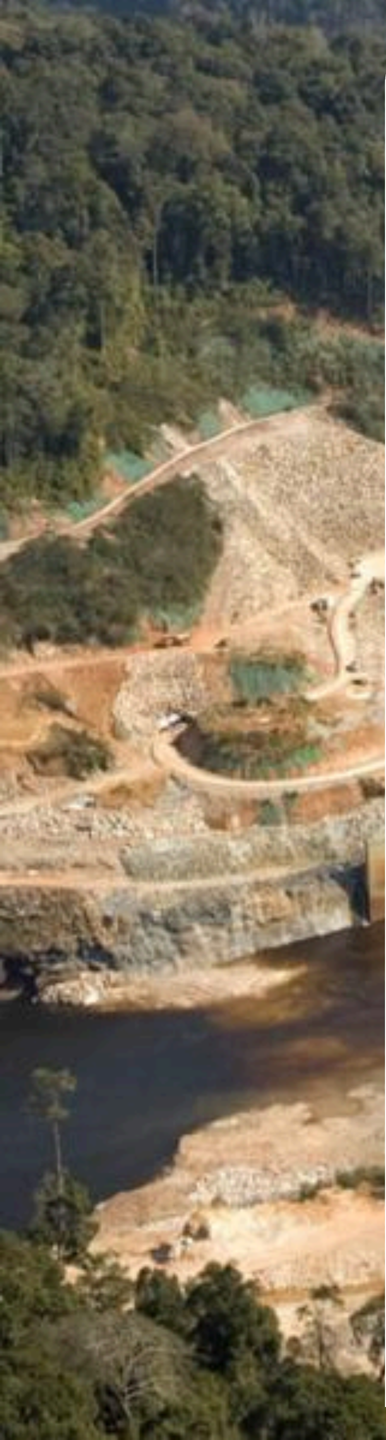

Introduction to sensitivity analysis

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Kolkata, December, 2018

Using « Best-estimate » computer codes in engineering

Exploratory studies: understand a phenomenon, an experim/indust. process

Safety studies: compute a failure risk (margins, rare events) and prioritize the risk indicators, with validated computer models

Design studies: optimize and manage the system performances

Uncertainties

- **Environnemental variables**
- **Physical parameters**
- **Parameters of process**

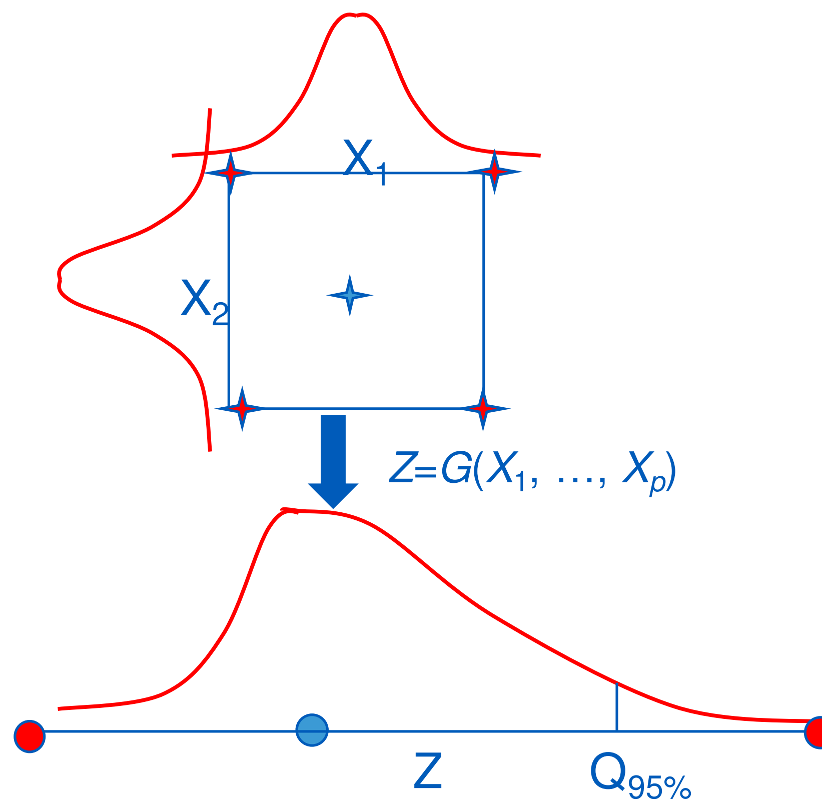
Design of experiments

Computer code
or
Experiment

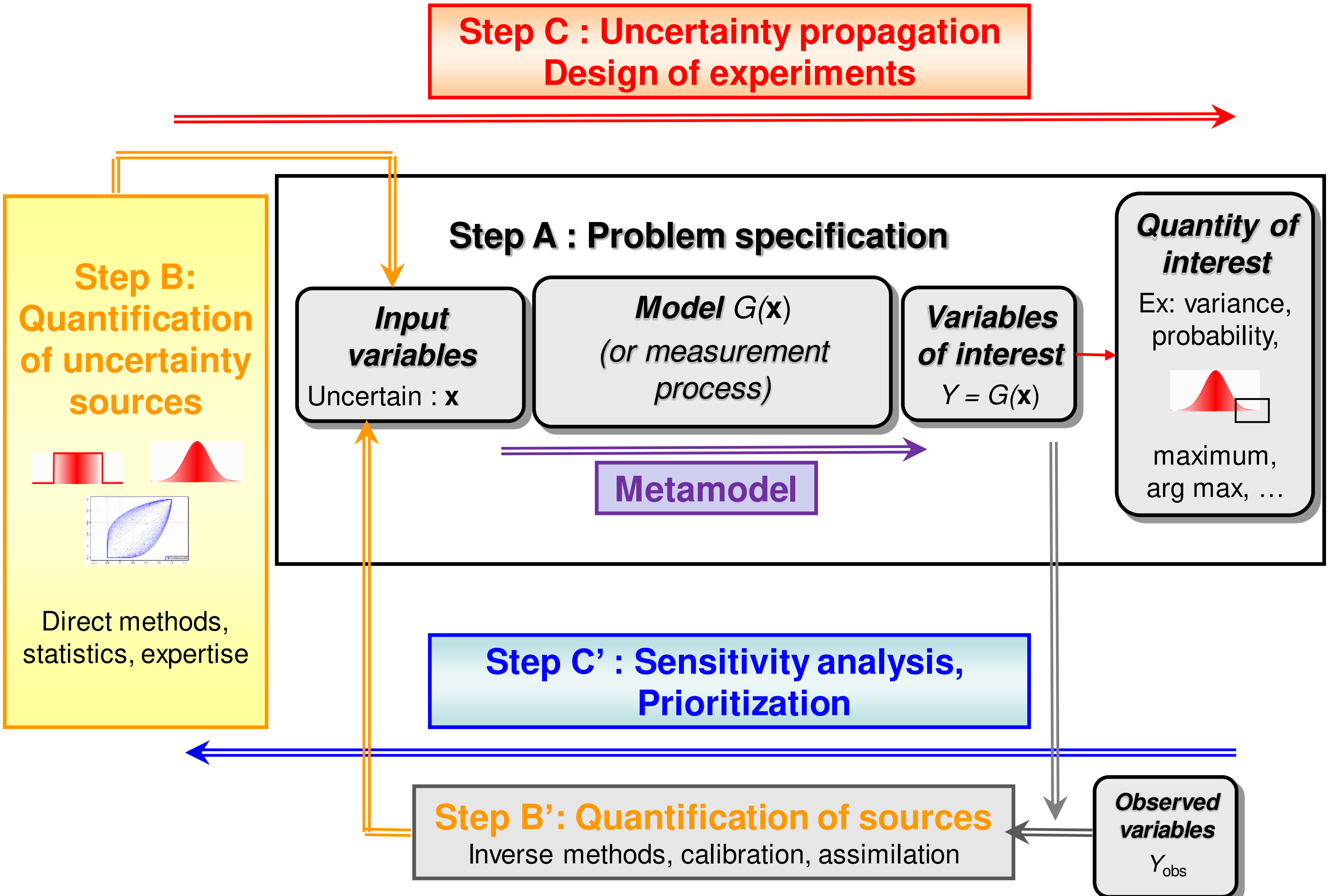
Cost ($€, cpu, …) potentially large

Possible needs of metamodel

- **Distributions of outputs**
- **Probability of failure**
- **Most influent inputs**
- **Calibrating the input parameters**



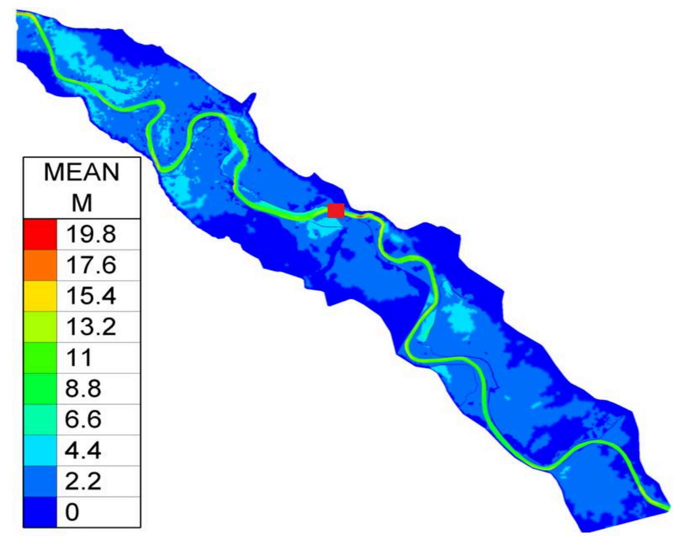
UQ methodology

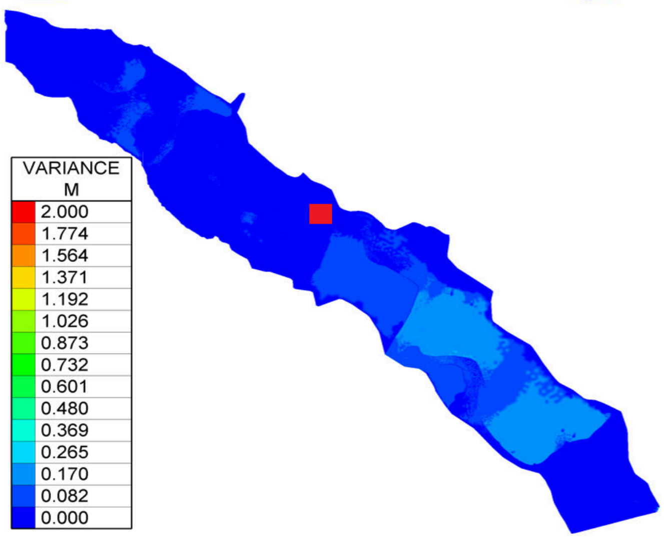


Example: Garonne flood risk (TELEMAC code
Uncertainty analysis

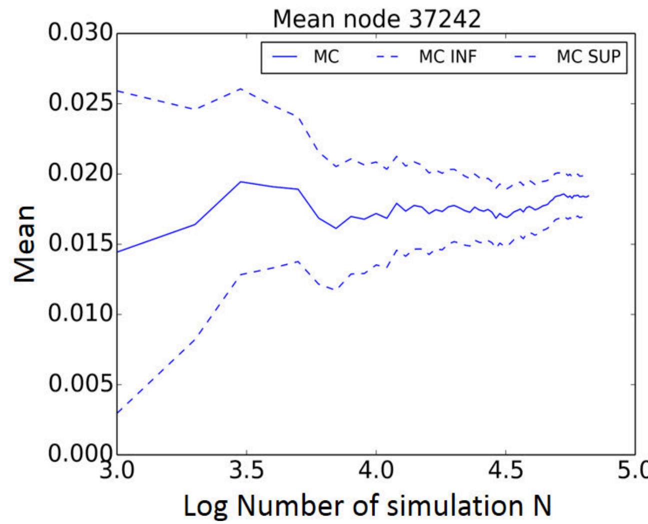
[Goeury et al., 2015]

Global analysis

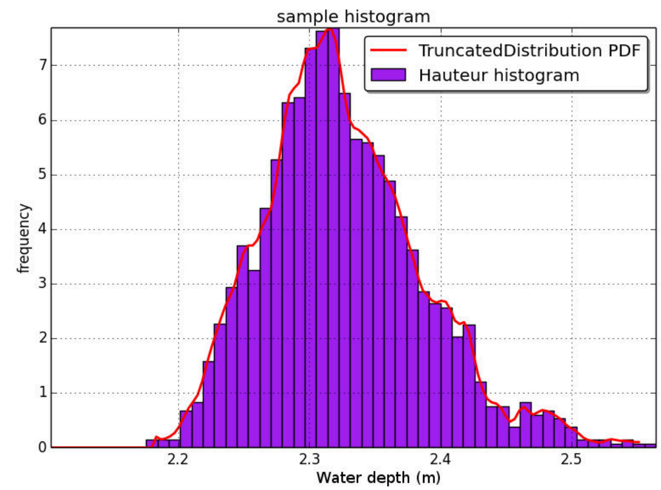




local analysis



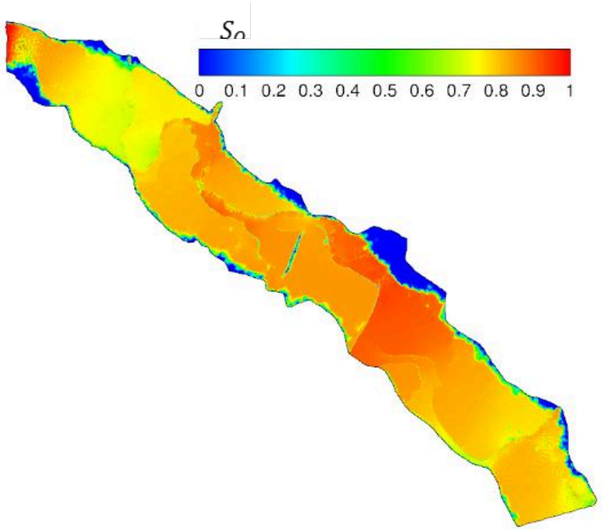
Convergence and confidence intervals at 95 % of the estimated mean

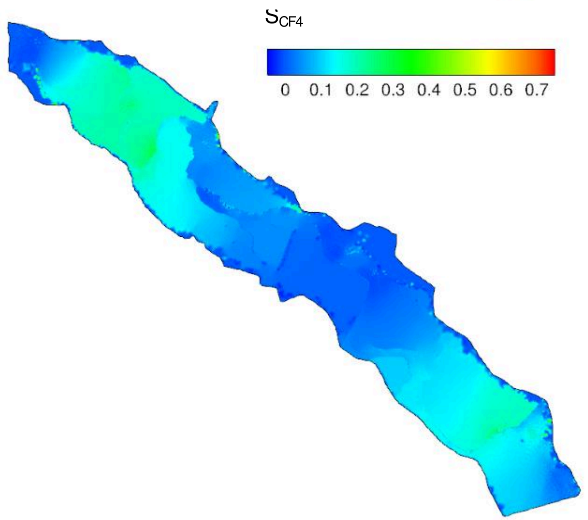


Empirical PDF based on 70 000 simulations

Industrial example: Garonne flood risk
Sensitivity analysis

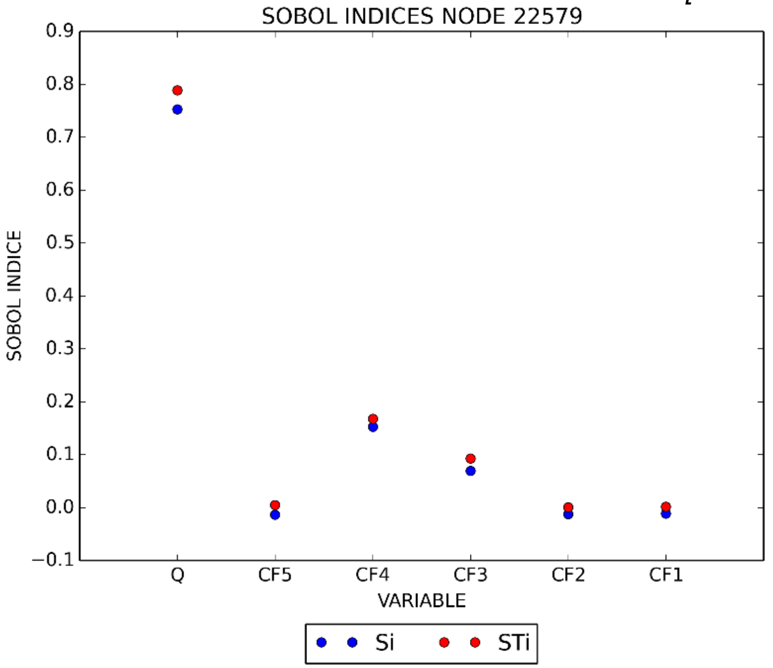
Global analysis :
First order indices of Sobol





local analysis

[*Goeury et al., 2015*]



- The flowrate input factor explains about 80% of the variance of the output variable
- Few interactions between the uncertain variables

Sensitivity analysis notions

The model :

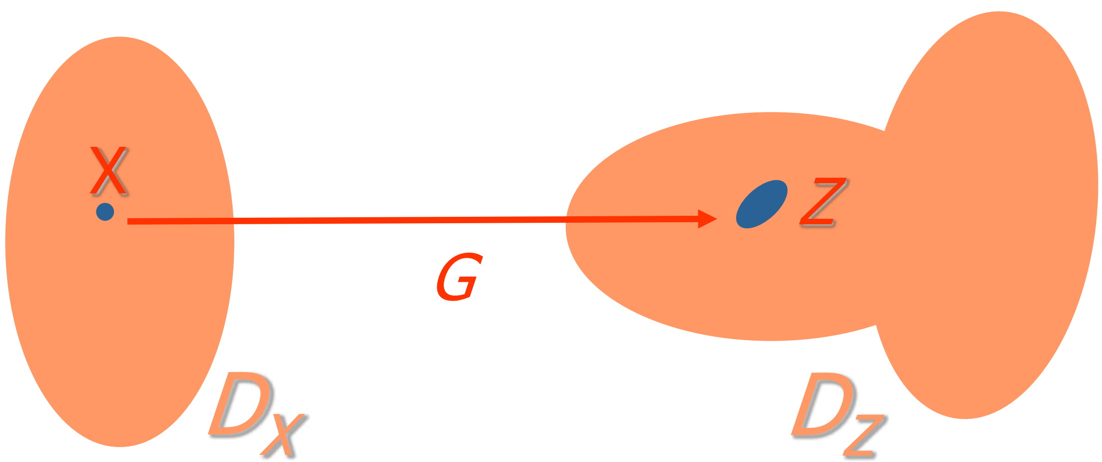
$Z = G(X_1, \ldots, X_p)$

- **Sensitivity,** for example $\partial Z / \partial X_i$

Answer to: how the output varies with respect to potential perturbations of the inputs?

- **Contribution = sensitivity x importance,** for example $\dfrac{\partial Z}{\partial X_i} \sigma(X_i)$

Gives the weight of an input (or group of inputs) on the uncertainty of the output



The input contributions are often called "importance measures", "sensitivity indices", …

Quadratic combination method
Independent case

If the X_i ($i = 1 \ldots p$) are independent, by using the first-order Taylor decomposition of Z around μ (linearization), we obtain:

$$\mathrm{Var}(Z) = \sum_{i=1}^{p} \left(\left. \frac{\partial G}{\partial X_j} \right|_{X=\mu} \right)^2 \sigma_i^2$$

Quadratic summation formula

Contribution of each input variable to the uncertainty of the output variable

Computation of derivatives via finite differences, exact differentiation, or automatic differentiation

$$\eta_i^2 = \frac{1}{\mathrm{Var}(Z)} \left(\left. \frac{\partial G}{\partial X_i} \right|_{X=\mu} \right)^2 \sigma_i^2$$

Sensitivity indices (normalized)

Sensitivity analysis is directly obtained

Main objectives of sensitivity analysis (SA)

1. Understand the behaviour of the model (decompose input-output relations)

2. Simplify the computer model (dimension reduction)

Screening

- **Determine the non-influent variables** (that can be fixed)
- Determine the non-influent phenomena (to skip in the analysis)
- Build a simplified model, a metamodel

3. Prioritize the uncertainty sources to reduce the model output uncertainty

Quantitative partitioning

- Variables to be fixed to obtain the **largest output uncert. reduction**

- Most influent variables in a given output domain

4. Analyze the robustness of the quantity of interest (QoI) with respect to the input uncertainty laws

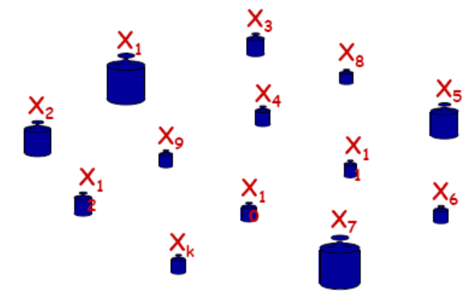
Robustness analysis

Methodology of SA

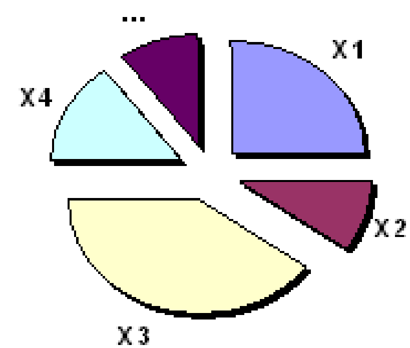
(QoI= variability of the output)

[Kleijnen 2008, Saltelli et al. 2008, Storlie et al. 2010, …]

Three types of answers:

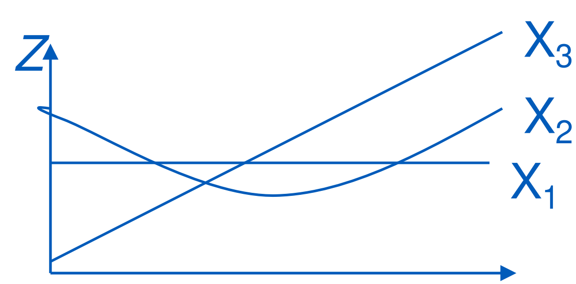


1. Screening (qualitative information: influent/non influent)
   - classical design of experiments,
   - numerical design of experiments (Morris, sequential bifurcation)



2. Quantitative measures of global influence
   - correlation/regression on values/ranks,
   - functional variance decomposition (Sobol)

3. Deep exploration of sensitivities
   - smoothing techniques (param./non parametric)
   - metamodels



Outline

1. Design of experiments

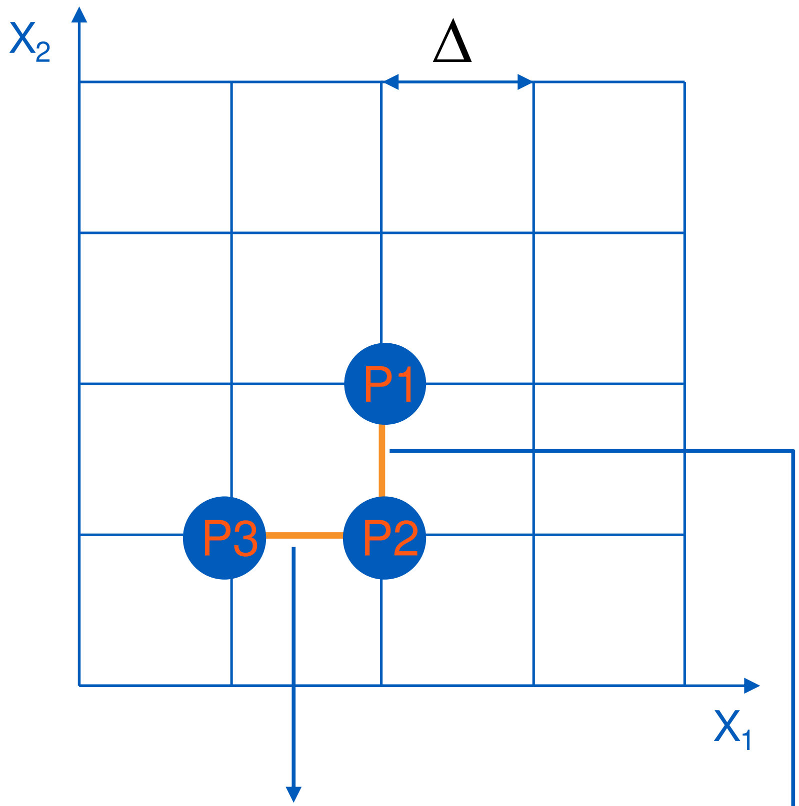
2. Global sensitivity analysis

   2.1 Screening

   2.2 Sampling-based approaches

   2.3 More advanced methods

Screening without hypothesis on function: Morris' method

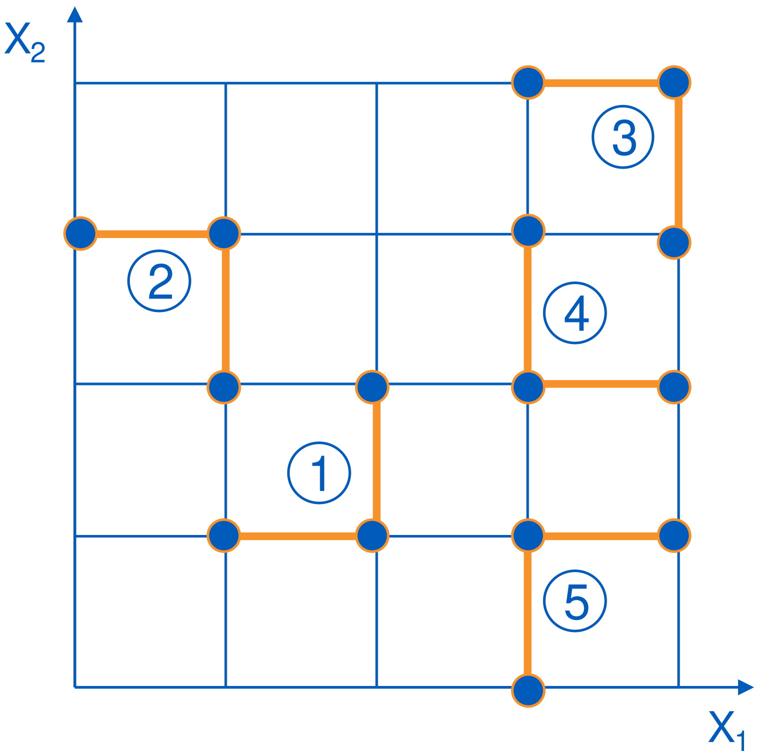


- Discretization of input space
- Needs $p + 1$ experiments
- OAT (One-at-A-Time)
- Computation of one elementary effect for each input

$$d_{X_1} = \frac{f(P3) - f(P2)}{\Delta}$$

$$d_{X_2} = \frac{f(P2) - f(P1)}{\Delta}$$

Morris' method



- OAT design is repeated *R* times (total: $n = R^*(p+1)$ experiments)
- It gives an *R*-sample for each elementary effect

$$\left\{d^i_{X1}\right\}_{i=1\ldots R}$$
$$\left\{d^i_{X2}\right\}_{i=1\ldots R}$$

- Sensitivity measures:

$$\mu_i^* = \mathrm{E}(|d_{X_i}|)$$
$$\sigma_i = \sigma(d_{X_i})$$

Morris: Sensitivity measures

- $\mu_i^* = \mathrm{E}(|d_{X_i}|)$ is a measure of the sensitivity:

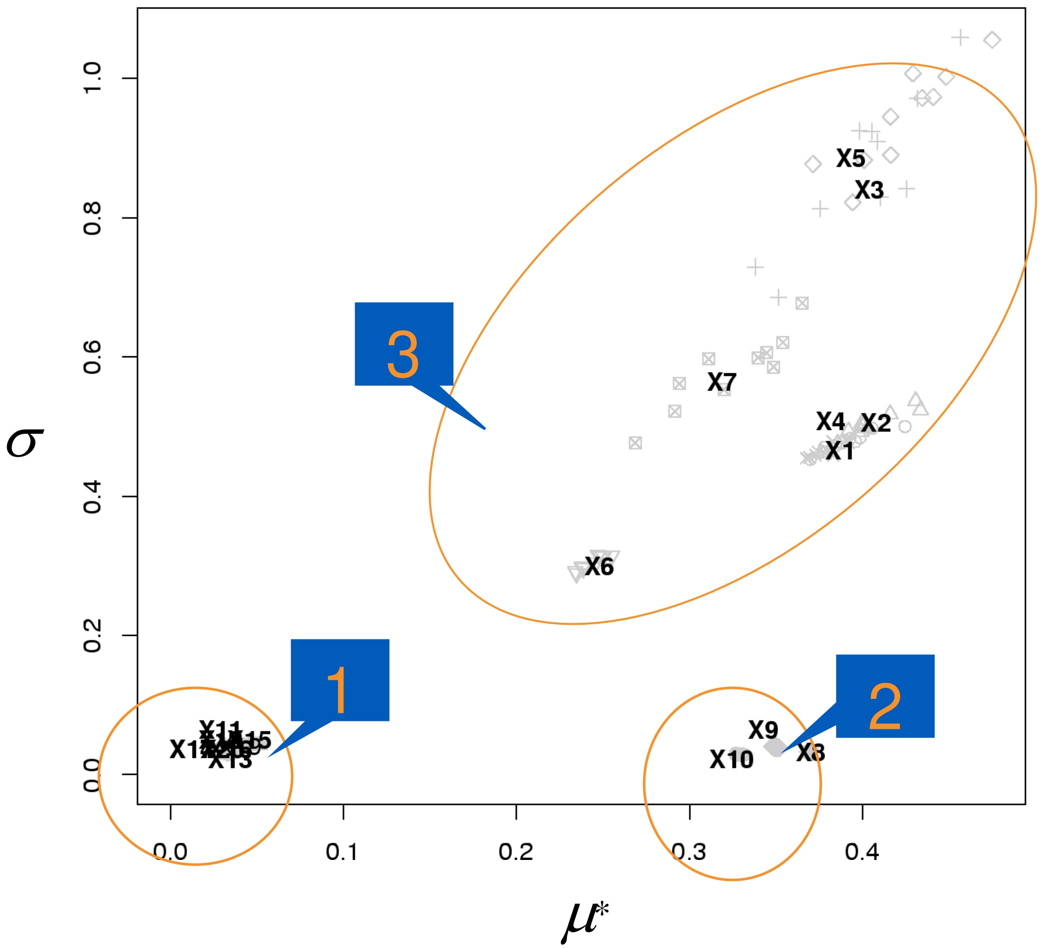
  Important value → important effects (in mean)
  → sensitive model to input variations

- $\sigma_i = \sigma(d_{X_i})$ is a measure of the interactions
  and of the non linear effects:

  important value → different effects in the *R*-sample
  → effects which depend on the value:
  - of the input X_i => non linear effect
  - or of the other inputs => interaction

  (the distinction between the two cases is impossible)

Morris: Example



20 factors
210 simulations
→ Graph (μ^*, σ)

Distinction between 3 groups:

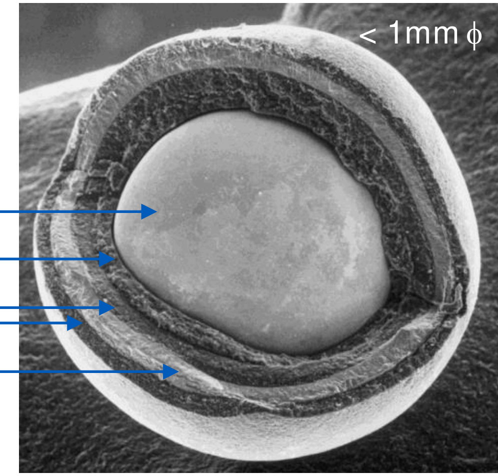
1. Negligible effects
2. Linear effects
3. Non linear effects and/or with interactions

Test case: non monotonic funtion of Morris

Example : fuel irradiation computation in HTR

Computer code ATLAS (CEA) : simulation of the HTR fuel (fuel particles) behaviour under irradiation

Noyau de matière fissile
Carbone pyrolytique poreux
Carbone pyrolytique dense
Carbure de Silicium



Number of particles inside a reactor : 10^9 to 10^{10} !

Contamination sources: failure of particles

→ Reliability studies are needed

The failure of a particle can be caused by the failure of the external thick layers (IPyC, SiC, OPyC)

Output variables are reprsentative of failure phenomena: maximal orthoradial strains in external layers

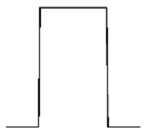
3 uncertainty types for the inputs

- 10 parameters of fuel particle manufacturing process (thickness, …)

  Specifications ➡ truncated Gaussian distributions

- 5 parameters of irradiation (temperature, …)

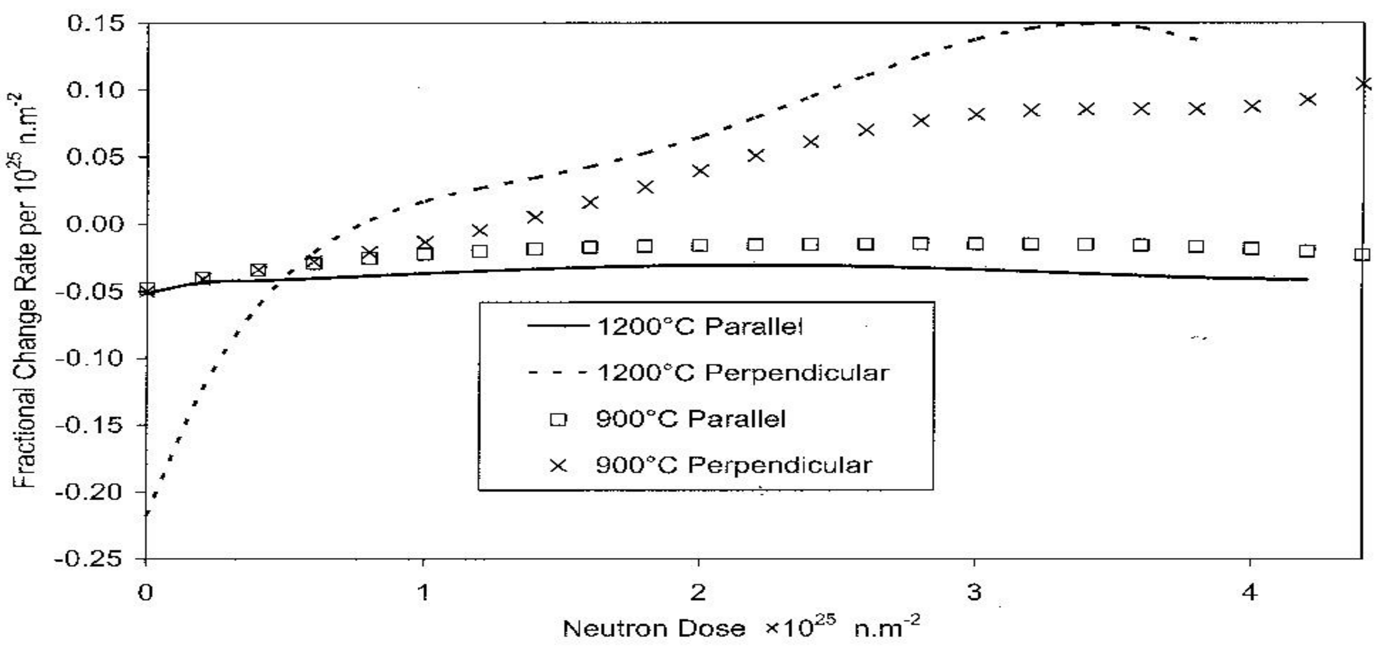
  Interval [min,max] ➡ uniform distributions



- 28 behaviour laws (functions of temperature, flux, …)

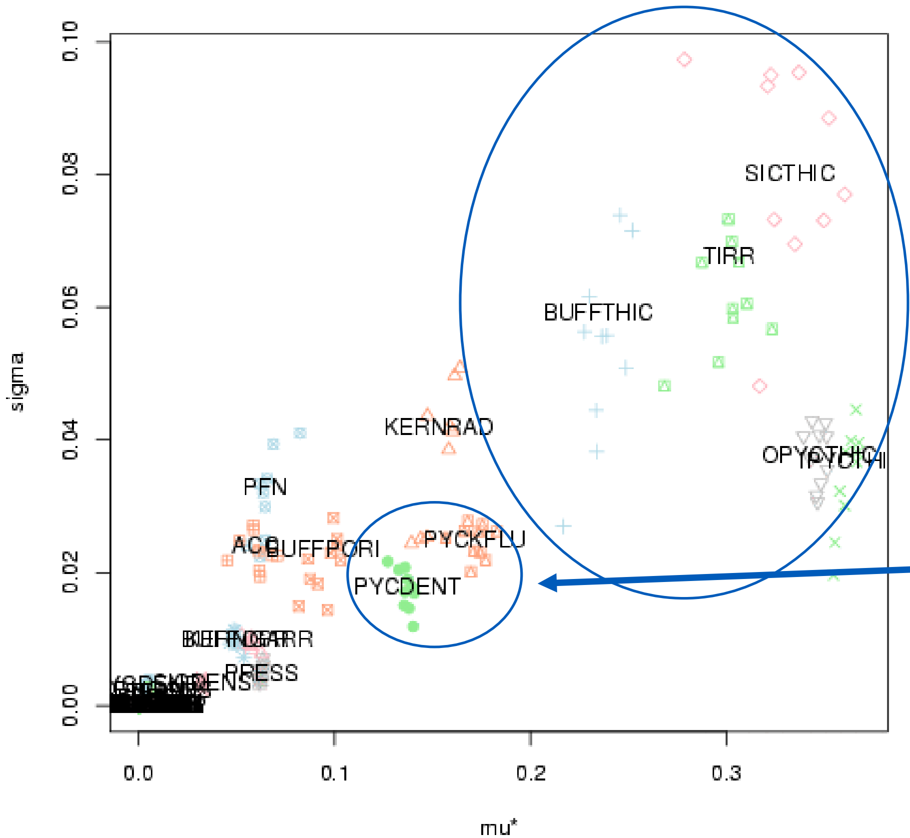
  Expert judgment ➡ multiplicative constants (~ U[0.95,1.05])

Example : Law of PyC densification



Results of Morris

p = 43 inputs, 20 repetitions, *n* = 860 runs, unitary cost ~ 1 mn => total=14h



Large sensitivities to these inputs (thickness, irradiation temperature)
Small interaction effects

Influence of creep and densification laws of PyC

Conclusion:

Morris method provides qualitative information about output variations due to potential variations of inputs

Useful in order to identify the potential influent inputs

Outline

1. Design of experiments

2. Global sensitivity analysis

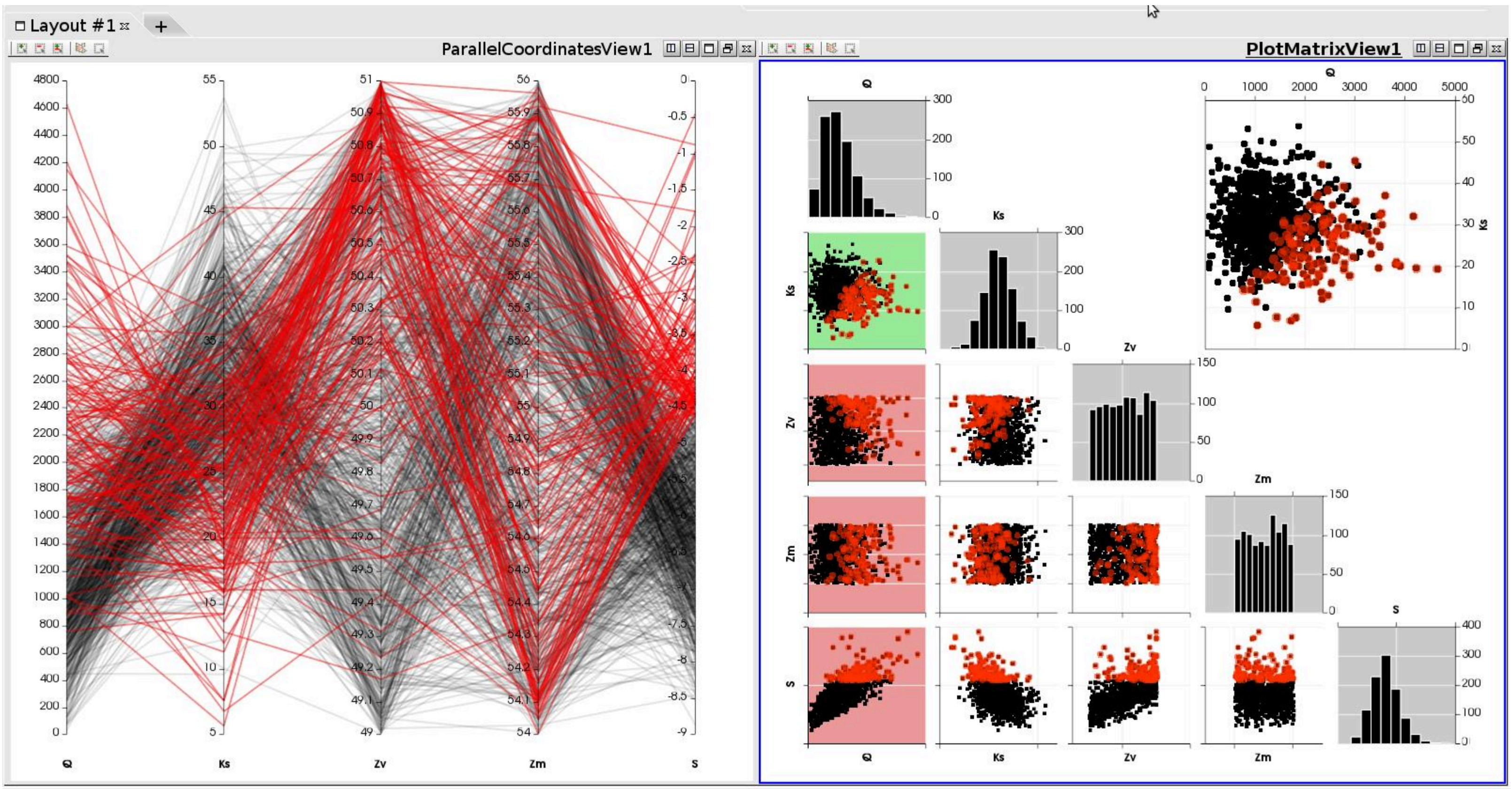
   2.1 Screening
   2.2 Sampling-based approaches
   2.3 More advanced methods

The sampling-based approaches

Sample ($X \in \Re^p$, $Z(X) \in \Re$) of size $N > p$ *[…, Saltelli et al. 2000, Helton et al. 2006, …]*

Monte Carlo sample, space filling design, …

Preliminary step : graphical visualization (for ex: scatterplots, but also cobweb…)



Example of the OpenTURNS graphical interface

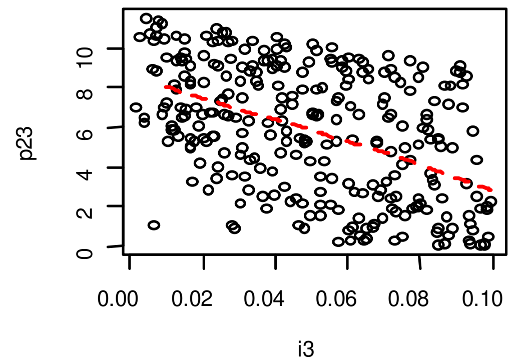
First visualization: scatterplots

Measure the linear character of the cloud

N runs

Graphs: Output with respect to each input

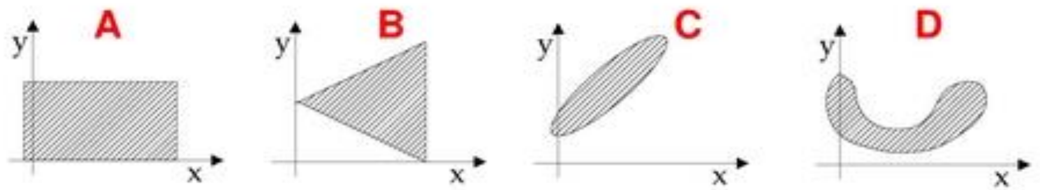
Example:
N = 300



$$\rho = \frac{\mathrm{cov}(X,Y)}{\sigma_X \sigma_Y}$$

$$\hat{\rho} = \frac{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{s_x s_y}$$

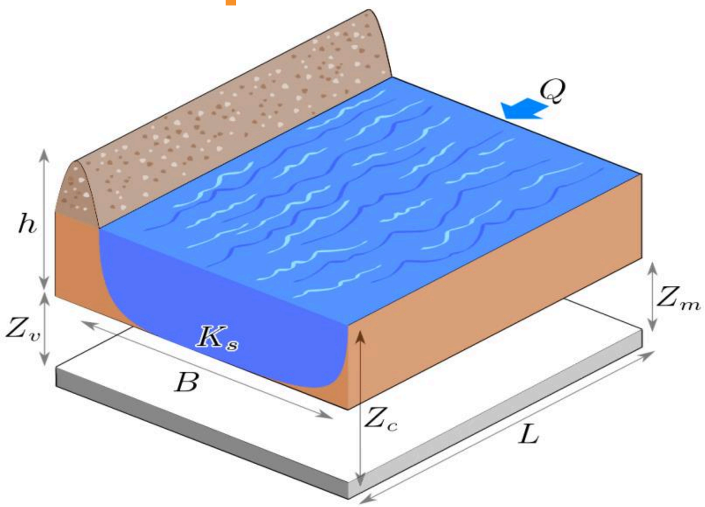
- Nuage de points : exemples



?

- 1 : corrélation non linéaire
- 2 : absence de liaison en moyenne mais pas en dispersion
- 3 : corrélation linéaire
- 4 : absence de liaison

Example: Flood model - Scatterplots – Output S



Q = river flowrate ~ Gumbel on [500,3000]

Ks = friction coefficient ~ normal on [15,50]

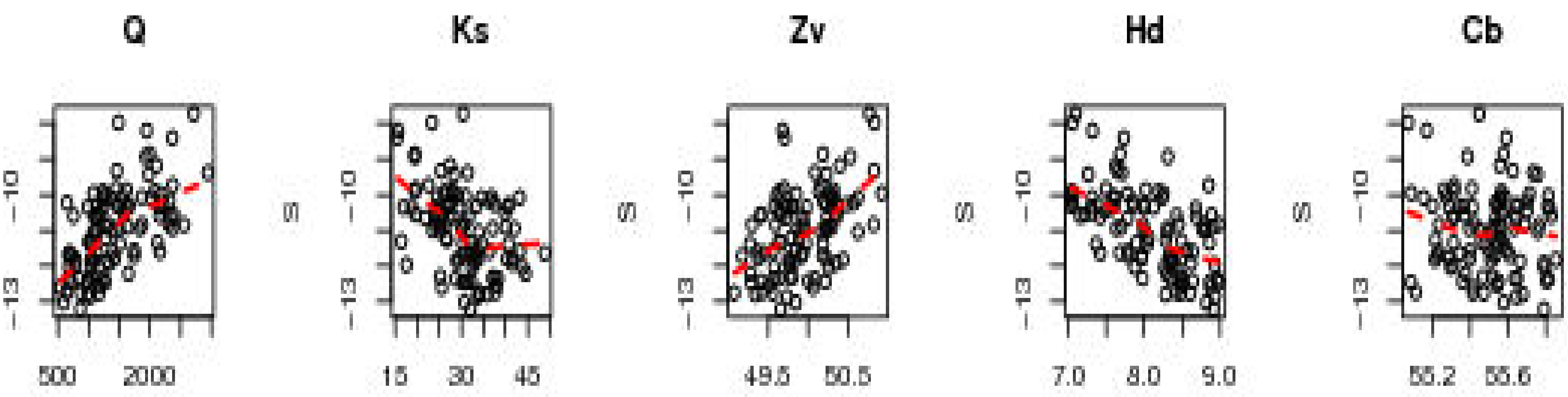
Zv = downstream river bed heigth ~ triangular on [49,51]

Hd = dyke heigth ~ triangular on [7,9]

Cb = bank heigth ~ triangular on [55,56]

$$S = Z_v + H - H_d - C_b \quad \text{avec} \quad H = \left(\frac{Q}{BK_s\sqrt{\frac{Z_m - Z_v}{L}}} \right)^{0.6}$$

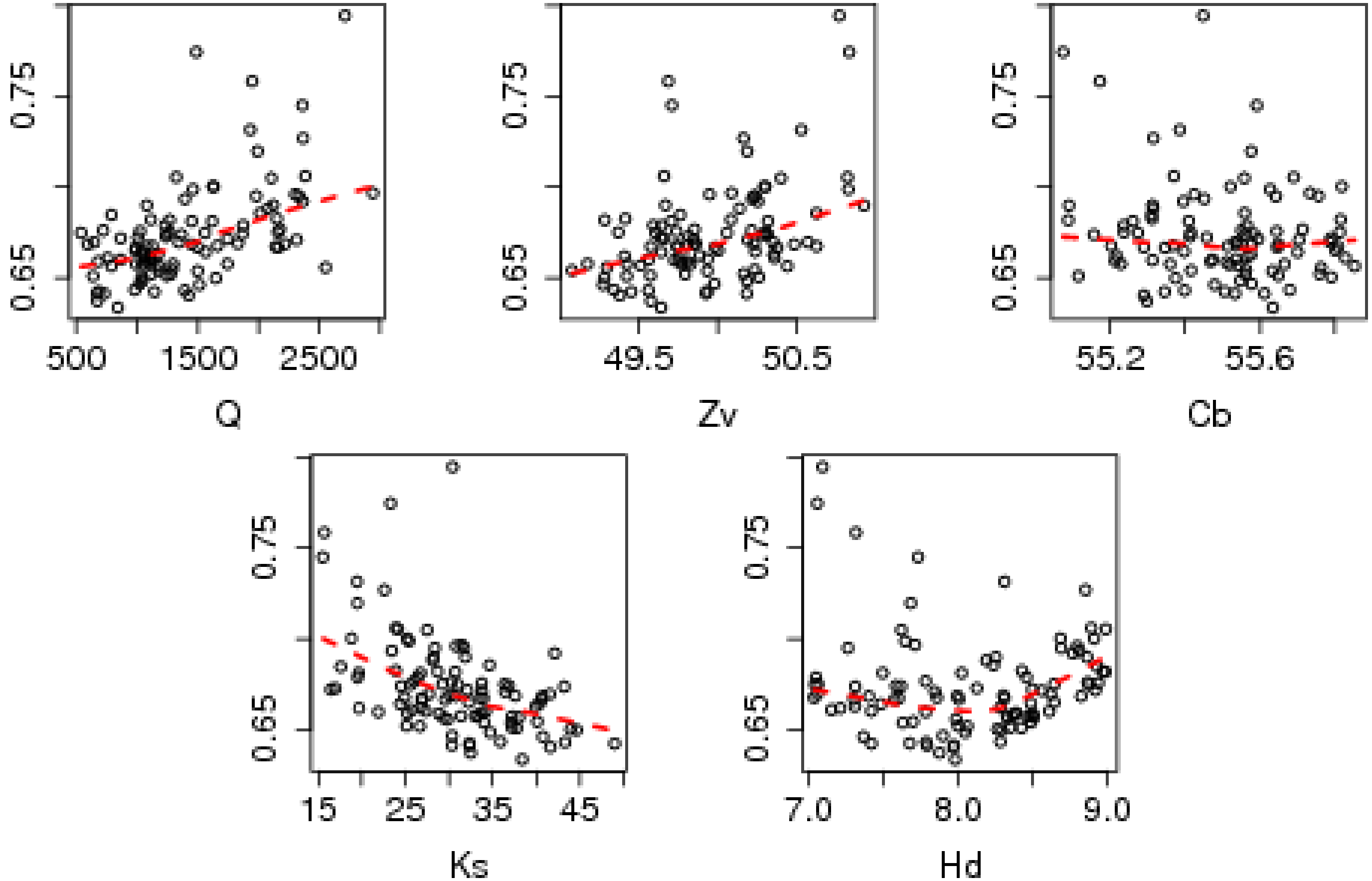
Monte Carlo sample – $N = 100$



Example: Flood model - Scatterplots – Output Cp

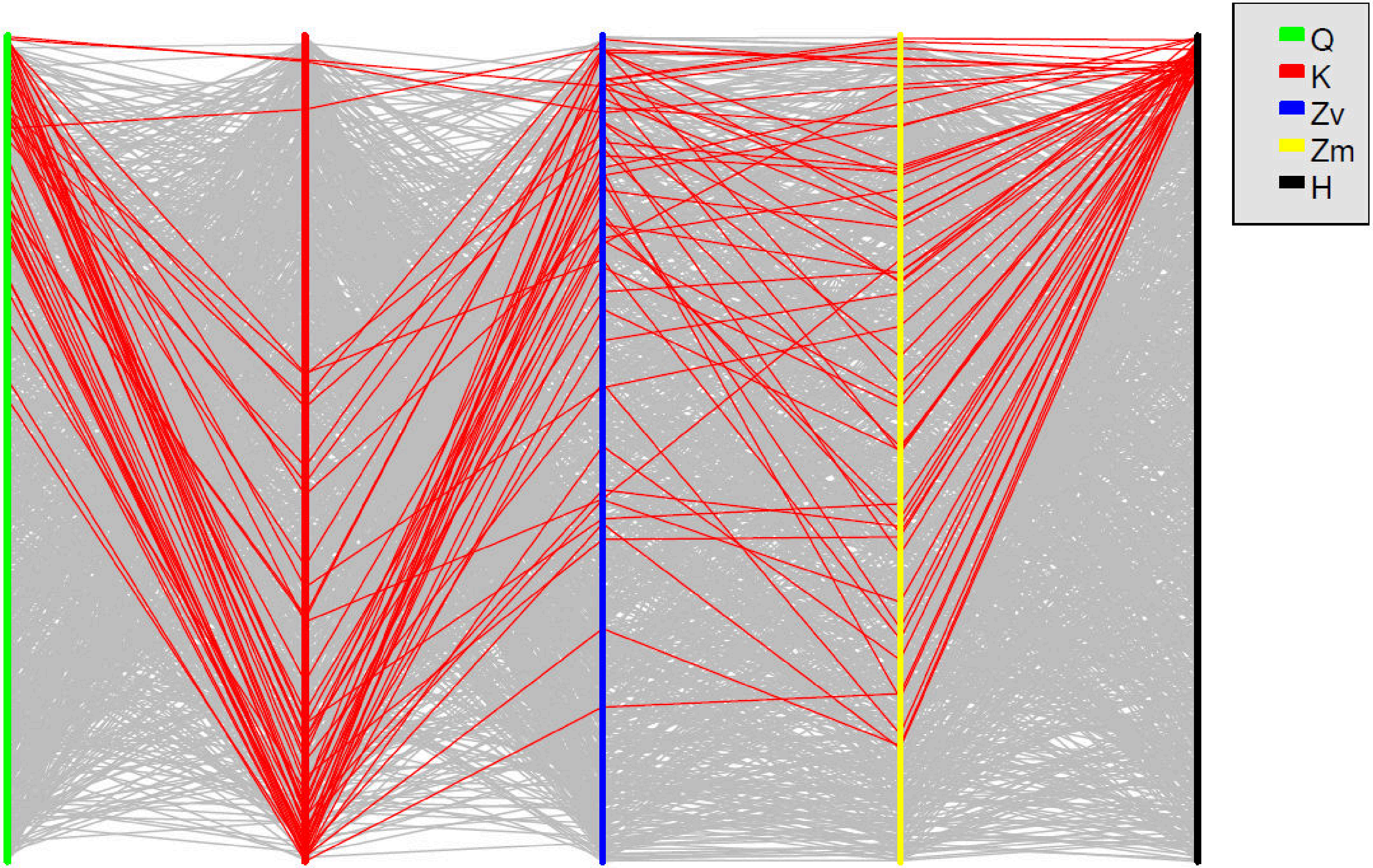
$$C_p = 1\!\!1_{S>0} + \left\{0.2 + 0.8\left[1 - \exp\left(-\frac{1000}{S^4}\right)\right]\right\} 1\!\!1_{S\leq 0} + \frac{1}{20}\left(H_d 1\!\!1_{H_d>8} + 8 1\!\!1_{8\leq H_d}\right) ,$$

Monte Carlo sample – *N* = 100



Major drawback: only first order relations between inputs are analyzed and not their interactions (=> needs of other data anlysis tools)

Second visualization: the cobweb plot



Interactive use with the OpenTURNS graphical interface

Sensitivity indices in case of linear inputs/output relation

Independent inputs $\mathbf{X} = \left(X_1, \ldots, X_p\right)$

Sample: *n* realizations of $\left(\mathbf{X}, Y\right)$

Linear regression model: $Z = \beta_0 + \sum_{i=1}^{p} \beta_i X_i$

- Standard Regression Coefficients (SRC) : $\mathrm{SRC}\left(X_i\right) := \beta_i \sqrt{\dfrac{\mathrm{Var}(X_i)}{\mathrm{Var}(Z)}}$

  Sign of β_i gives the direction of variation of Z / X_i

- Similar to the linear correlation coefficient (Pearson)

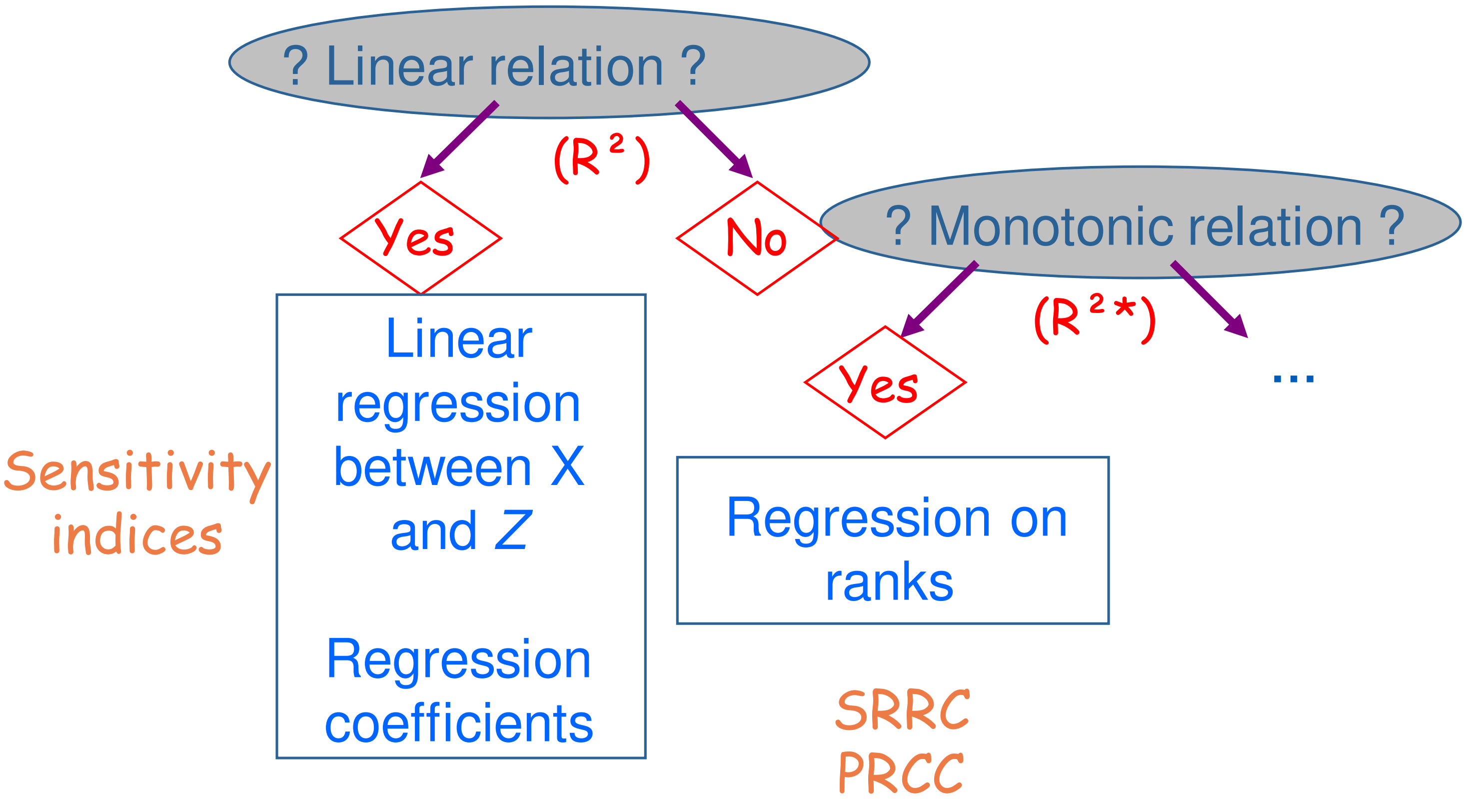
- Validity of the linear model

  via regression diagnostics and R^2: $R^2 = 1 - \dfrac{\sum_{i=1}^{n} \left(\hat{Y}_i - Y_i\right)^2}{\sum_{i=1}^{n} \left(Y_i - \bar{Y}\right)^2}$

- Theoretically, we have $R^2 = \sum_{i=1}^{p} \mathrm{SRC}^2\left(X_i\right)$, useful to interpret SRC

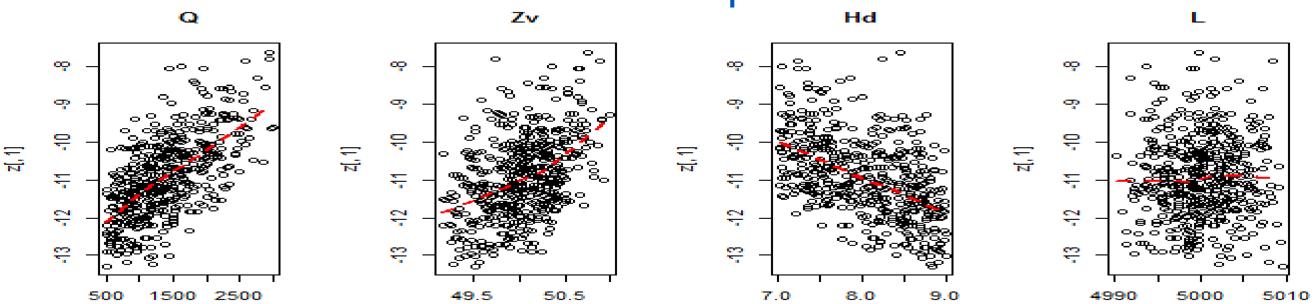
A classical view on the « Sampling-based approaches »

Sample ($X \in \Re^p$, $Z(X) \in \Re$) of size $N > p$ *[…, Saltelli et al. 2000, Helton et al. 2006, …]*

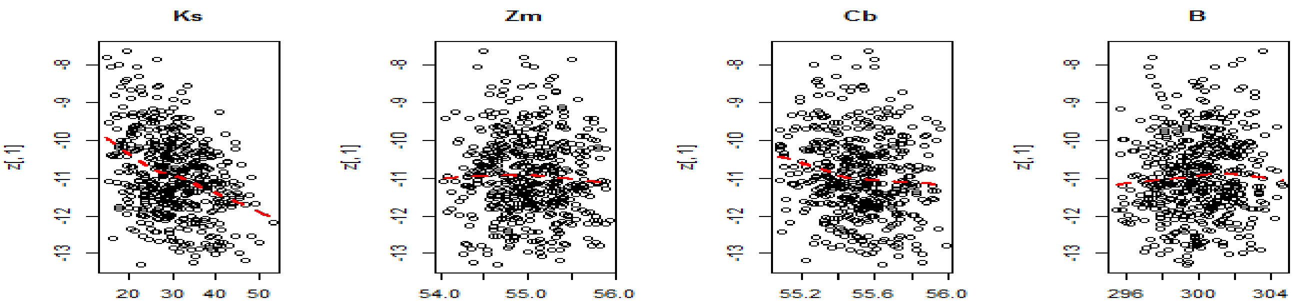


Application on the flood model – Output S

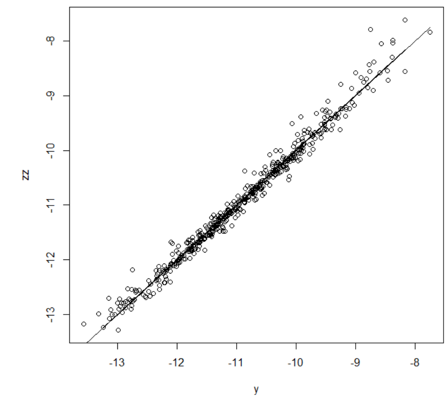
Monte Carlo sample – N = 500



36% 16% 29% 0%



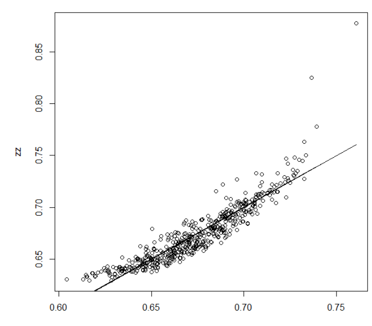
12% 0% 3% 0%



R^2 = 0.98
=> Linear model is valid

⇒ **SRC are valid sensitivity indices; global sensitivity analysis is achieved**

<u>Cp output :</u> R^2 = 0,83 ;

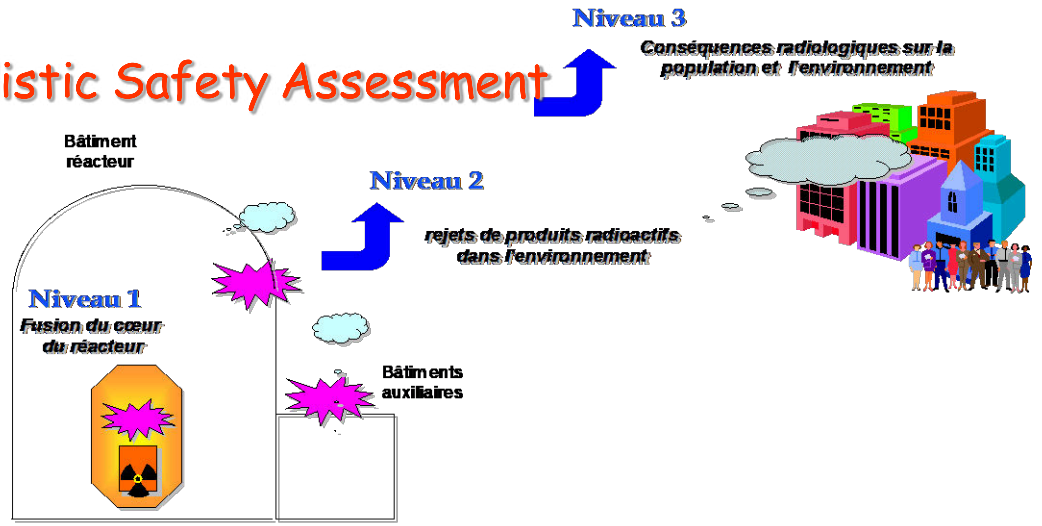


=> linear model is not sufficiently valid

Industrial example: Nuclear PWR severe accident study

Important issue for the safety core of the industrial operator

Probabilistic Safety Assessment



<u>Main questions after a severe accident (PSA level 2):</u>

- Is there some radionuclide release outside the containment (and what is the time of this release) ?
- What is the efficiency of mitigation actions?
- Influence of the large parameter uncertainties on the failure probability?

⇒ Development of a soft simulating severe accident scenarios (corium behavior)

Main objectives of these uncertainty studies

Complex and coupled numerical models which require sensitivity analysis to:

- Identify the main sources of uncertainties
- Understand the relations between uncertain inputs and interest outputs

Scenario :

Core degradation, corium transfer and interaction (inside in-vessel and ex-vessel)

23 Interest output variables :

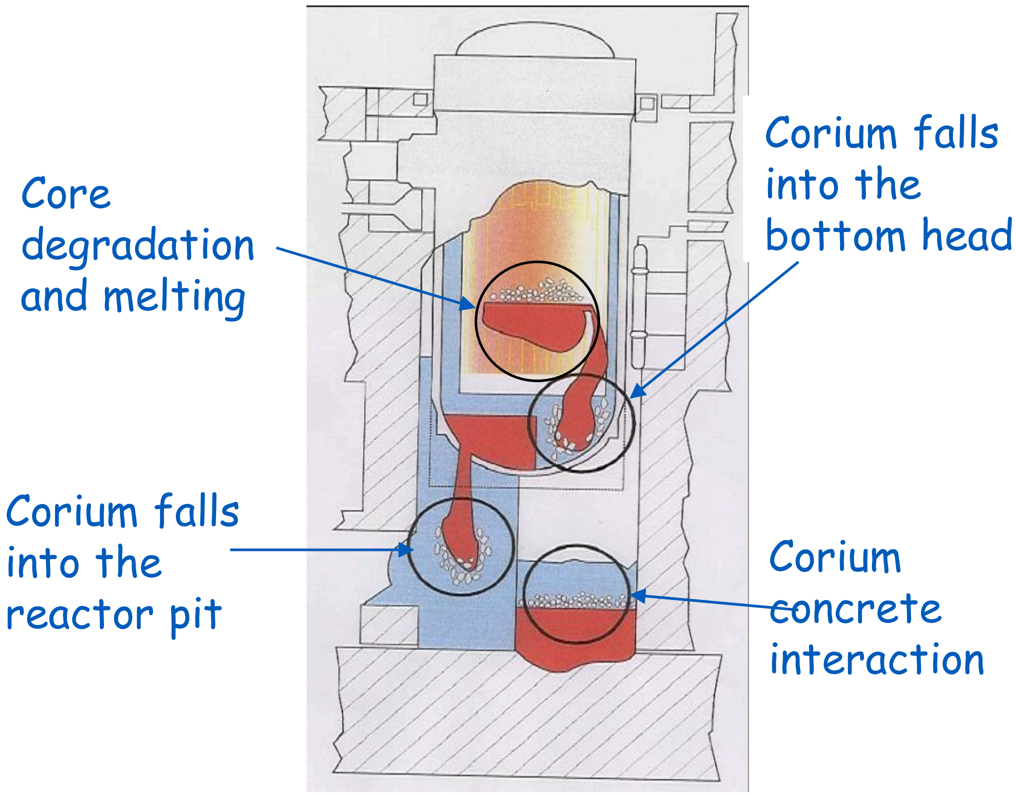
Corium masses

Time of vessel failure

Time of reactor pit failure

32 input random variables (uniform laws) :

Water management, physical properties, …



Additional problem: provide tools for engineers knowing nothing little about stats

3 use levels of the soft – 1) Precise and punctual needs

Example: relative study about the failure probability

Goal: **understand** the influence of water injection in the vessel and/or in the reactor pit during the accident

Random var. **X** : 22 uniform - 1 binary (for the water presence/absence)

N =500 random simulations (LHS design)

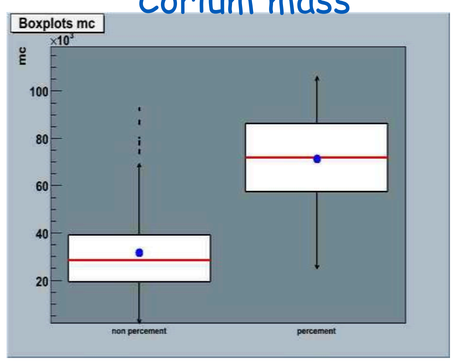
Failure probability is estimated by Monte-Carlo method

P(failure without water injection)/ P(failure with water injection) = 2.5

This kind of result could help the definition of management strategies

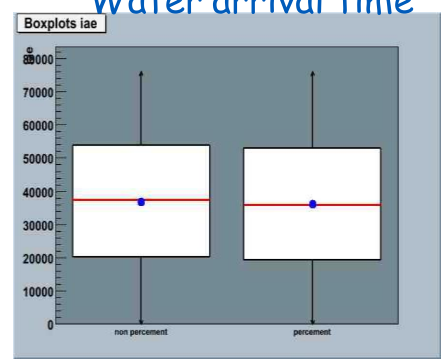
Sensitivity analysis via boxplots

Corium mass



No failure Failure

Water arrival time



No failure Failure

3 use levels – 2) Detailed analyses for confirmed users

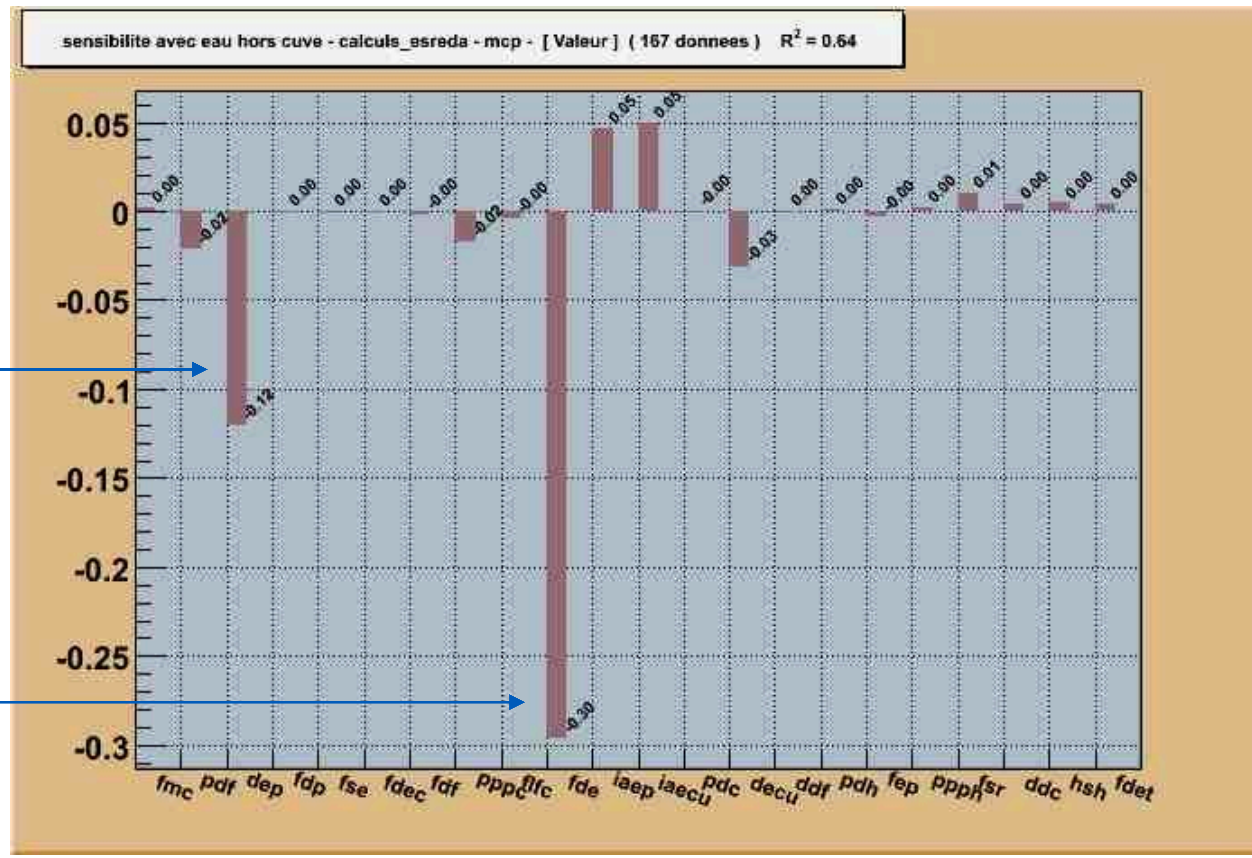
<u>Example: sensitivity analysis related to an output variable</u> (corium mass) with 22 inputs

<u>Goal</u>: **understand** which input variables strongly affect the corium mass

$$SRC_i^{\text{sgn}} = \text{sgn}(\beta_i)\,\frac{\beta_i^2 \text{Var}(X_i)}{\text{Var}(Z)}$$

Water flow in the pit

Rubble fraction in the water

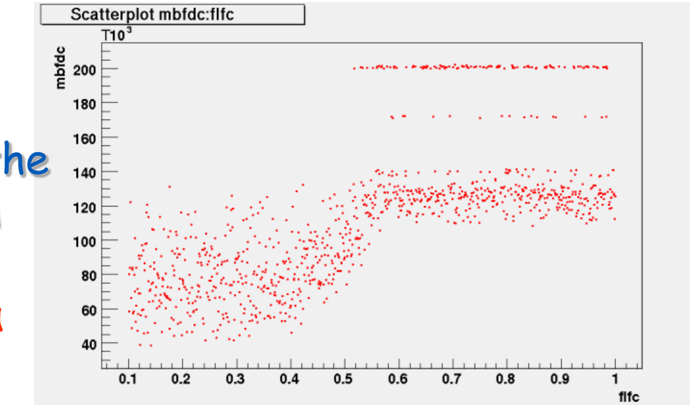


3 use levels – 3) Help to physical model developers

Example: detection of anomalies

500 MC simulations

Corium mass in the vessel bottom



Limit factor of the critical flux

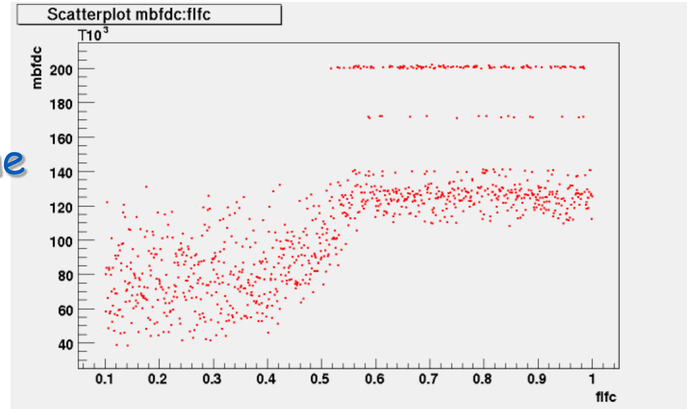
It has allowed to detect that important phenomena related to the corium transfer were not included in the physical model

3 use levels – 3) Help to physical model developers

Example: detection of anomalies

500 MC simulations

Corium mass in the vessel bottom

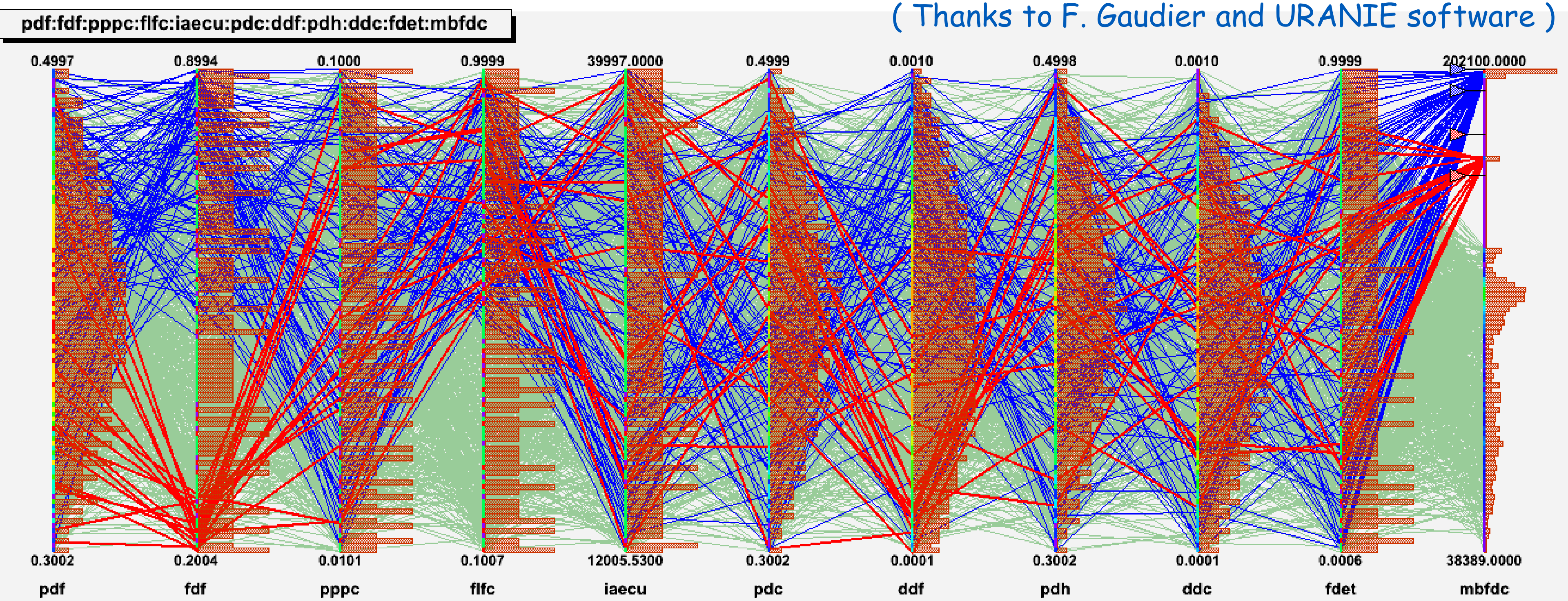


Limit factor of the critical flux

It has allowed to detect that important phenomena related to the corium transfer were not included in the physical model

Deep analysis using cobweb plots

(Thanks to F. Gaudier and URANIE software)



Outline

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    2.1 Screening

    2.2 Sampling-based approaches

    2.3 More advanced methods

Issues in sensitivity analysis

Numerical model' issues

P1) G (.) is complex: interactions, non linear, discontinuous, …

=> Sobol indices, moment-independent measures

P2) G (.) is costly (several minutes - days to compute one evaluation)

=> Metamodel

Model inputs' issues

P3) p is large : $p > 10 \ldots 100 \ldots$ **=> Quantitative screening (HSIC, DGSM)**

P4) Dependence between inputs **=> Shapley indices**

P5) Uncertainty on inputs' pdfs (epistemic) **=> Robustness analysis (PLI)**

Model outputs' issues

P6) Z is not a single scalar, but a high-dimensional vector, a temporal function, a spatial field, … **=> Ubiquitous and aggregated indices**

P7) Inputs or outputs are too voluminous

=> Iterative sensitivity analysis

P8) The Quantity of Interest (QoI) is not the variance (e.g. a quantile)

=> GOSA (Goal-Oriented Sensitivity Analysis)

P1) Sensitivity analysis for one scalar output

Sample ($\mathbf{X}$, Z ($\mathbf{X}$)) of size $n > p$, preferably $n >> p$

Preliminary step : graphical vizualisation (for ex: scatterplots)

Quantitative sensitivity analysis methodology

? Linear relation ?

(R^2)

Yes

No

? Monotonic relation ?

(R^{2*})

Yes

No

Sensitivity indices

Linear regression between X_i and Z

Regression coefficients

Regression on ranks

Sobol' indices

$$S_i = \frac{\mathrm{Var}[\mathrm{E}(Z|X_i)]}{\mathrm{Var}(Z)}$$

P1) Sensitivity indices without model hypotheses

Functional ANOVA *[Efron & Stein 81]* (hyp. of independence between X_i , i=1…p):

$$\mathrm{Var}(Z) = \sum_{i=1}^{p} V_i(Z) + \sum_{i<j}^{p} V_{ij}(Z) + \cdots + V_{12\ldots p}(Z)$$

$$\text{where } V_i(Z) = \mathrm{Var}[\,\mathrm{E}(Z \mid X_i\,)]$$

$$V_{ij} = \mathrm{Var}\big[\mathrm{E}(Z \mid X_i X_j\,)\big] - V_i - V_j\ , \ldots$$

Sobol indices definition:

- First order sensitivity indices: $$S_i = \frac{V_i}{\mathrm{Var}(Z)}$$
- Second order sensitivity indices: $$S_{ij} = \frac{V_{ij}}{\mathrm{Var}(Z)}$$
- ...

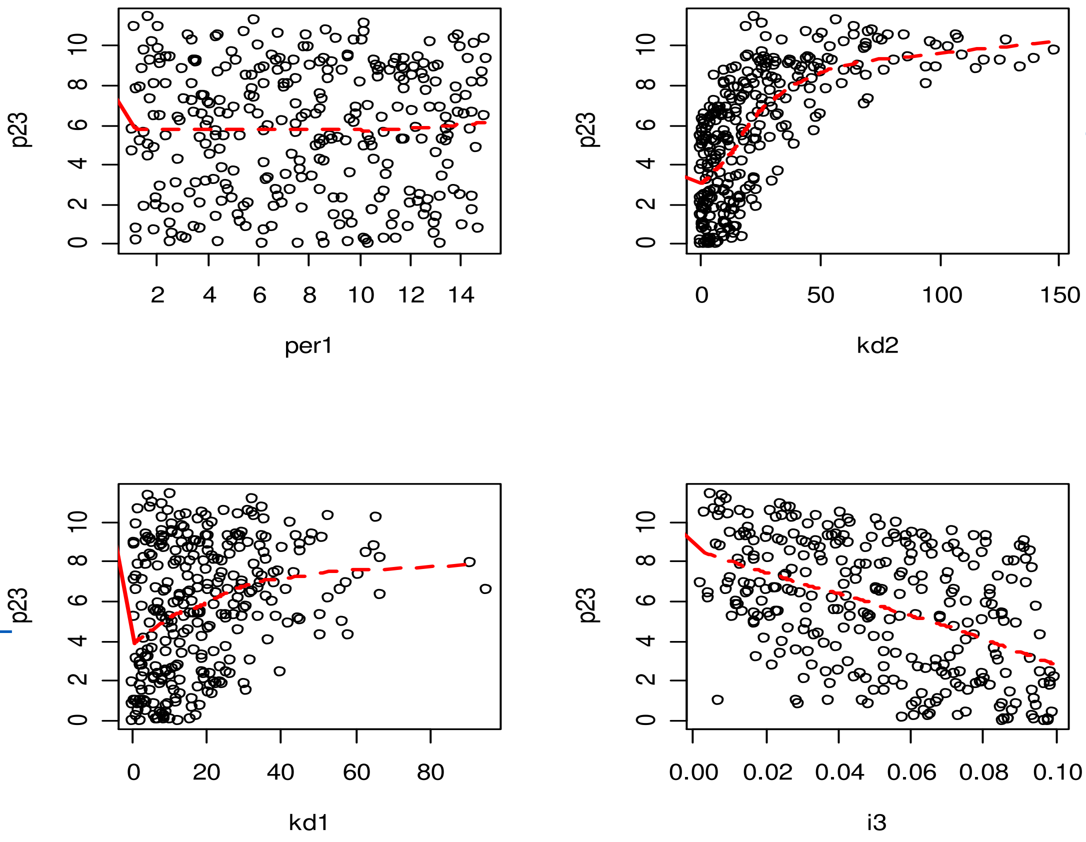
P1) Graphical interpretation

First order Sobol' indices measure the variability of conditional expectations (mean trend curves in the scatterplots)

Null index

High index

Small index



P1) Sobol' indices properties

$$\boxed{1 = \sum_{i=1}^{p} S_i + \sum_i \sum_j S_{ij} + \sum_i \sum_j \sum_k S_{ijk} \ldots + S_{1,2,\ldots,k}}$$

$\sum_i S_i \leq 1$ Always

$\sum_i S_i = 1$ Additive model

$1 - \sum_i S_i$ Measure the degree of interactions between variables

Examples : $p = 4$ gives 4 indices S_i, 6 indices S_{ij} , 4 indices S_{ijk}, 1 indice S_{ijkl}

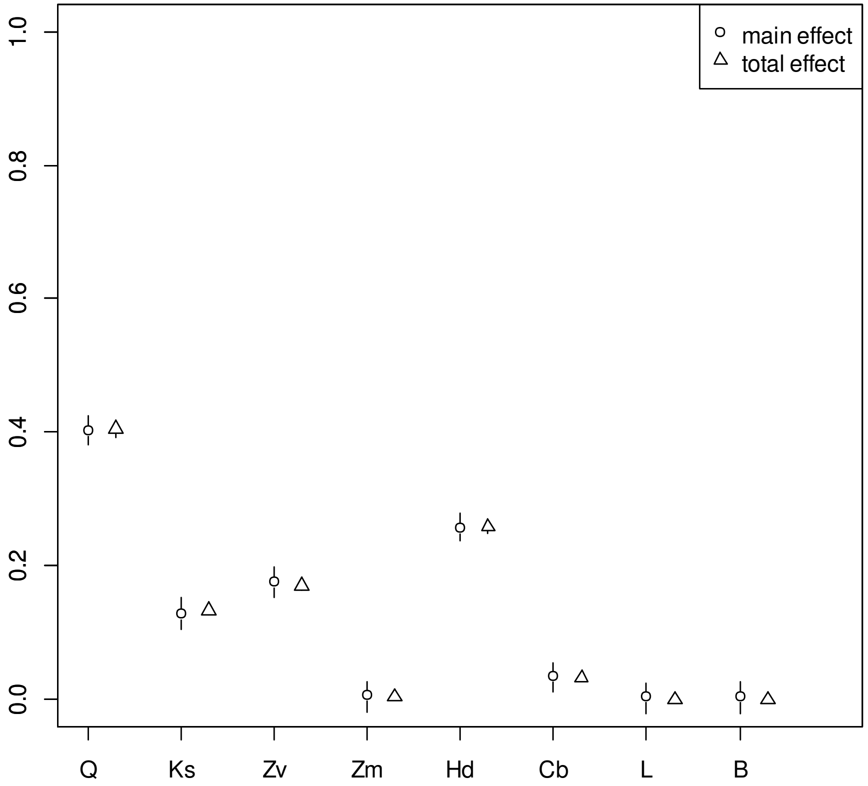
General case : 2^p-1 indices to be estimated

Total sensitivity index: $$S_{Ti} = S_i + \sum_j S_{ij} + \sum_{j,k} S_{ijk} + \ldots = 1 - S_{\sim i}$$

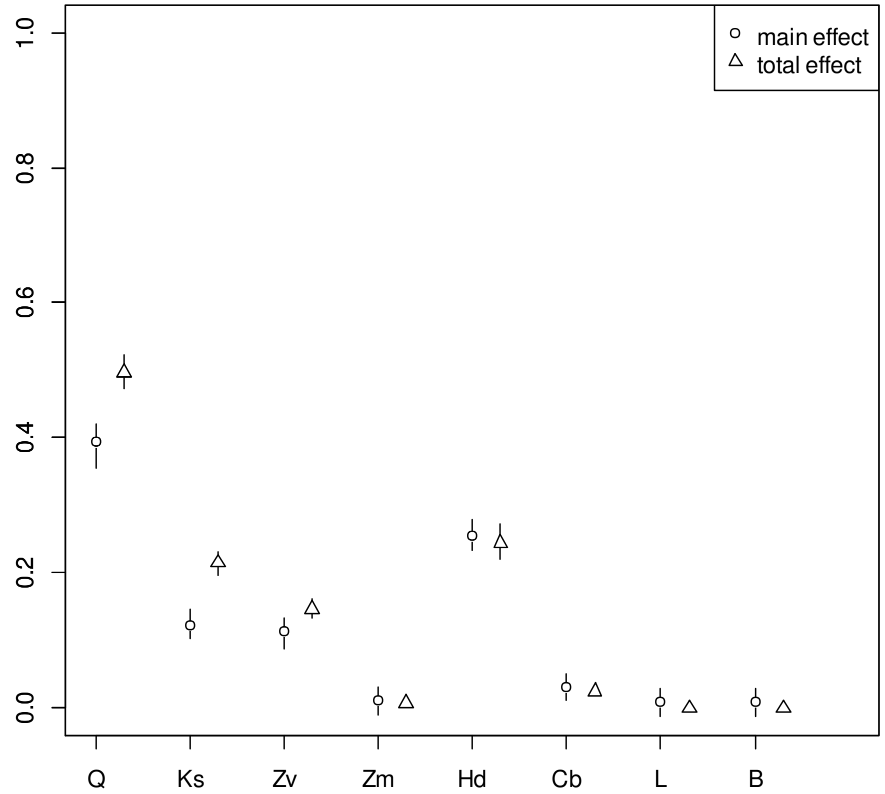
[Homma & Saltelli 1996]

Come-back to the flood model example

Overflow S

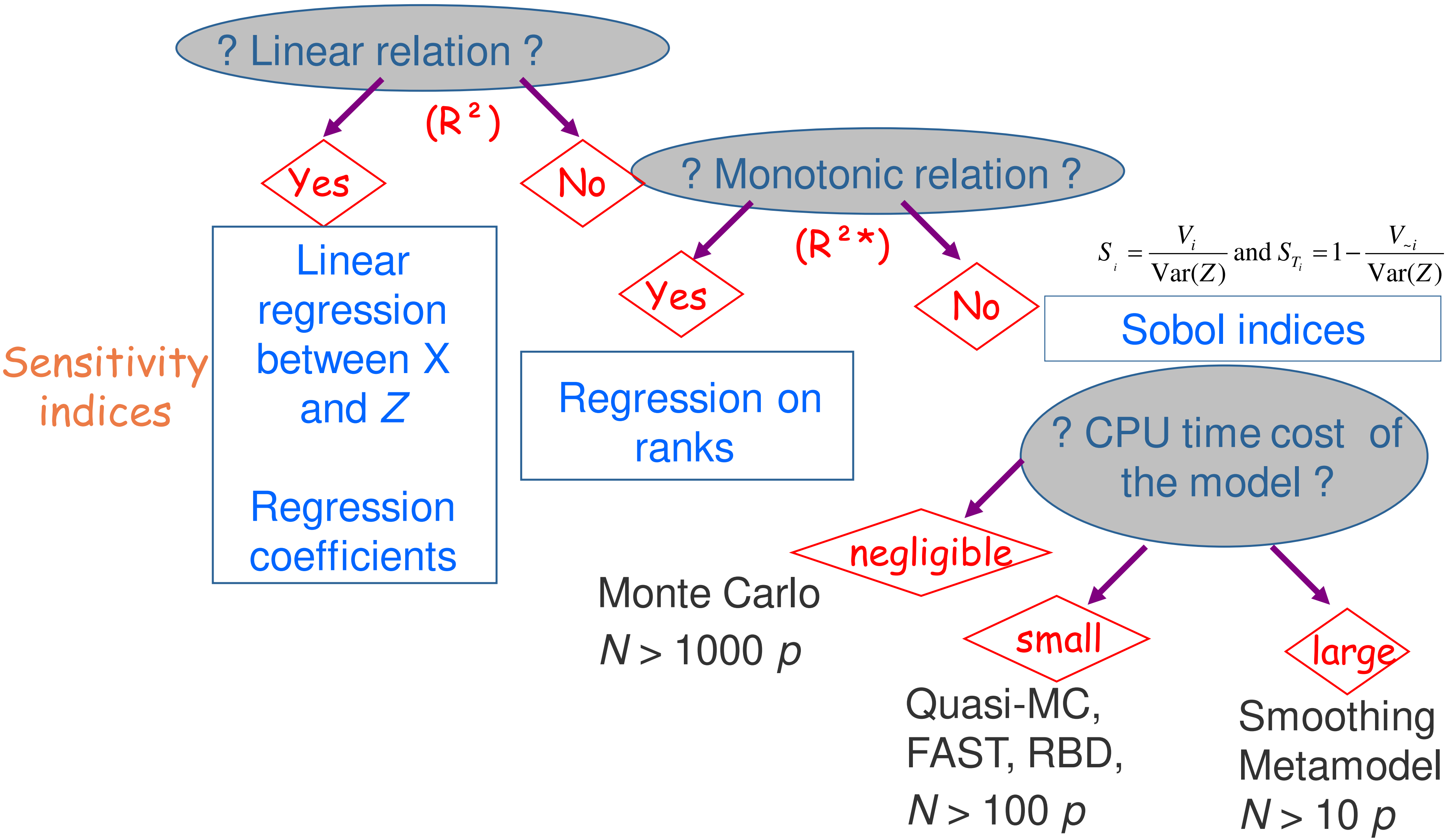


Cost C_p



P1) The sampling-based approaches

Sample ($X \in \Re^p$, $Z(X) \in \Re$) of size $N > p$



Issues in sensitivity analysis

Numerical model' issues

P1) *G* (.) is complex: interactions, non linear, discontinuous, …

=> Sobol indices, moment-independent measures

P2) *G* (.) is costly (several minutes - days to compute one evaluation)

=> Metamodel

Model inputs' issues

P3) p is large : $p > 10 \dots 100 \dots$ **=> Quantitative screening (HSIC, DGSM)**

P4) Dependence between inputs **=> Shapley indices**

P5) Uncertainty on inputs' pdfs (epistemic) **=> Robustness analysis (PLI)**

Model outputs' issues

P6) *Z* is not a single scalar, but a high-dimensional vector, a temporal function, a spatial field, … **=> Ubiquitous and aggregated indices**

P7) Inputs or outputs are too voluminous

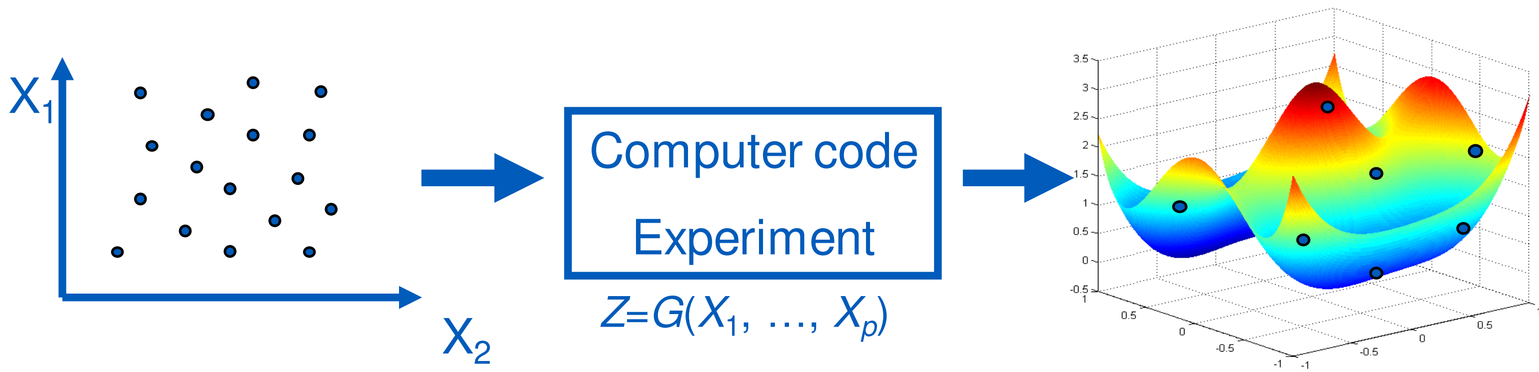
=> Iterative sensitivity analysis

P8) The Quantity of Interest (QoI) is not the variance (e.g. a quantile)

=> GOSA (Goal-Oriented Sensitivity Analysis)

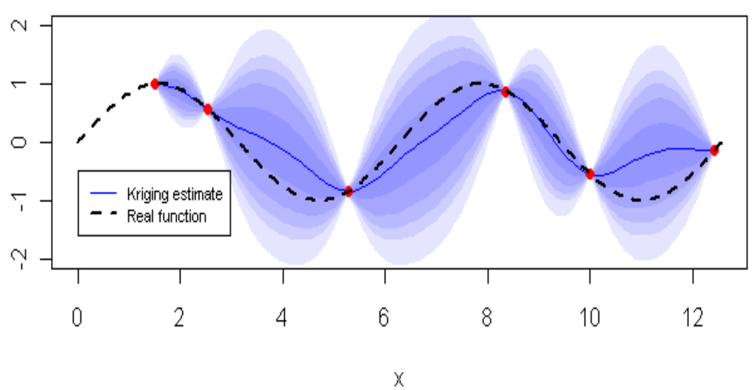
P2) Sensitivity analysis by metamodels

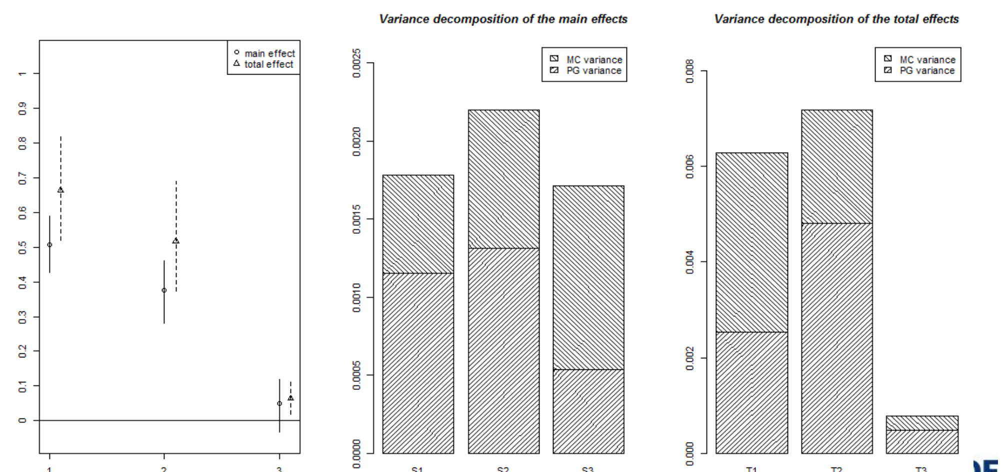
- Mathematical function (polynomial, polyn. chaos, neural network, kriging, …) representative of the computer model with negigible cpu cost (ex: 1 ms)
- Approximation from a design of experiments : N simulations ($N \sim 10\ p$)
- Important: control the approximation and prediction capabilities



Example: Kriging metamodel, which propagates the metamodel error on sens. indices

[Le Gratiet et al. 2014]





Come-back to the flood model example – Output Cp

From the 100-size Monte Carlo sample, a **Gaussian process metamodel** is fitted

Predictivity of the Gp metamodel : Q_2 = 99%

The previously shown formula are applied on the metamodel predictor

N=1e5

100 replicates

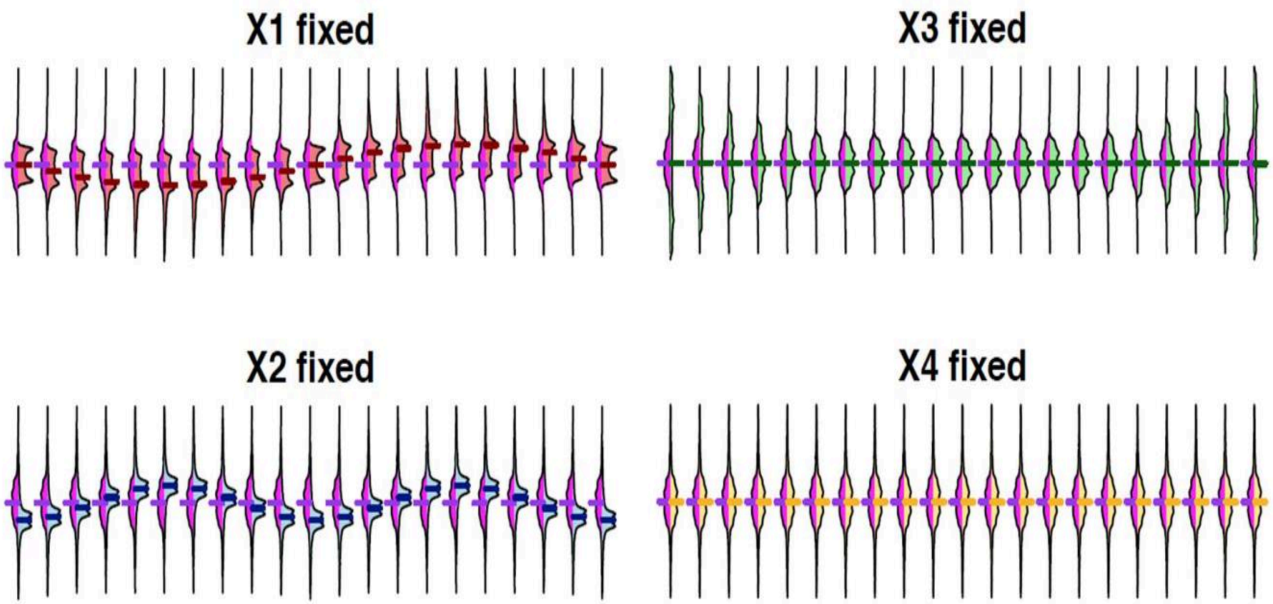
Indices (en %)	Q	K_s	Z_v	H_d	C_b
S_i modèle	35.5	15.9	18.3	12.5	3.8
S_i métamodèle	38.9	16.8	18.8	13.9	3.7
S_{T_i} modèle	48.2	25.3	22.9	18.1	3.8
S_{T_i} métamodèle	45.5	21.0	21.3	16.8	4.3

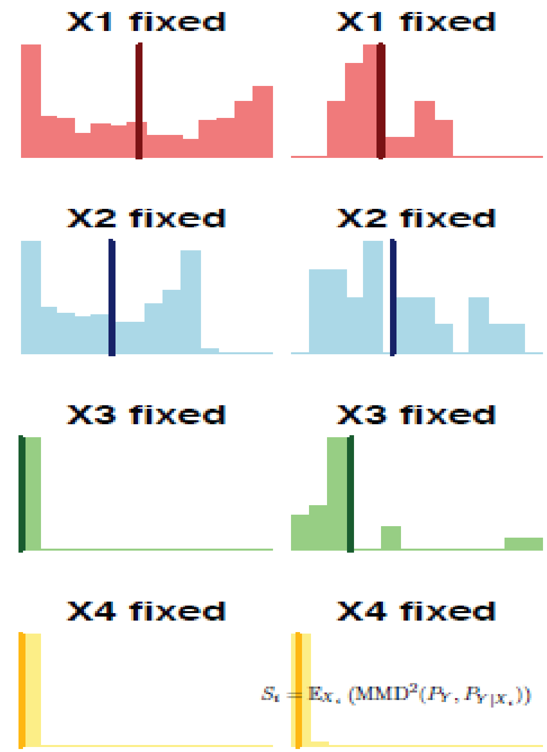
N x (p+2) x 100 = 7 x 10^7 metamodel evaluations

P3) Large *p* - Quantitative screening with HSIC

Generalized sensitivity indices **(HSIC)** which not limit SA to a single metric to compare change in the distribution of the output when there an input varies

[Da Veiga 2015]





A dependence measure is used to compare the joint distribution between Z and X_i and the product of marginals of Z and X_i with an infinity of metrics (kernel trick) **=> HSIC(X_i , Z)**, which is computed with a single sample

high-dimensional SA

Rem: classical indices (SRC, Sobol) only use the « mean » metric

P3) Large *p* - Quantitative screening with DGSM (Derivative-based Global Sensitivity Measures)

$$\nu_i = \int_{\mathrm{Supp}(\mathbf{X})} \left(\frac{\partial G(\mathbf{X})}{\partial X_i} \right)^2 \mu_{\mathbf{X}} d\mathbf{X}$$

[Kucherenko et al. 2009]

- Mix stochastic and deterministic approaches as global and local approaches

- Inequality between DGSM and Sobol' indices:

$$\boxed{0 \leq S_{Ti} \leq \frac{C(f_{X_i})}{\mathrm{Var}(Z)} \nu_i} \qquad C\left(f_{X_i}\right) : \text{ constant}$$

[Lamboni et al. 2013
Roustant et al. 2017]

- Cost (empirical) of evaluating the integral: N ~ 100 (smaller than for Sobol' indices)

- Cost of evaluating the derivatives:
  - $p + 1$ by finite differences
  - Independent of *p* if the adjoint models of *f* is available

$\Rightarrow$ high-dimensional SA

P5) Robustness analysis : PLI (Perturbed-Law based indices)

- The idea is to pertub the pdf (prob. measure μ_i) of X_i by a statistics (e.g. the mean, variance, …). With a δ perturbation, it gives the perturbed measure $\mu_{i\delta}$

[Lemaître et al. 2015]

- **Sensitivity indices on the QoI (e.g. a probability of failure *P*):**

$$\mathbf{S}_{i\delta} = \left(\frac{P_{i\delta}}{P} - 1\right)\mathbf{1}_{P_{i\delta} \geq P} + \left(1 - \frac{P}{P_{i\delta}}\right)\mathbf{1}_{P_{i\delta} < P}$$

- Two main technical ingredients:

- Minimisation of the Kullback-Leibler divergence

- Reverse importance sampling to estimate P and $P_{i\delta}$ using the same model outputs

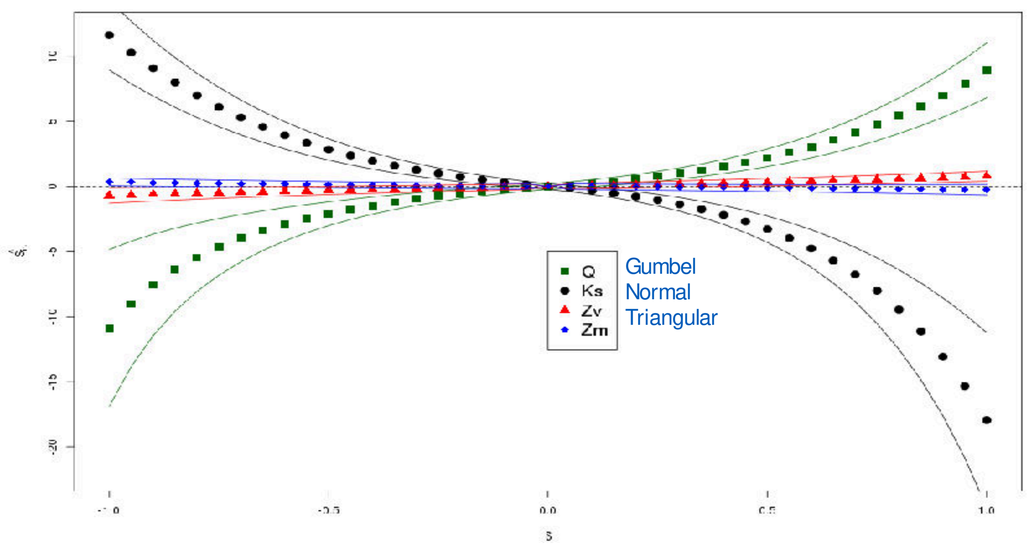
$$P_{i\delta} = \int 1_{G(\mathbf{X})<0} \frac{\mu_{i\delta}(X_i)}{\mu_i(X_i)} \mu_{\mathbf{X}} d\mathbf{X}$$

Example :

P = Proba(model output > threshold)

Monte Carlo : $N = 10^5$, $P = 9*10^{-4}$

Perturbation on the mean of each input pdf



P5) Examples of perturbed probability laws

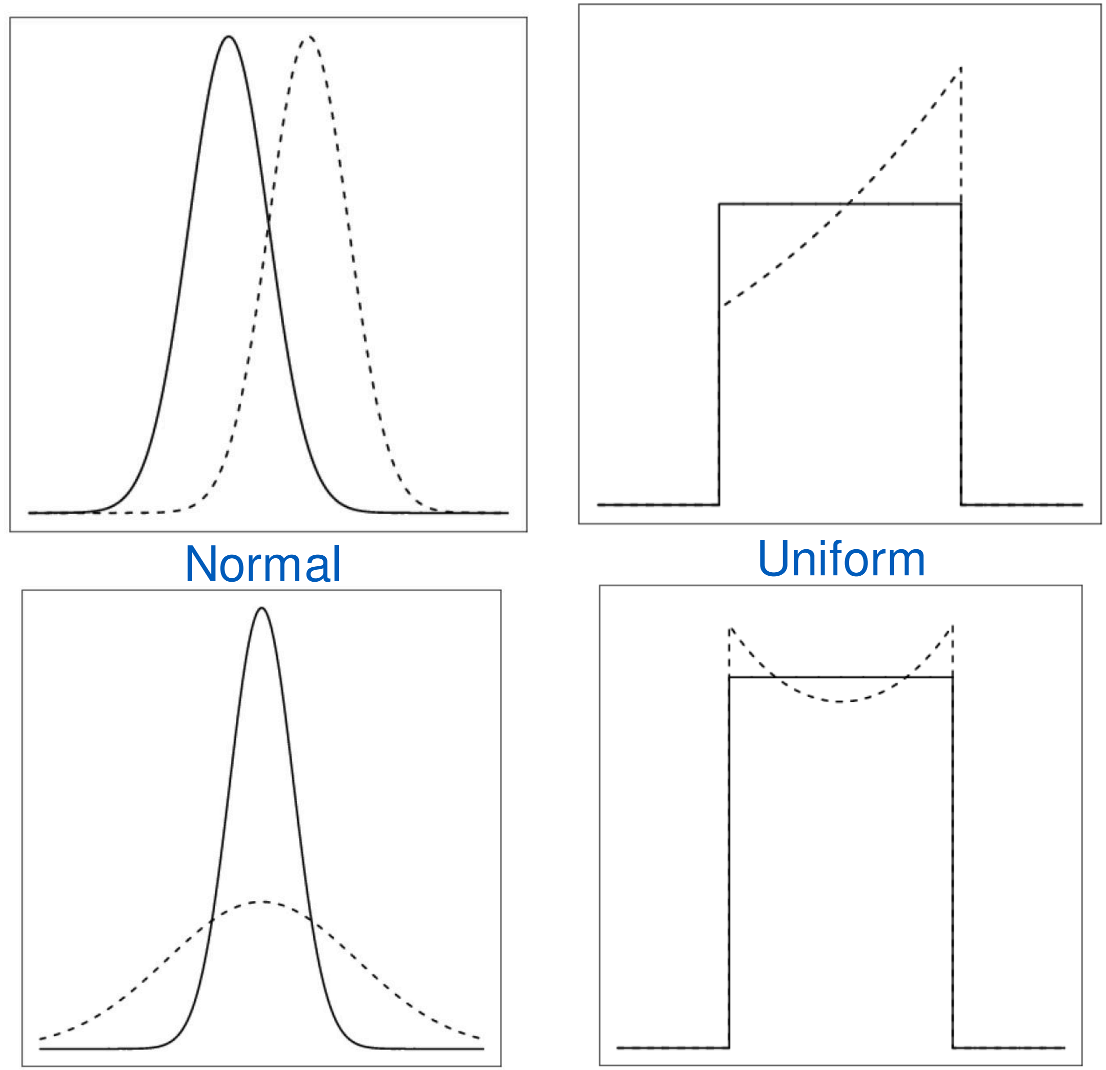
Mean perturbation:

$$\mathbb{E}[X_i] = \mu + \delta$$

Variance perturbation

$$\mathbb{E}[X_i] = \mu$$

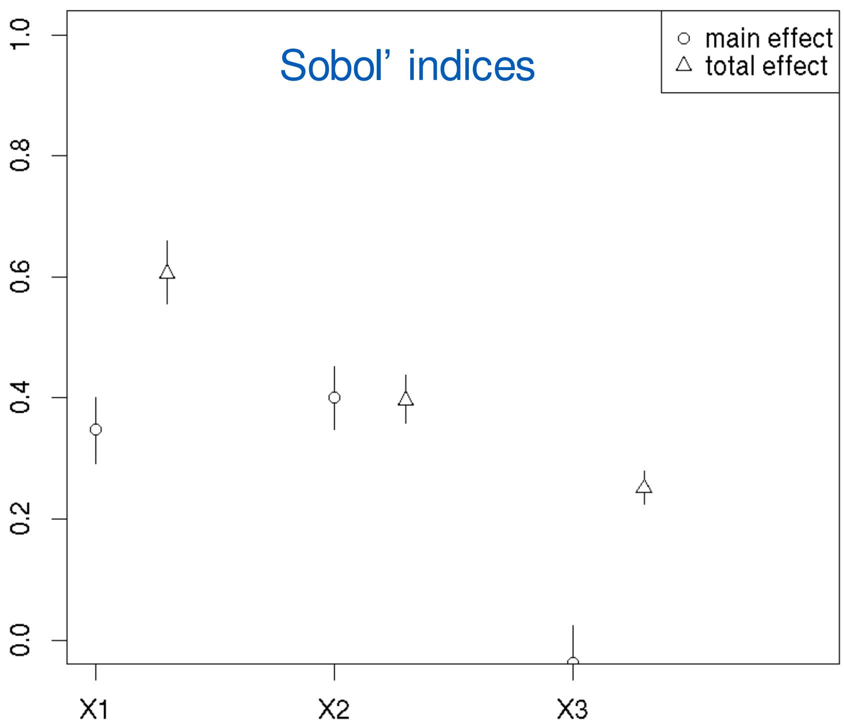
$$\mathrm{Var}[X_i] = \sigma^2 + \delta$$

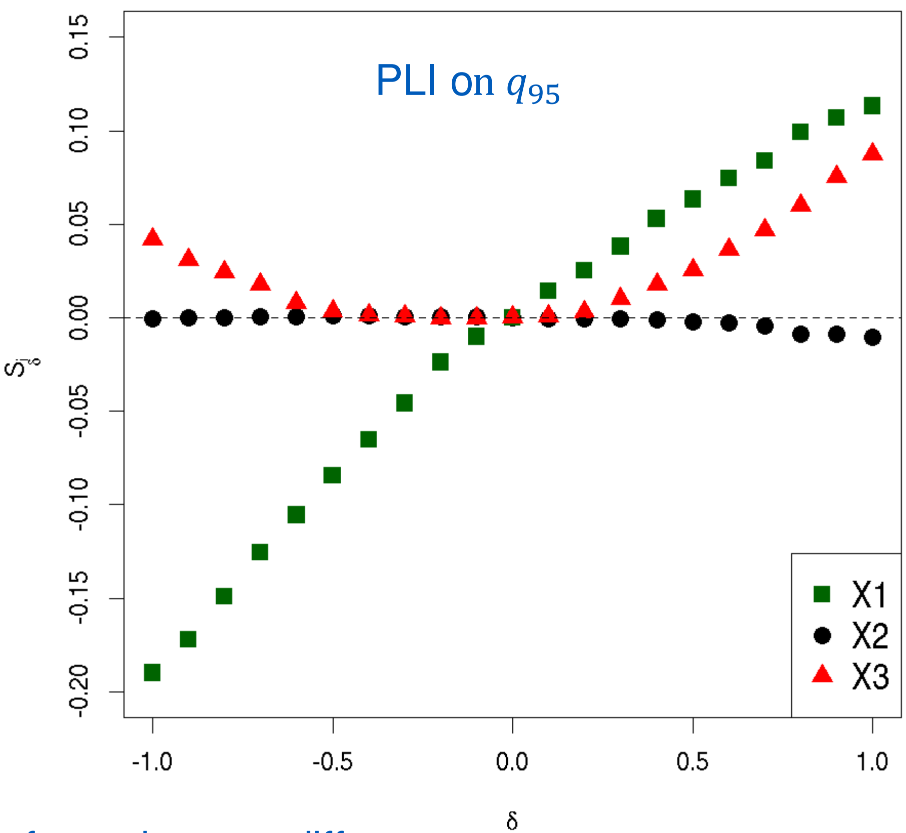


P5) Quantile-PLI – Analytical example

[Sueur et al. 2017]

$G(X) = sin(X_1) + 7 * sin^2(X_2) + 0,1 * X_3^4 * sin(X_1)$; $X_i \sim U(-\pi,\pi)$ independent

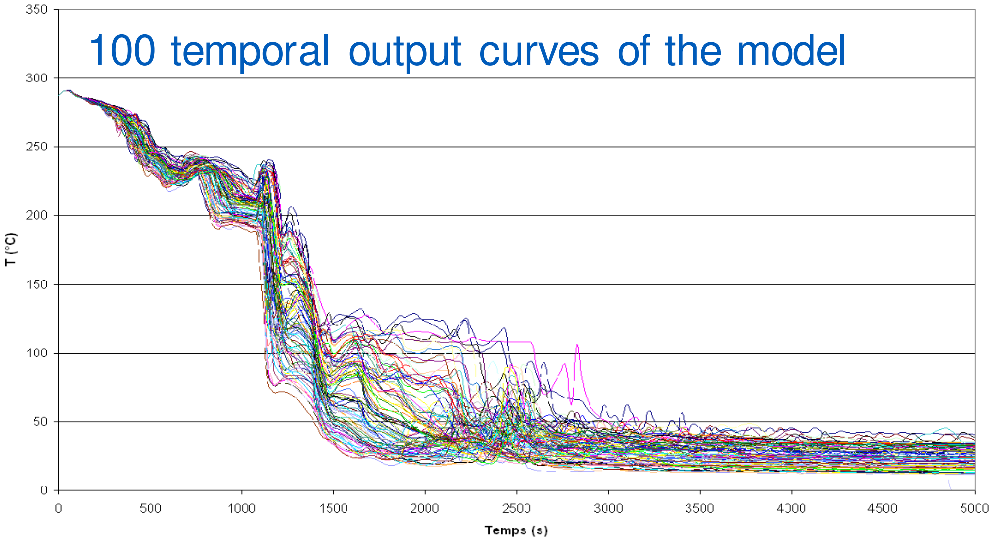




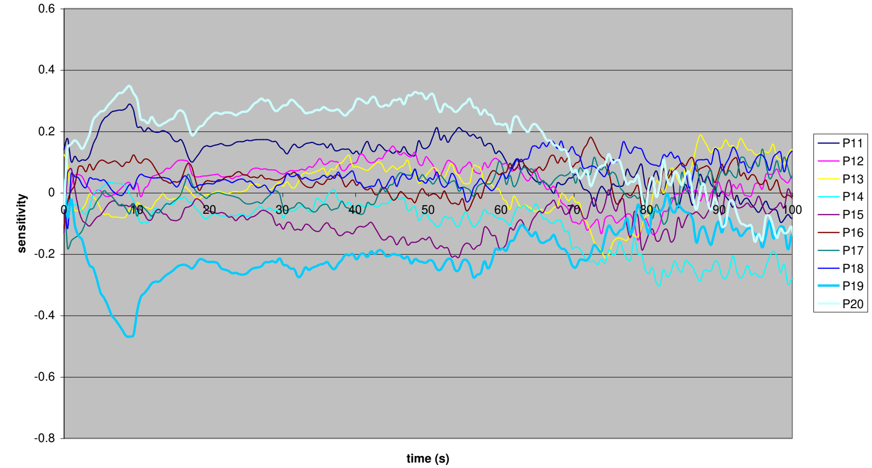
The provided information are different

=> Robustness analysis for BEPU methodolology

P6) Ubiquitous and aggregated indices



Functional (in time) sensitivity indices



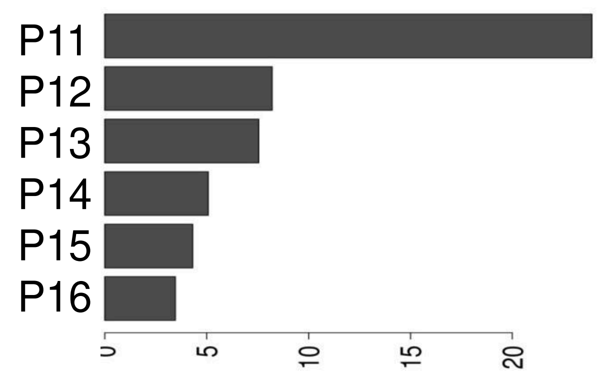
Synthesize the information on sensitivity indices:

Aggregated sensitivity indices:

Output variance at time t_k

$$GSI_i = \sum_{k=1}^{q} \frac{I_k}{I} S_i(t_k)$$

Total output variance



[Lamboni et al. 2009; Gamboa et al. 2014]

Conclusion: choosing the right method?

- Requested information (qualitative/quantitative)
- Number of inputs
- Regularity of the model (linearity/monotony/continuity)
- Cpu cost of one model evaluation
- Number of outputs
- Additional constraints, for example :
  - Uncertainty/sensitivity joint analysis,
  - Dependency between inputs, …

Recall: Main objectives of sensitivity analysis

1. **Understand the behaviour of the model (decompose input-output relations)**

2. **Simplify the computer model (dimension reduction)**

Screening

- **Determine the non-influent variables** (that can be fixed)
- Determine the non-influent phenomena (to skip in the analysis)
- Build a simplified model, a metamodel

3. **Prioritize the uncertainty sources to reduce the model output uncertainty**

Quantitative partitioning

- Variables to be fixed to obtain the **largest output uncert. reduction**

- Most influent variables in a given output domain

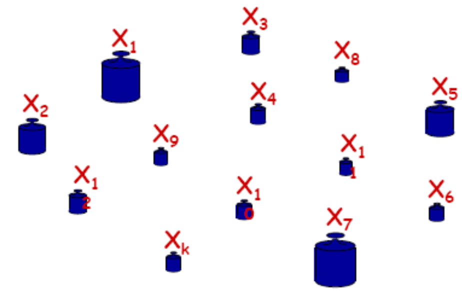
4. **Analyze the robustness of the quantity of interest (QoI) with respect to the input uncertainty laws**

Robustness analysis

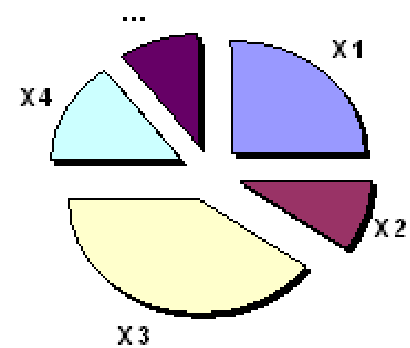
Recall: Methodology of SA

(QoI= variability of the output)

Three types of answers:

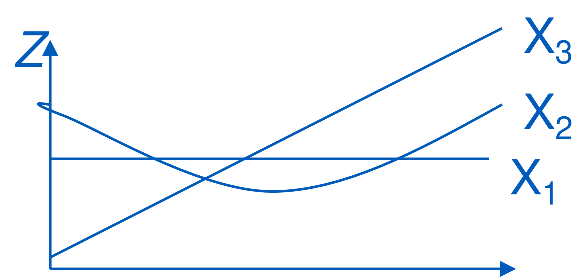


1. Screening (qualitative information: influent/non influent)
   - classical design of experiments,
   - numerical design of experiments (Morris, sequential bifurcation)

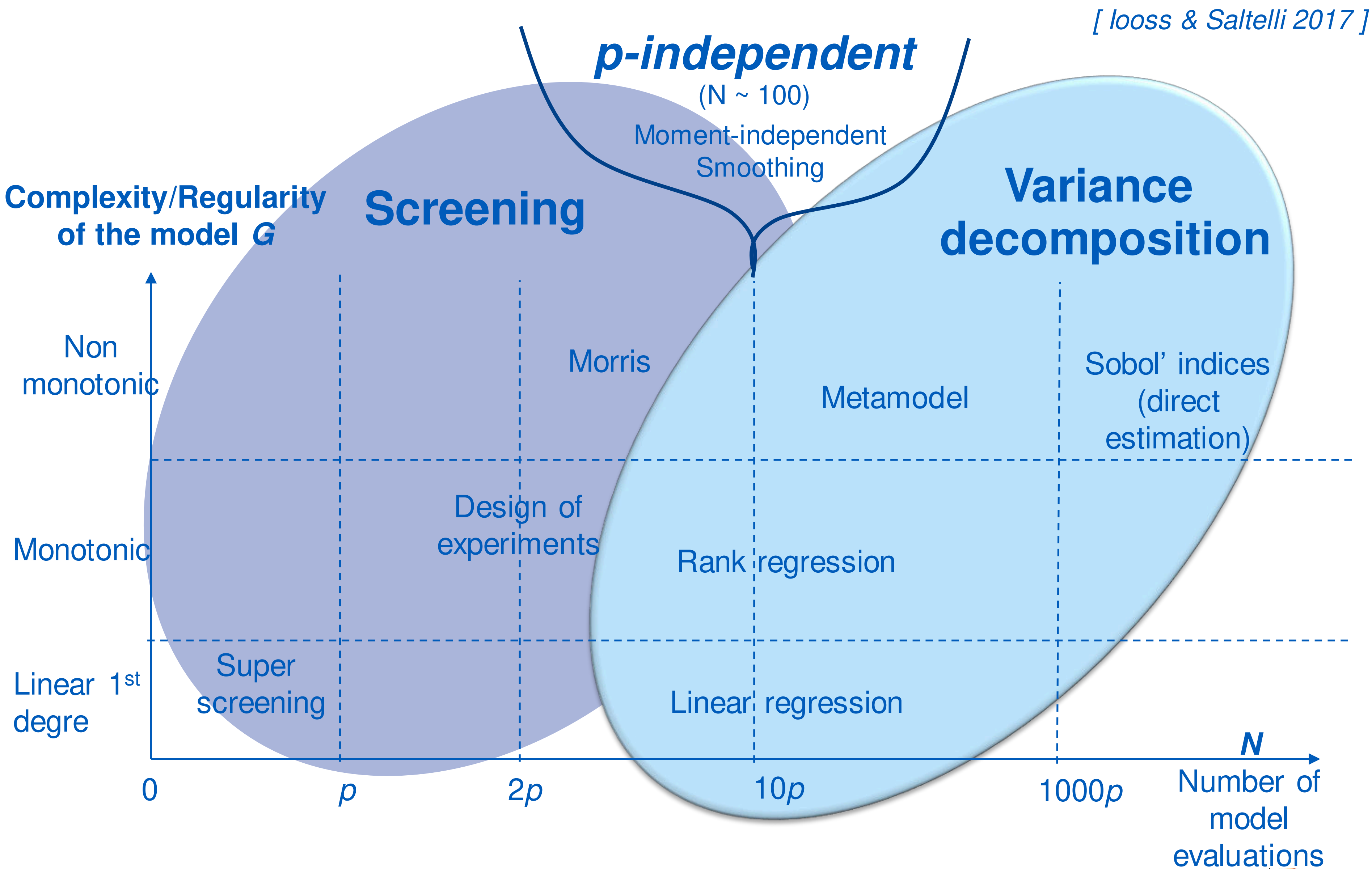


2. Quantitative measures of global influence
   - correlation/regression on values/ranks,
   - functional variance decomposition (Sobol)

3. Deep exploration of sensitivities
   - smoothing techniques (param./non parametric)
   - metamodels



Classification of methods



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