DE LA RECHERCHE À L'INDUSTRIE

www.cea.fr

# Particle MCMC for Bayesian microwave control

*P. Minvielle-Larrousse* [1] N. Malléjac [2]
A. Todeschini [3] F. Caron [3,4]
P. Del Moral [3,5]

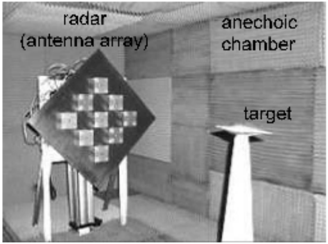
[1]**CEA-DAM**/CESTA [2]**CEA-DAM**/LR
[3]**INRIA Bx Sud-Ouest** [4]Univ. of Oxford
[5]UNSW

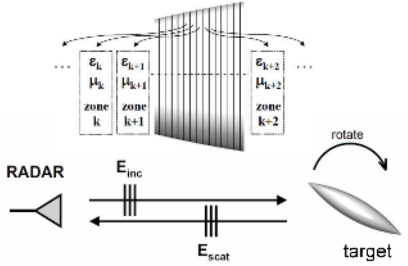
*Forum Incertitudes DAM - 1 oct 2014*

# Outline

# Goal : microwave control from global scattering

- **Goal :** ***estimate/check local electromagnetic material properties***[1] (i.e. permitivity ($\epsilon$) and permeability ($\mu$)) and ***"associated uncertainties", from global scattering measurements***.



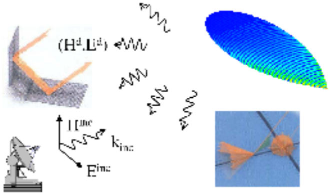




- **RCS measurement setup :** a Stepped Frequency Continuous Wave (SFCW) radar measuring complex scattering coefficents at $K_f$ frequencies, $K_\theta$ angles and *HH*/*VV* linear polarizations.

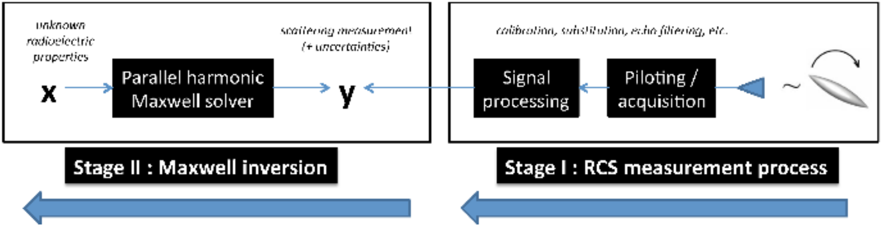
1. The materials are assembled and placed on the full-scaled object or system

# High dimensional ill-posed inverse problem

- Stage I : RCS measurement process
  - SFCW piloting/acquisition : motorized rotating positioning system, network analyser, etc.
  - Signal processing : calibration, substitution, echo filtering, etc.

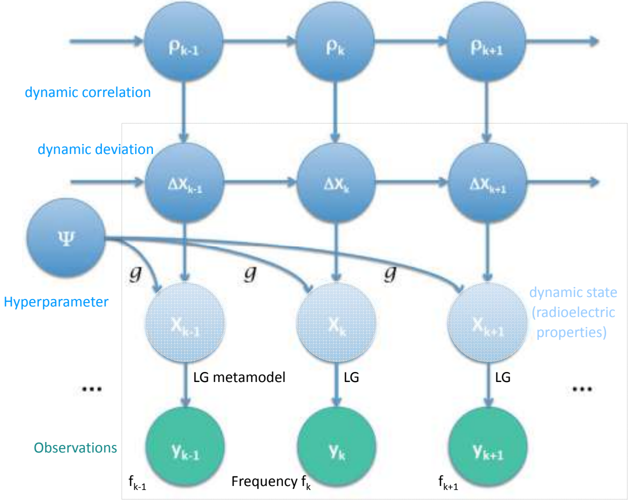


- **Stage II : go upstream a parallelized harmonic Maxwell solver** (volume finite element/integral equation) [SLMBS91] (i.e. numerical solution of Maxwell's equations in free space)[2].




2. Object shape perfectly known.

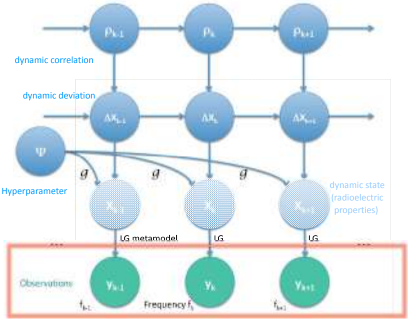
# Statistical modeling : A general HMM




A general Hidden Markov Model

# A general HMM : Observation

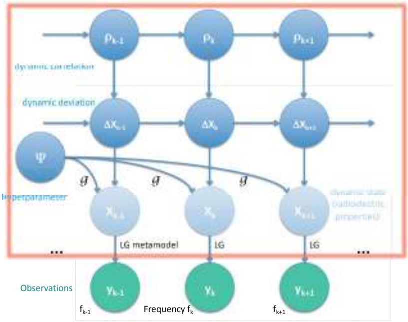
**Observation** : $\mathbf{y_k} = [\cdots]^T$, composed of the **complex scattering coefficients**, measured *at frequency* $f_k$ for various rotation angles $\theta_1, \cdots, \theta_{K_\theta}$ and polarizations $\{HH, VV\}$.



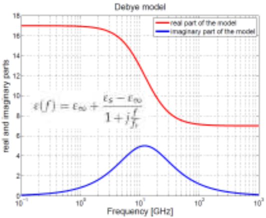
# A general HMM : State

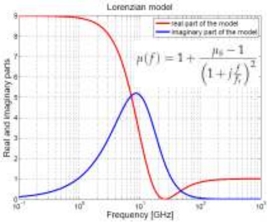
**Dynamic state** : $\mathbf{x_k} = \begin{bmatrix} \underline{\epsilon}'_{\mathbf{k}} & \underline{\epsilon}''_{\mathbf{k}} & \underline{\mu}'_{\mathbf{k}} & \underline{\mu}''_{\mathbf{k}} \end{bmatrix}^T$, composed of the real ($'$) and imaginary ($''$) **material property components** *at frequency* $f_k$ ($N$ elementary areas),

$$\mathbf{x}_k = \underbrace{g(f_k, \Psi)}_{\text{material model}} + \underbrace{\Delta\mathbf{x}_k}_{\text{deviation}}$$



- **Parametric material model** : $g(f_k, \boldsymbol{\Psi})$ ($\sum$ Debye relaxation/Lorenzian resonant terms [BJGD92]) $\rightarrow$ hyperparameter $\boldsymbol{\Psi}$.




- **Spatial/frequency deviation (from model)** : $\Delta\mathbf{x}_k \sim$ AR(1) (with spatial correlation [GMM13]) given frequential correlation $\rho_k$ ($\sim$ random walk) :

$$\Delta\mathbf{x}_1 \sim \mathcal{N}(\mathbf{0}, \mathbf{P}_1) \quad \text{and} \quad \Delta\mathbf{x}_{k+1} = \mathbf{M}_k^{\rho} \cdot \Delta\mathbf{x}_k + \mathbf{w}_k$$

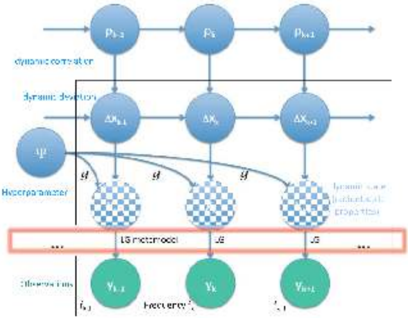
where $\mathbf{w}_k$ : Gaussian noise ($\mathbb{E}(\mathbf{w}_k) \neq 0$).

# A general HMM : Likelihood (Meta)model

**(Approximate) Likelihood model : Linear Gaussian (LG)**

- **Intensive training** (Maxwell solver computations *at each frequency* $f_k$ on a **petaflopic supercomputer**).
- **Valid in a limited check subdomain of the state space** around the expected values [GMM13].

$$\mathbf{y}_k \approx \left[\mathbf{A}_k \cdot \mathbf{x}_k + \mathbf{y}_k^0\right] + \mathbf{v}_k,\ \mathbf{v}_k \sim \mathcal{N}(0, \mathbf{R_k})$$



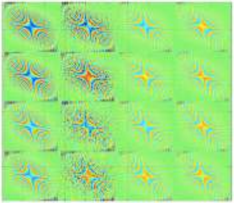



Illustration : matrix $\mathbf{A}_k$
(dim : $400 \times 200$) for $f_k = 2$ GHz

# Sequential Bayesian analysis

**The problem** : **estimation on a general state-space model**

- **Static & dynamic estimation** : frequent in econometrics, robotics, telecommunications, etc.
- **Here : estimate fixed hyperparameter $\Psi$, dynamic state $\Delta \mathbf{x}_1, \cdots, \Delta \mathbf{x}_K$ and dynamic correlations $\rho_1, \cdots, \rho_K$.**

**Bayesian computational approach** : **Particle Markov Chain Monte Carlo**

- **Emerging class of techniques** in signal processing & Bayesian statistics, that combine MCMC & SMC samplers [ADH10].
- **Central idea** : "exact approximations" of idealized MCMC algorithms, targeting either **the joint or marginal posterior distributions**.

# Particle Marginal Metropolis-Hastings with Interacting Kalman filters

## PMCMC choice : Particle Marginal Metropolis Hastings

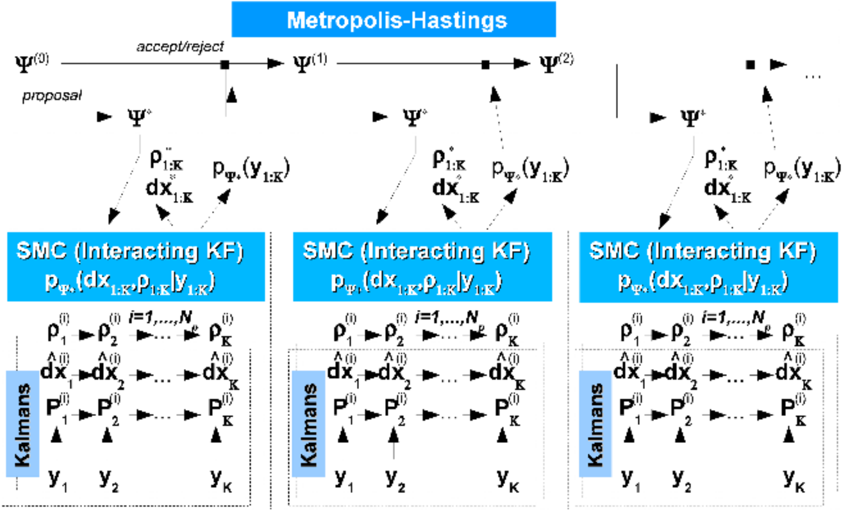
- Approximation of the ideal MMH (Marginal Metropolis Hastings).
- Targets the *joint posterior distribution* $p(\boldsymbol{\Psi}, \Delta \mathbf{x}_{1:K}, \rho_{1:K} | \mathbf{y}_{1:\mathbf{K}})$, invariant distribution of PMMH Markov chain.

## Design : a "multilevel" stochastic algorithm

1. **Upper level** : *Markov Chain Monte Carlo* (Metropolis-Hastings)
   - *Sample* $\boldsymbol{\Psi}^\star$
2. **Lower level** : *Sequential Monte Carlo*
   - ***Rao-Blackwellization*** **(conditionally Linear Gaussian HMM structure)** $\rightarrow$ bank of **Interacting Kalman filters** [DGA00]
   - Compute marginal likelihood $p(\mathbf{y}_{1:\mathbf{K}} | \boldsymbol{\Psi}^\star)$ and *sample* $(\Delta \mathbf{x}^\star_{1:K}, \rho^\star_{1:K}) \sim p_{\boldsymbol{\Psi}^\star}(\cdot | \mathbf{y}_{1:\mathbf{K}})$.

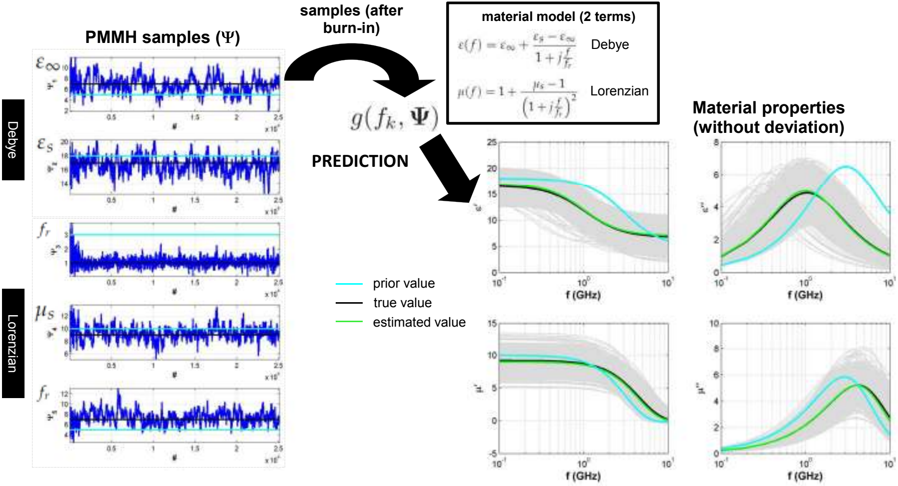
**Implementation** : adaptive MH [PHH10], tempering phase, kernel mixture, SMC (SIR)

# PMMH with Interacting Kalman Filters



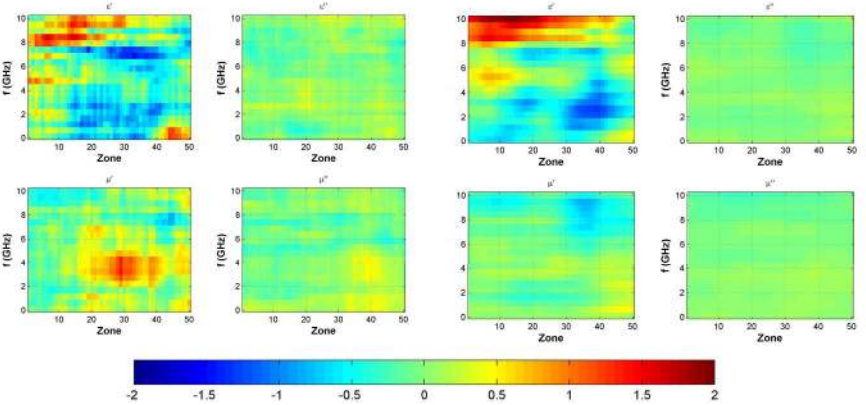
# Illustration (synthetic data)

$K = 20$ ($f \in$ [0.1-12.4 GHz]), $K_\theta = 100$ ($\theta \in [0^\circ - 180^\circ]$), dim($\mathbf{x_k}$)=$50 \times 4$, dim($\mathbf{y_k}$)=$100 \times 4$, **Adaptive PMMH** [$\sim$ 100 “Kalman” particles].



# Illustration : dynamic part (deviation)

**True deviation** **Estimated deviation**



# Conclusion

- **Bayesian inference for material microwave control**.
- Particle MCMC simultaneously **estimates model parameters** and **spatial/frequency deviations**, and **"associated uncertainties"**.
- **Computationally intensive** : **HPC** and/or **high dimension approximations** $\rightarrow$ PMMH with Interacting Ensemble Kalman Filters.

References :


- **P. Minvielle, A. Todeschini, F. Caron and P. Del Moral, Particle MCMC for Bayesian Microwave Control, arXiv 1405.2673, 14/05/2014**
- [GMM13] F. Giraud, P. Minvielle and P. Del Moral, Advanced interacting sequential Monte Carlo sampling for inverse scattering, IOP Inverse Problems, vol. 29(9), 2013
- F. Giraud, PhD *Analyse des modèles particulaires de Feynman-Kac et application à la résolution de problèmes inverses en électromagnétisme*, 29/05/2013

# References


- Christophe Andrieu, Arnaud Doucet, and Roman Holenstein.
  Particle markov chain monte carlo methods.
  *Journal of the Royal Statistical Society : Series B (Statistical Methodology)*, 72(3) :269–342, 2010.
- James Baker-Jarvis, Richard G Geyer, and Paul D Domich.
  A nonlinear least-squares solution with causality constraints applied to transmission line permittivity and permeability determination.
  *Instrumentation and Measurement, IEEE Transactions on*, 41(5) :646–652, 1992.
- Arnaud Doucet, Simon Godsill, and Christophe Andrieu.
  On sequential monte carlo sampling methods for bayesian filtering.
  *Statistics and computing*, 10(3) :197–208, 2000.
- F Giraud, P Minvielle, and P Del Moral.
  Advanced interacting sequential monte carlo sampling for inverse scattering.
  *Inverse Problems*, 29(9) :095014, 2013.
- Gareth W Peters, Geoff R Hosack, and Keith R Hayes.
  Ecological non-linear state space model selection via adaptive particle markov chain monte carlo (adpmcmc).