failure/non-failure class

The formulation of the classification model is based on a Gaussian Process (GP) surrogate

GAUSSIAN PROCESS CLASSIFIER (GPC) FORMULATION

- A GPC is based on a latent GP Z conditioned on observations $(\mathcal{X}, \mathcal{Y})$ (as $Z_n = (Z(x_1), \dots, Z(x_n))$ is not available)



$$Z(x) \sim GP(m_n(\cdot), k_n(\cdot, \cdot))$$

$m_n(\cdot), k_n(\cdot, \cdot)$:

conditioned mean and kernel of $Z(x)$

GAUSSIAN PROCESS CLASSIFIER (GPC) FORMULATION

The GPC model allows to predict the probability of non failure of a simulation:

$$p_n(x) = \mathbb{P}[Y_n(x) = 1] = \mathbb{P}[Y(x) = 1 | \mathcal{X}, \mathcal{Y}]$$

This probability $p_n(x)$ is modeled on the basis of [Bachoc et al., 2020] by using the sign of the latent process Z :

$$p_n(x) = \mathbb{P}[\mathbb{1}_{Z(x) > 0} = 1 | x, \mathcal{X}, \mathcal{Y}] = \int_{\mathbb{R}^n} \phi_{\mathcal{Y}}^{Z_n}(z_n) \bar{\Phi}\left(\frac{-m_n(x, z_n)}{\sqrt{k_n(x)}}\right) dz_n$$

with $\phi_{\mathcal{Y}}^{Z_n}(z_n)$ the conditioned p.d.f of Z_n truncated to respect $\text{sign}(Z_n) = \mathcal{Y}$, and:

$$\bar{\Phi}\left(\frac{a}{b}\right) = \begin{cases} 1 - \Phi\left(\frac{a}{b}\right) & \text{si } b \neq 0 \\ \mathbb{1}_{-a > 0} & \text{si } b = 0 \end{cases}$$

where Φ is the c.d.f. of the normal standard distribution.

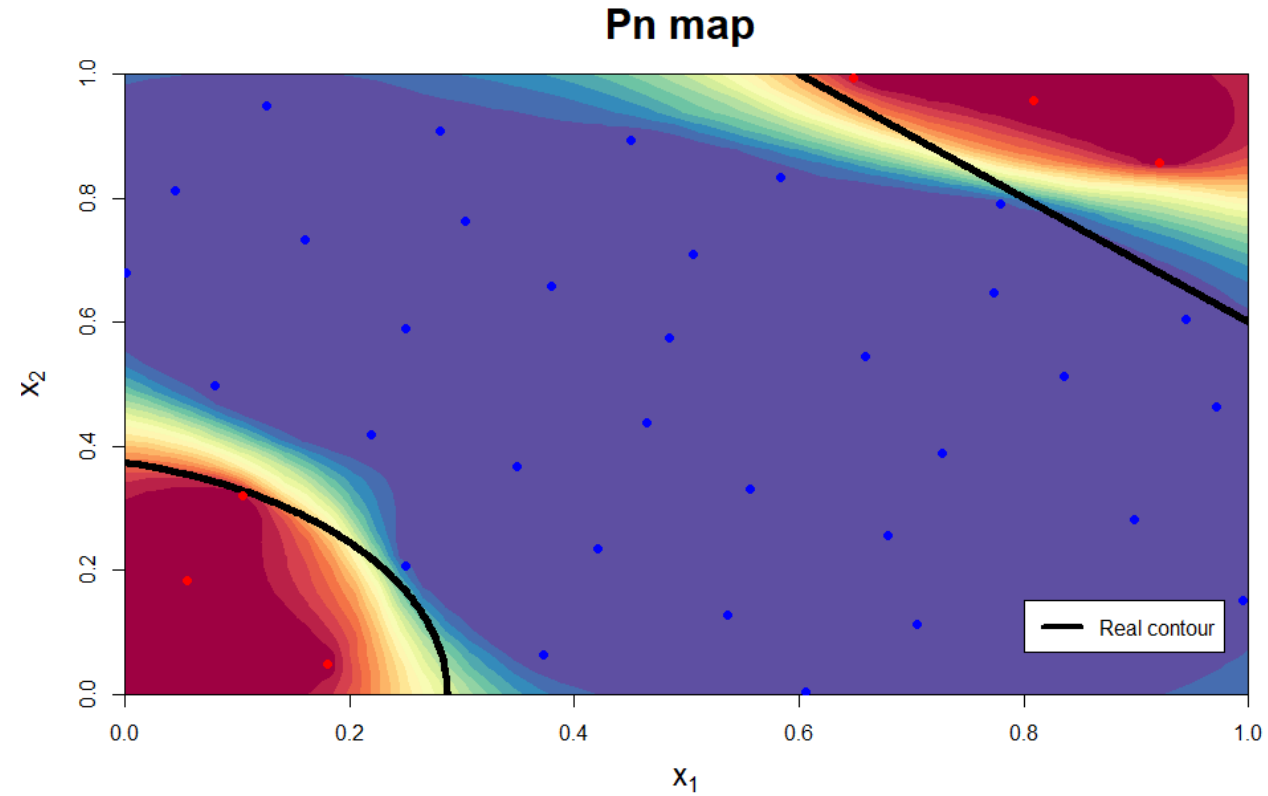
GAUSSIAN PROCESS CLASSIFIER (GPC) FORMULATION

Practical building of the GPC model $p_n(x)$ for any x :

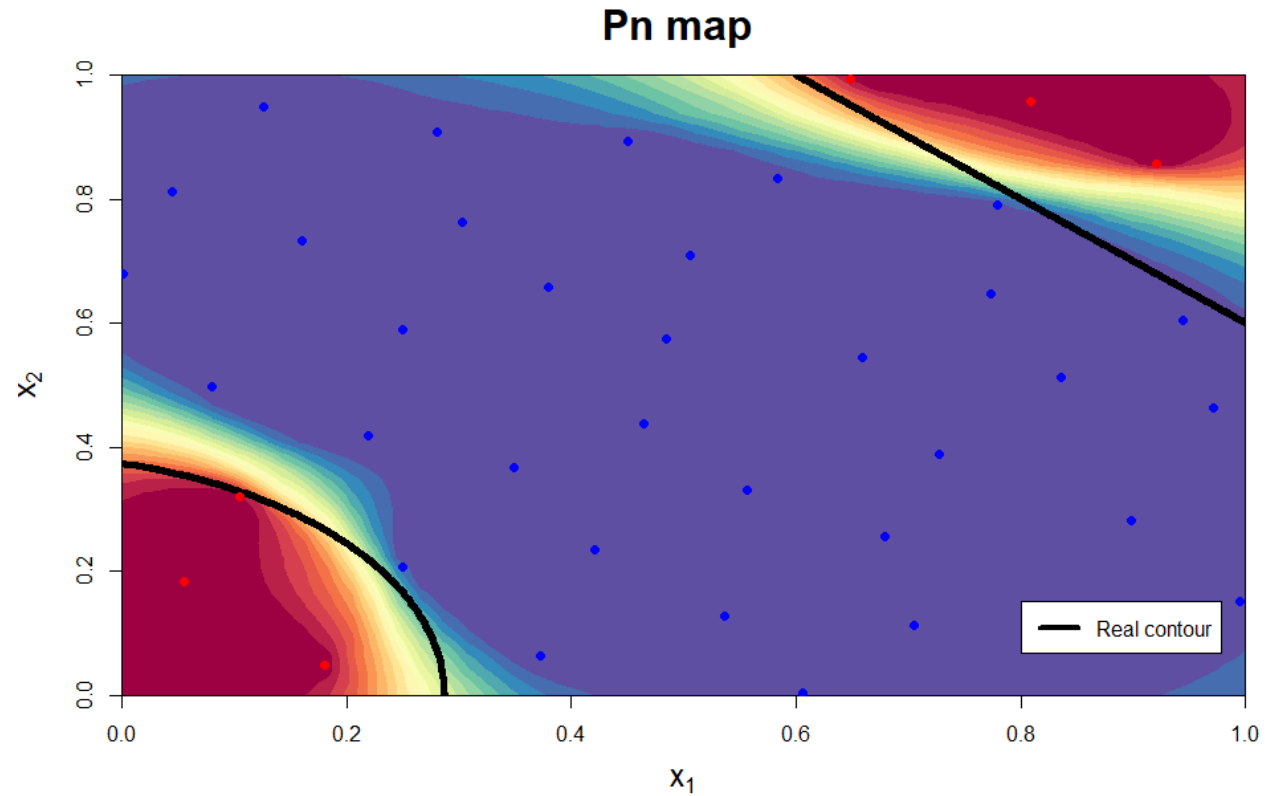
- Optimization of the hyperparameters of the latent GP to maximize the likelihood:
 $\mathbb{P}[\text{sign}(Z_n) = \mathcal{Y}]$
- Generation of realizations $z_n^{(i)}$ of $Z_n | \text{sign}(Z_n) = \mathcal{Y}$
→ Approximation of $p_n(x)$:

$$\hat{p}_n(x) = \frac{1}{N} \sum_{i=1}^N \bar{\Phi} \left(\frac{-m_n(x, z_n^{(i)})}{\sqrt{k_n(x)}} \right)$$

EXAMPLE OF A GPC FOR HIDDEN CONSTRAINT



EXAMPLE OF A GPC FOR HIDDEN CONSTRAINT



- Characterization of the feasible set by quantiles

$$Q_\alpha = \{x \in \Omega : p_n(x) \geq \alpha\}, \alpha \in (0, 1]$$

STEPWISE UNCERTAINTY REDUCTION STRATEGY

The *Stepwise Uncertainty Reduction* strategy based on the uncertainty defined by the **vorob'ev deviation** $Var_n(\Gamma)$ [Chevalier, 2013, El Amri et al., 2021, Vorobyev and Lukyanova, 2013] is based on the following learning criterion:

$$J_n(x_{n+1}) = \mathbb{E}_n[Var_{n+1}(\Gamma)]$$
$$J_n(x_{n+1}) = \mathbb{E}_{Z(x_{n+1})} \left[\int (1 - p_{n+1}(x)) \mathbb{1}_{p_{n+1}(x) \geq \alpha^*} \mu(dx) + \int p_{n+1}(x) \mathbb{1}_{p_{n+1}(x) < \alpha^*} \mu(dx) \right]$$

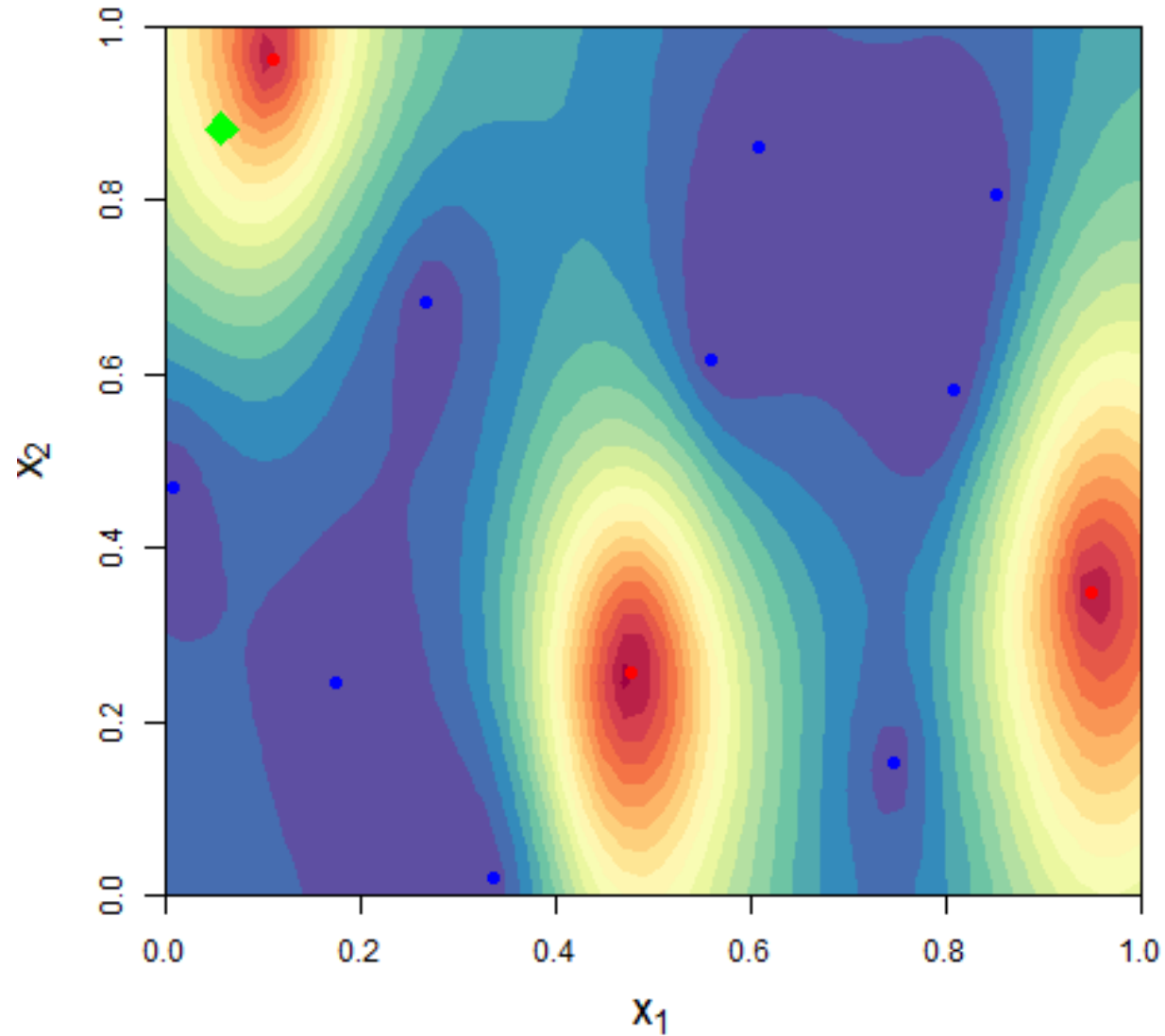
This expression can be developed using the expression of $p_n(x)$ given for the GPC model [Bachoc et al., 2020] and GP update formulae provided in [Chevalier, 2013]

- To reduce the computational time, we can get rid of the integration w.r.t. the realizations of the latent GP, using the conditional Bernoulli process $Y_n(x) = Y(x)|\mathcal{X}$, $\mathbf{Y}_n = \mathcal{Y}$

→ **ARCHISSUR criterion: Active Recovery of Constrained and Hidden Subset by SUR**

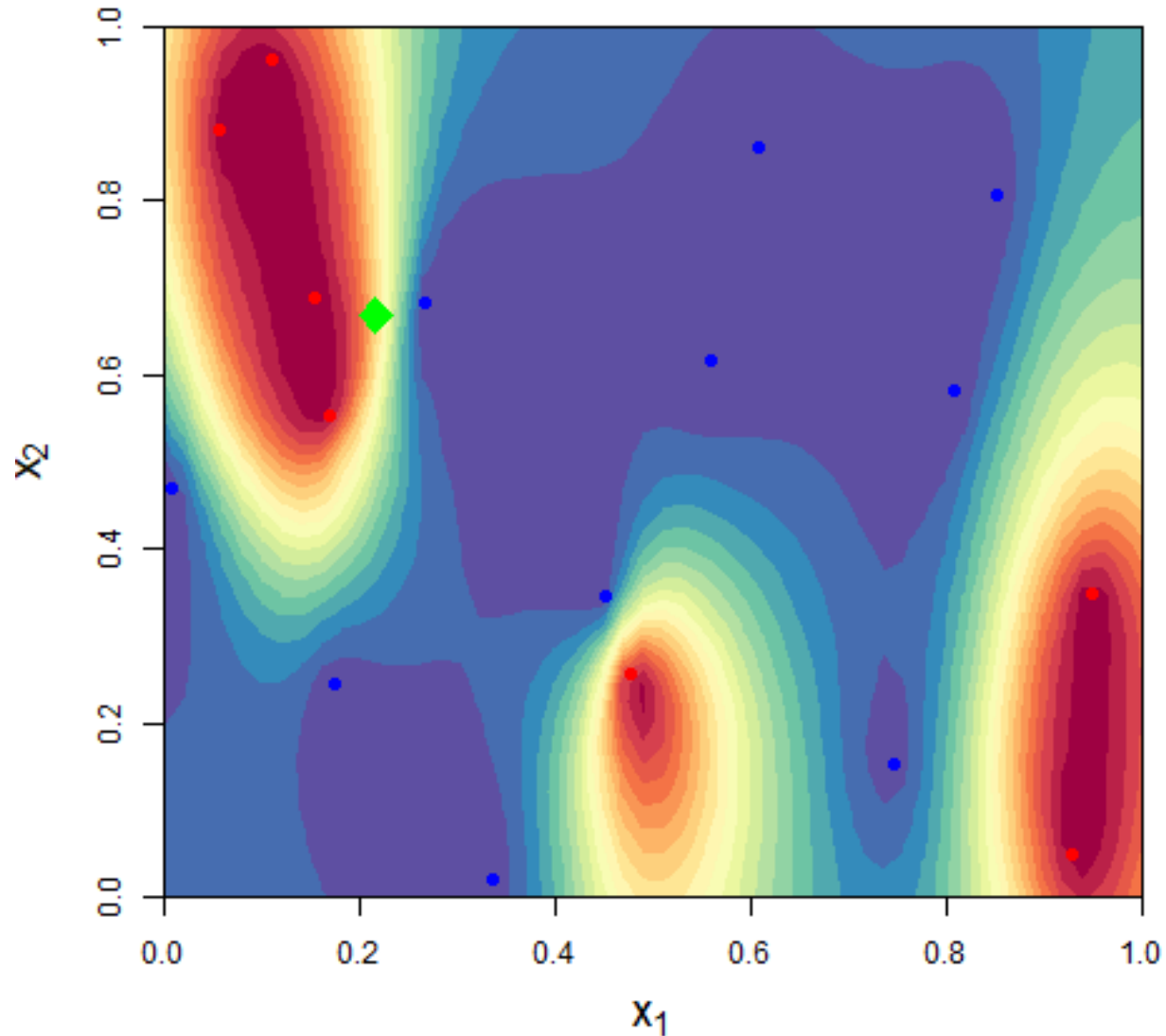
ARCHISSUR ON A 2D EXAMPLE (Branin function)

Pn map - iteration 0



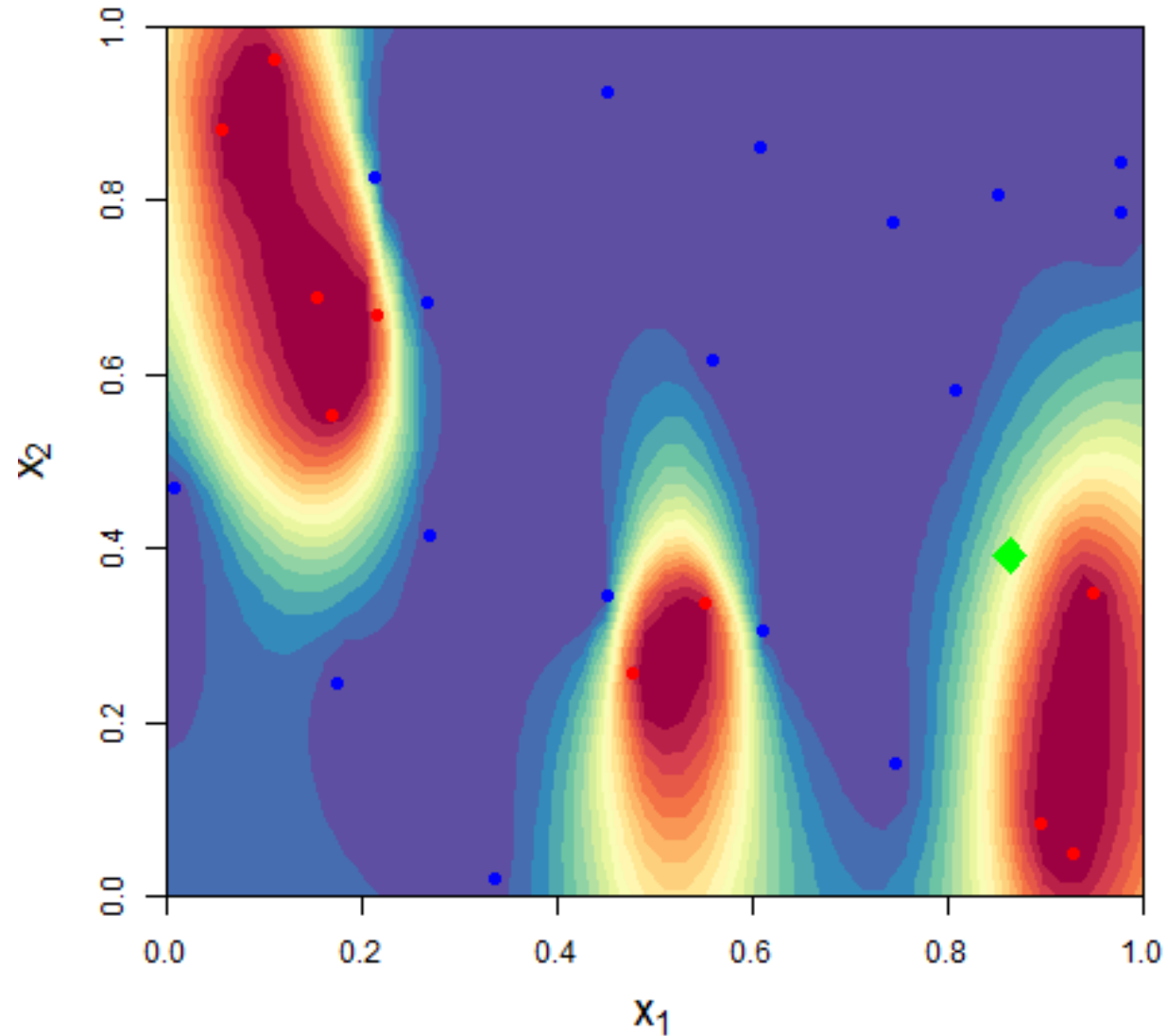
ARCHISSUR ON A 2D EXAMPLE (Branin function)

Pn map - iteration 5



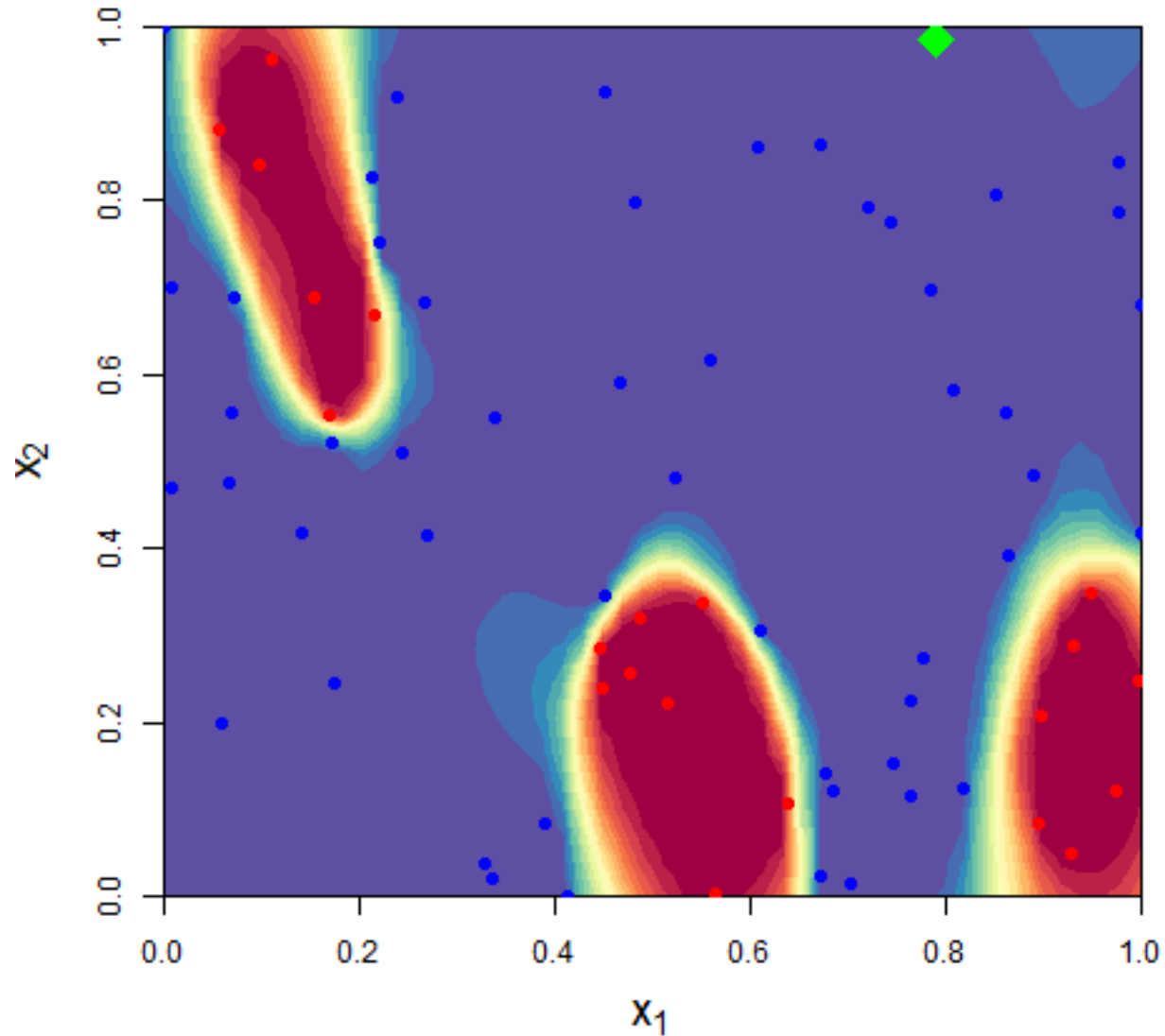
ARCHISSUR ON A 2D EXAMPLE (Branin function)

Pn map - iteration 15



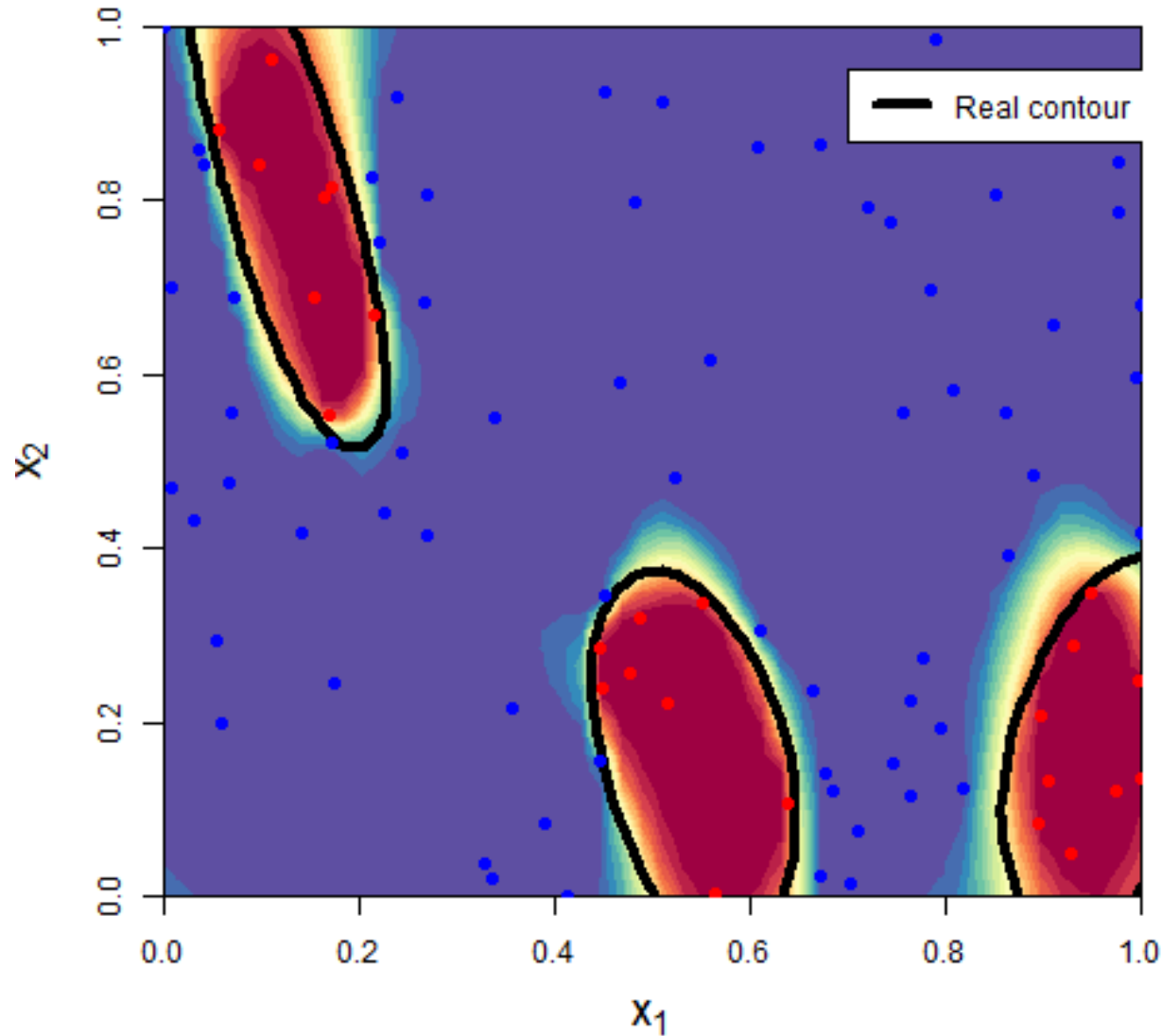
ARCHISSUR ON A 2D EXAMPLE (Branin function)

Pn map - iteration 60



ARCHISSUR ON A 2D EXAMPLE (Branin function)

Pn map - iteration 80



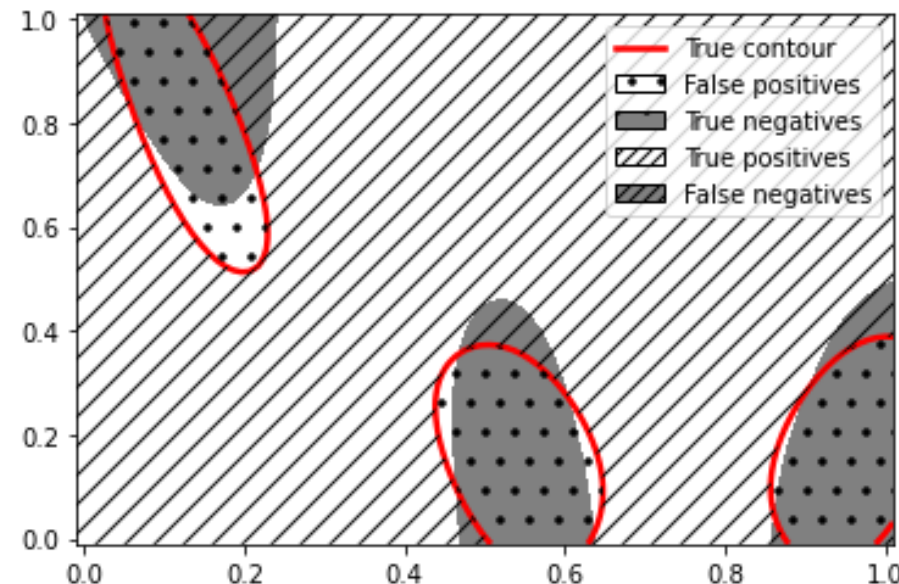
COMPARISON OF DIFFERENT ENRICHMENT CRITERIA

Compared strategies

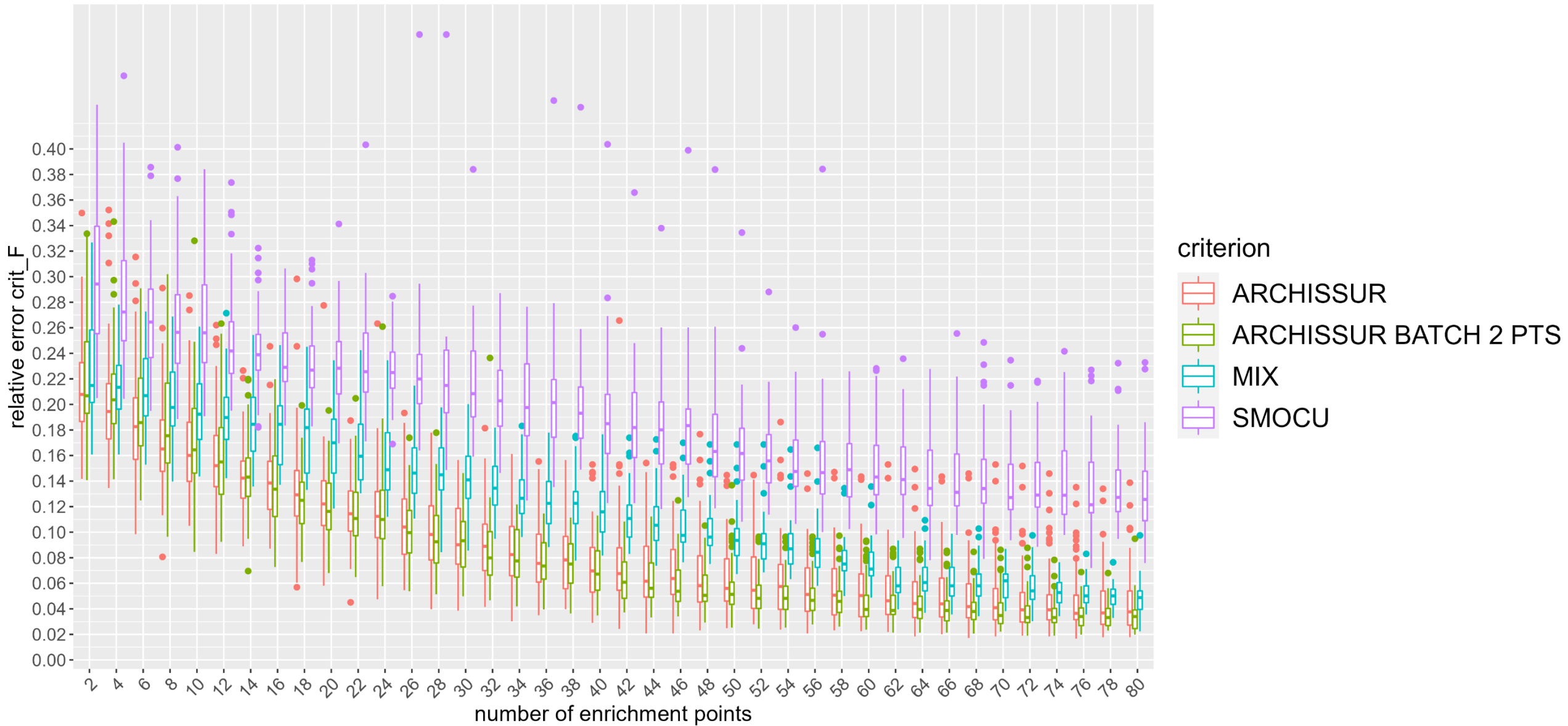
- ARCHISSUR criterion: *Active Recovery of Constrained and Hidden Subset by SUR*
- Mixed enrichment criterion: add the point corresponding to the maximum of the GP variance (exploration) and the one where $p_n(x)$ value is the closest to $\frac{1}{2}$ (exploitation) simultaneously
- SMOCU enrichment measure: Soft-MOCU (Mean Objective Cost of Uncertainty) method [Zhao et al., 2021]

Comparison criterion

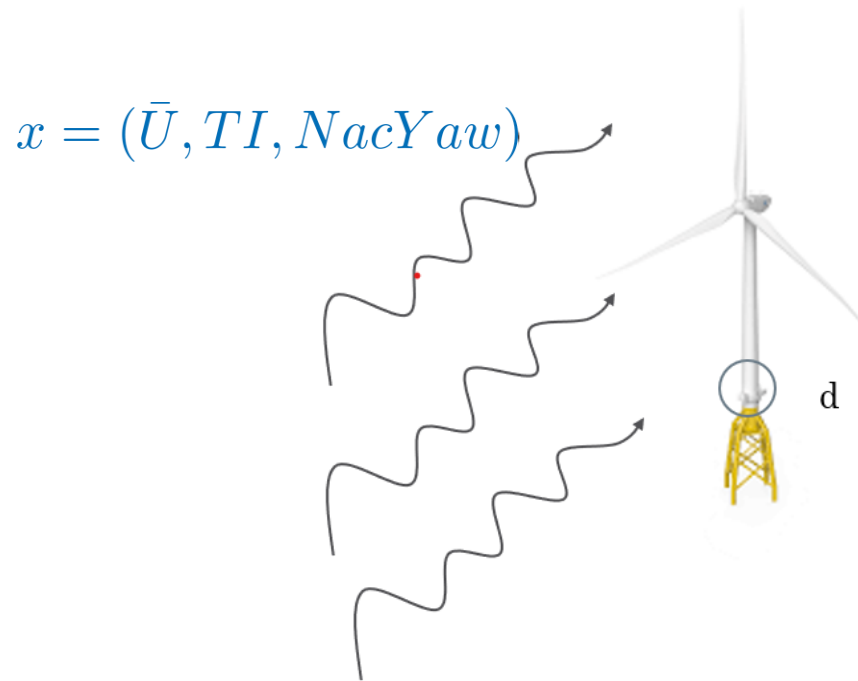
$$\frac{\mu(\Gamma^* \Delta Q_{\alpha^*})}{\mu(\Gamma^*)} = \frac{FN + FP}{TP + FN}$$



RESULTS ON BRANIN FUNCTION (2D)



RESULTS FOR DAMAGE PREDICTION OF A WIND TURBINE



Wind turbine subject to wind loads described by 3 parameters:
 \bar{U} mean of wind speed (10mn), TI turbulence intensity,
 $NacYaw$ misalignment angle

TurbSim to simulate multiple realizations
($\bar{U}, TI, NacYaw$)

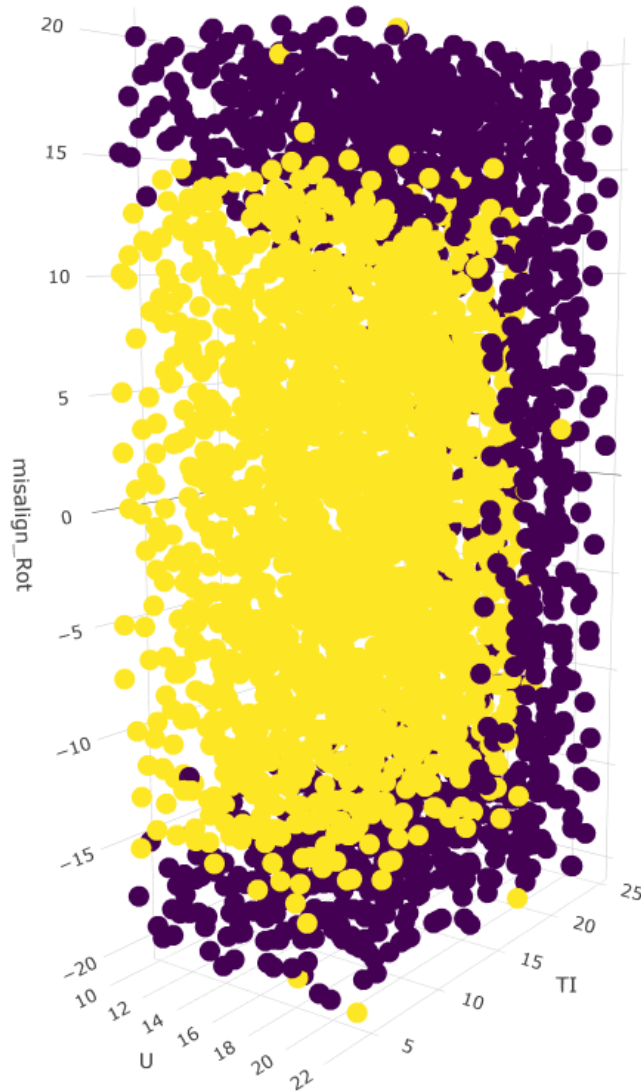
+

FAST simulator
+ Python scripts

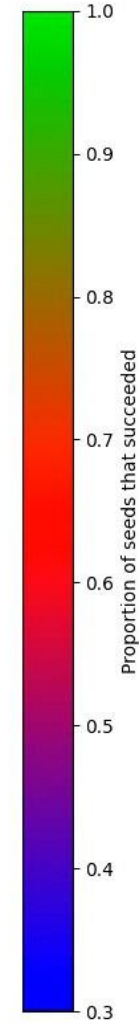
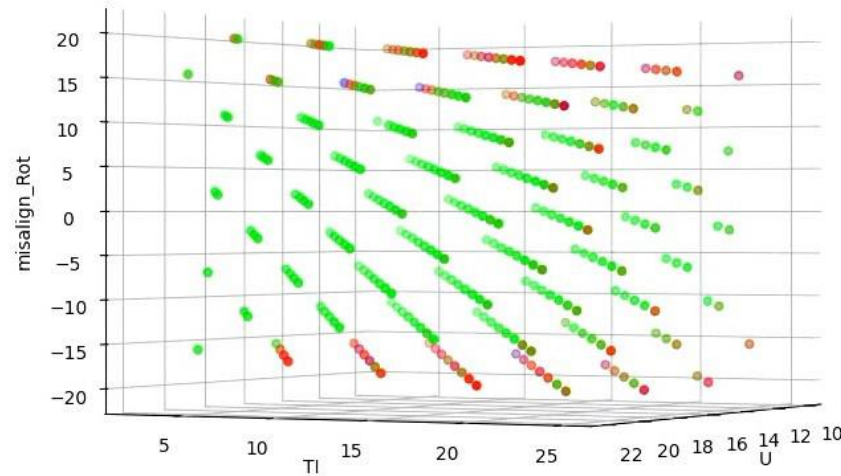
↓

Predictions of damage at the bottom of the tower

RESULTS FOR DAMAGE PREDICTION OF A WIND TURBINE



Scatterplot of experiment plan - 299 points
Proportion of seeds that succeeded, among the 20 tested at each point



TurbSim to simulate multiple realizations (\bar{U} , TI , $NacYaw$)

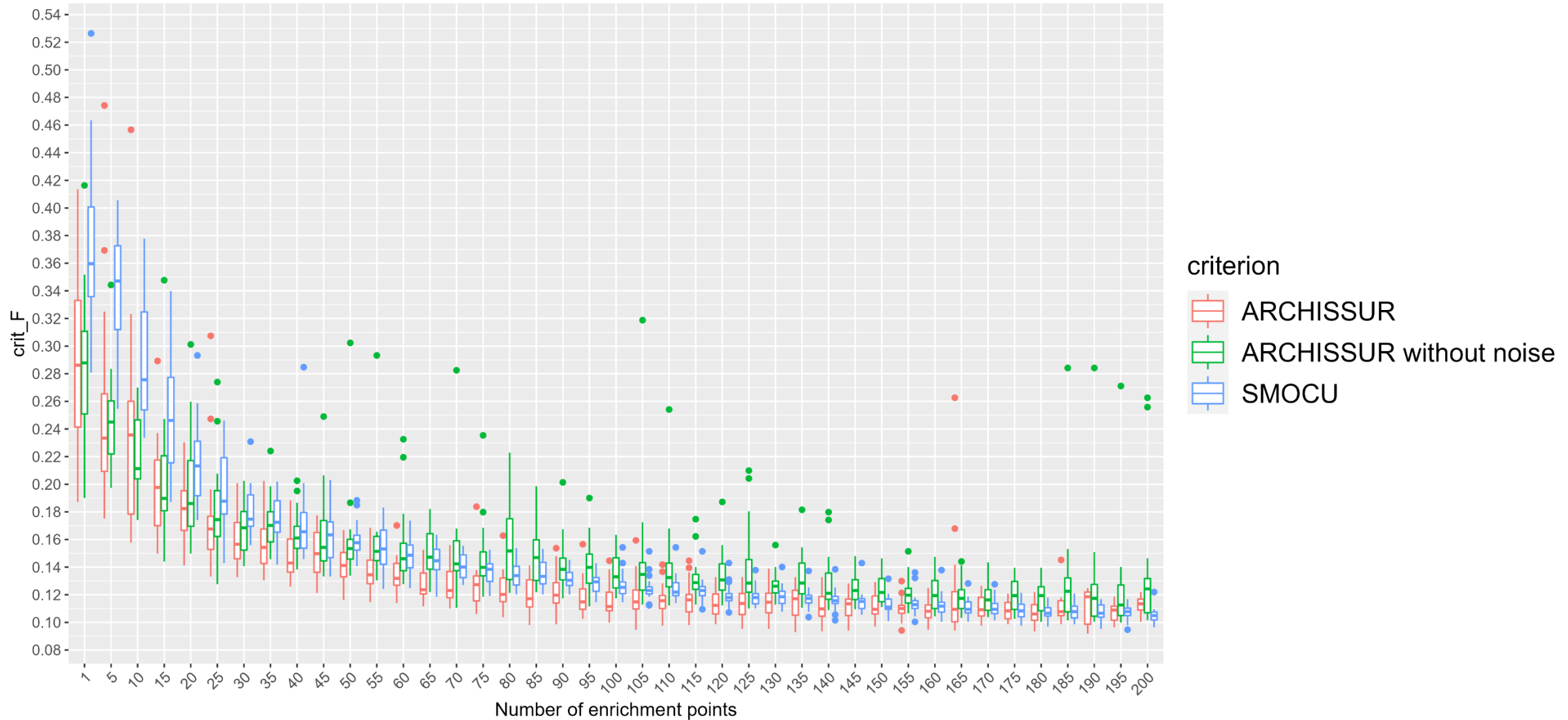


FAST simulator + Python scripts

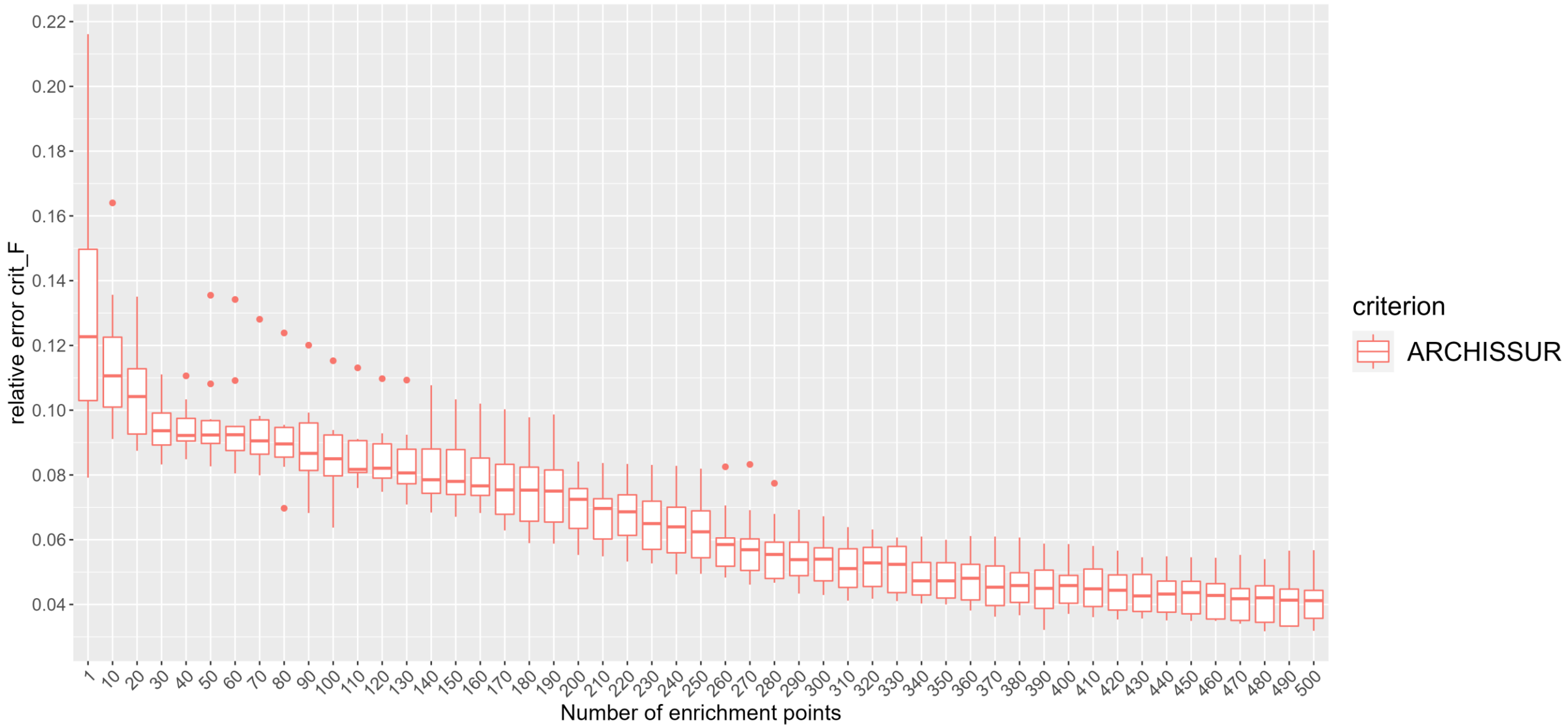


Predictions of damage at the bottom of the tower

RESULTS FOR DAMAGE PREDICTION OF A WIND TURBINE



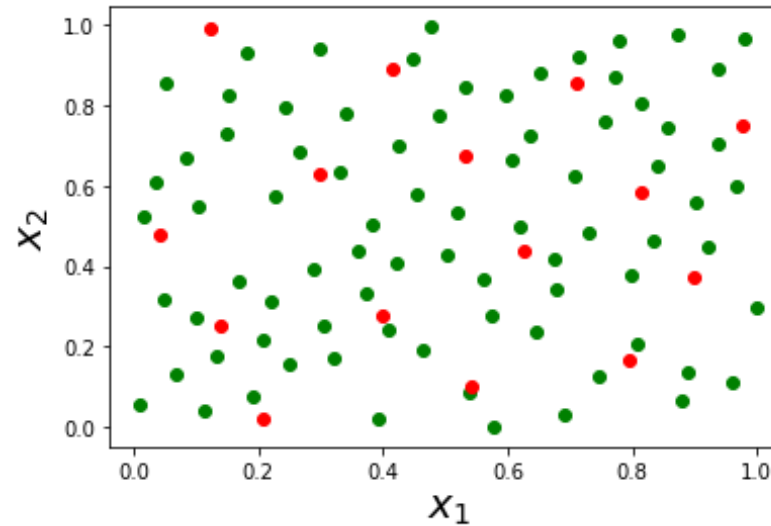
RESULTS ON A 10D FUNCTION



CONCLUSIONS

- Archissur has a good potential to learn non-convex feasible sets regarding crash constraints
- Optimization of the computation time for the criterion

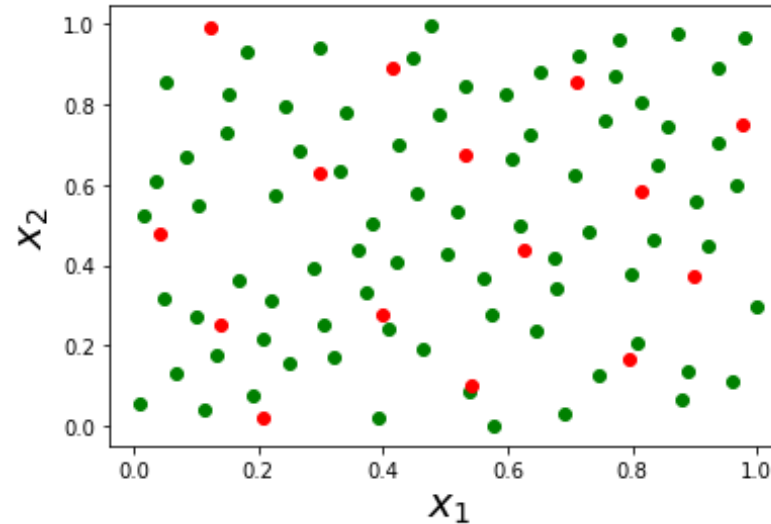
But Archissur is not suited for



CONCLUSIONS

- Archissur has a good potential to learn non-convex feasible sets regarding crash constraints
- Optimization of the computation time for the criterion

But Archissur is not suited for



Perspectives

- Coupling with optimization: Bayesian optimization and Direct Search methods
 - ANR Samourai (with Polytechnique Montréal)
- More than 2 classes: multiple robustness or convergence levels

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