A weather generator for simulating multivariate climatic series

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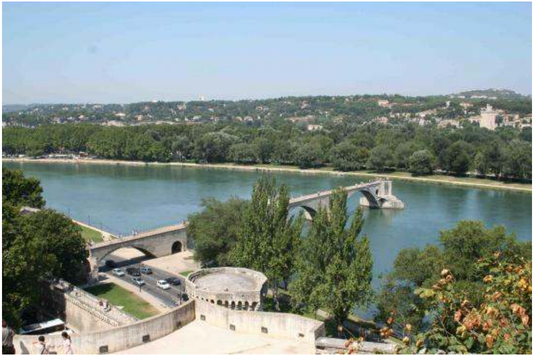
Outline

What is a Weather generator (WG)?



Sur le Pont d'Avignon

What is a Weather generator (WG)?

December 2003

What is a Weather generator (WG)?

March 2010

What is a Weather generator (WG)?

December 2003

Summer

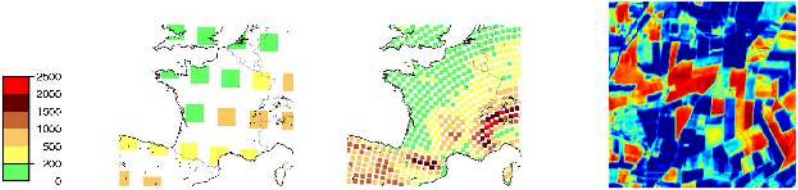
March 2010

- Stochastic model for generating series of daily climatic variables (here, P, Tn, Tx, R, W)
- Calibrated on recorded series

For what purpose?

- Impact studies when climate is involved: e.g. crop models for agriculture
- Explore unmeasured climates
- Disaggregating (downscaling) climatic variables, in particular for GCM outputs (not yet done)

Disaggregation of climatic variables



- Resolution of GCM outputs is at best $\sim$ at a 50 km scale
- But impact studies need climatic variables at the appropriate spatial resulation and time step
- Some plant models need hourly rainfall, which are rarely measured

Outline

Some existing weather generators (WG)

Data-base oriented (non parametric)

- Pros: compatibility between climatic variables is guaranteed
- statistical features are reproduced
- Cons: cannot create unobserved meteorological situations (impact studies !)

Model based (parametric)

- Pros: can create non recorded situations
- Cons: existing WG are limited two a wet/dry classification; non flexible classes of densities

WACS-Gen

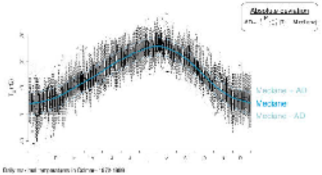
Weather-state Approach Conditionally Skewed- generator

- Parametric, model-based approach
- Accounts for seasonality and inter-annual trend
- Several dry and wet states
- Mixture of multivariate skew-normal densities
- With temporal correlation

PhD Thesis of Cédric Flécher, (co-supervision with Philippe Naveau, LSCE-CNRS, and Nadine Brisson, AgroClim, INRA)

Removing the trend

- For P, Tn, Tx, R, W: build standardized residuals using smoothed median and absolute standard deviations



- Create 4 seasons: MAM, JJA, SON, DJF
- Apply Gamma transform on P for each season

Now work on residuals, for each season independently

Weather States

For each season

- Model-based clustering (Mclust, Fraley & Raftery, 2002) for dry and wet days
  - estimate # states (using BIC)
  - provides a soft classification of days

  Weather states as Markov Chain with transition matrix estimated from soft classification

Multivariate density

For each seaon and each weather state, residuals ($k = 4$ or 5) are distributed according to Complete Skew-Normal distribution

$$f_k(y) = 2^{-k}\phi_k(y;\mu,\Sigma)\Phi_k(S\Sigma^{-1/2}(y-\mu);0,I_k - S^2)$$

- μ vector of location parameter
- Σ is a matrix of co-variation
- S is a diagonal matrix of skewness

Outline

The Closed Skew-Normal (CSN) distribution

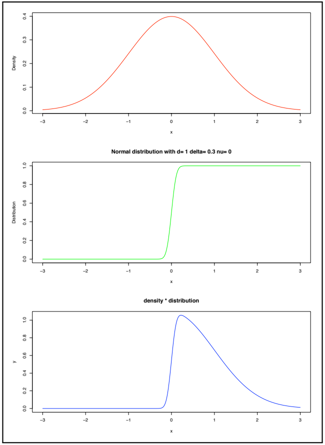
CSN pdf $CSN_{n,m}(\mu, \Sigma, D, \nu, \Delta)$

$$f(y) = \frac{1}{\Phi_m(0; \nu, \Delta + D^t \Sigma D)} \phi_n(y; \mu, \Sigma) \Phi_m(D^t(y - \mu); \nu, \Delta)$$

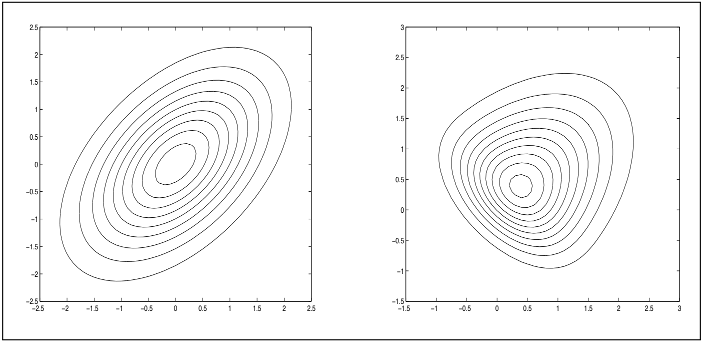
- If $D = 0$: $N_n(\mu, \Sigma)$
- If $m = 1$: skew-normal distribution [Azzalini, 2005; Arellano–Valle, and Azzalini, 2008; Dominguez-Molina and gonzales-Farias, 2003]

Example

$$m = n = 1; \mu = 0, \sigma^2 = 1, d = 1, \nu = 0.3, \Delta = 0.3$$



Gaussian and CSN bivariate density



Some properties of CSN distributions

Linearity

$$A \times CSN_{n,m}(\mu, \Sigma, D, \nu, \Delta) \sim CSN_{r,m}(A\mu, \Sigma_A, D_A, \nu, \Delta_A)$$

where

$$\Sigma_A = A\Sigma A^T, \quad D_A = D\Sigma A^T \Sigma_A^{-1}, \quad \Delta_A = \Delta + D\Sigma D^T - D_A \Sigma_A D_A^T$$

Sum (particular case)

$$N(\mu, \Sigma) + CSN_{n,m}(\psi, \Omega, D, \nu, \Delta) \sim CSN_{n,m}(\psi + \mu, \Omega + \Sigma, D, \nu, \Delta)$$

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Some properties of CSN distributions

Conditioning

Consider $Y = (Y_1, Y_2) \sim CSN_{n,m}(\mu, \Sigma, D, \nu, \Delta)$.
Then, $Y_2 | Y_1 = y_1$ is

$$CSN(\mu_2 + \Sigma_{21}\Sigma_{11}^{-1}(y_1 - \mu_1), \Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}, D_2, \nu - D_1 y_1, \Delta)$$

Moment generating function

$$M(t) = \frac{\Phi_m(D^t\Sigma t; \nu, \Delta + D\Sigma D^T)}{\Phi_m(0; \nu, \Delta + D\Sigma D^T)} \exp\{\mu^T t + \frac{1}{2}(t^T \Sigma t)\}$$

From the MGF one can derive first and second moments

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Hierarchical construction

$$\begin{pmatrix} Y \\ X \end{pmatrix} \sim N_{n+m}\left(\begin{pmatrix} 0 \\ \nu \end{pmatrix}, \begin{pmatrix} \Sigma & -D^t\Sigma \\ -\Sigma D & \Delta + D^t\Sigma D \end{pmatrix}\right),$$

Then

$$\mu + (Y|X \leq 0) = CSN_{n,m}(\mu, \Sigma, D, \nu, \Delta)$$

Simulation algorithm

1. simulate a vector $X \sim N_m(\nu, \Delta + D^t\Sigma D)$, conditional on $X \leq 0$
2. simulate a vector Y conditionally on X, according to the bivariate model above
3. return $\mu + Y$

Outline

CSN for WACS-gen

To simplify the model, we set

- $m = n = k$
- $D = \Sigma^{-\frac{1}{2}} S$
- $\Delta = I_k - S^2$
- $S = \text{diag}(\delta_1, \ldots, \delta_K)^t$.

$CSN_k^*(\mu, \Sigma, S)$:

$$f_k(y) = 2^{-k} \phi_k(y; \mu, \Sigma) \Phi_k(S\Sigma^{-1/2}(y - \mu); 0, I_k - S^2)$$

Hence

$$\tilde{X} = \Sigma^{-1/2}(X - \mu) \sim CSN_5^*(0, I_5, S)$$

Estimation of the parameters

Maximum likelihood is known to be difficult and non-robust
Estimation is done by weighted moments

Using as third moment the quatity $E[\Phi_k(Y, 0, I_k)]$ with

$$E[\Phi_k(Y,0,I_k)] = 2^k \Phi_{2k}\left(0; \begin{bmatrix} -mu \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma + I_k & \lambda\Sigma^{1/2} \\ \lambda\Sigma^{1/2} & I_k \end{bmatrix}\right)$$

[Flecher, Allard and Naveau (2009) *Stat. Prob. Letters*]

Estimation workflow

- Remove inter-annual and seasonal trend for Tn, Tx, R, W
- For each season $\{$
    1. Gamma transform P
    2. Mclust classification of weather states (WS)
    3. Estimation of transition matrix
    4. For each WS, estimation of the CSN parameters
    5. Estimation of temporal correlation

  $\}$

Simulation algorithm

- Simulate the Markov Chain WS(t)
- Fort $t = 1, \ldots, T$ $\{$
  1. Conditionnally on WS simulate $\tilde{X}(t) \sim$ CSN given $\tilde{X}(t-1)$
  2. Transform $\tilde{X}(t)$ into $X(t)$
- Add seasonal and interannual trend

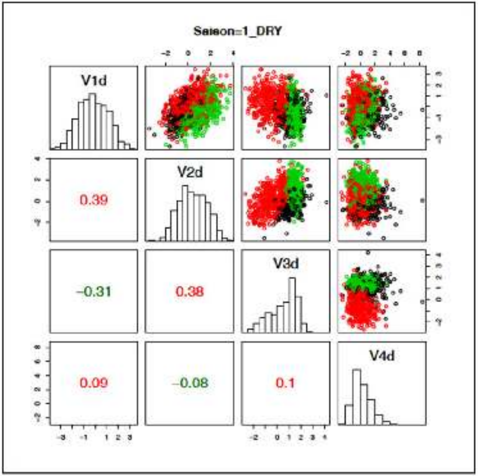
Outline

Illustration

30 year series in Colmar, France

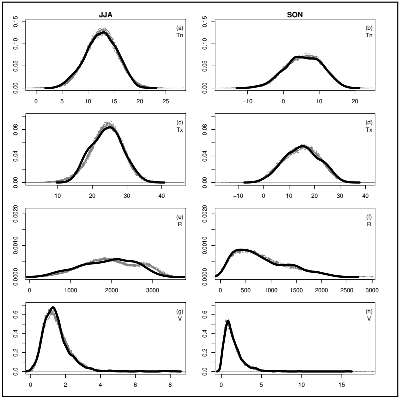
- Well marked seasonal cycle: cold winters ($\bar{T}n = 2^\circ$C in Jan.) and warm summers ($\bar{T}x = 25^\circ$C in Jul.)
- $\approx$ 1/4 wet days
- Average P is 530 mm/year
- 3 wet WS / season (exc. JJA)
- 3 dry WS / season

Classification of residuals



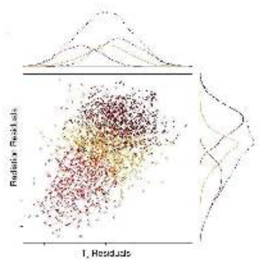
JJA dry days

Marginal densities



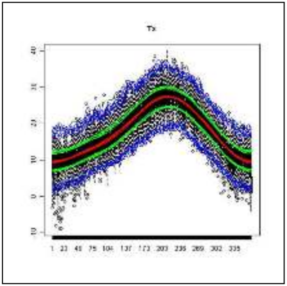
JJA and SON, dry days

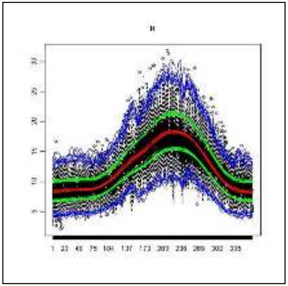
Marginal densities: pair (Tx, R)



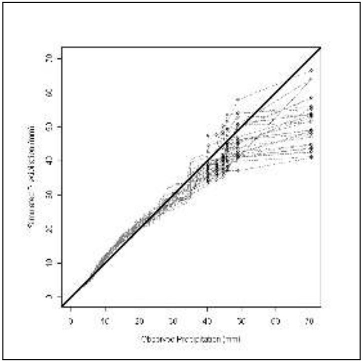
JJA dry days

More general picture: *Tx* and *H*

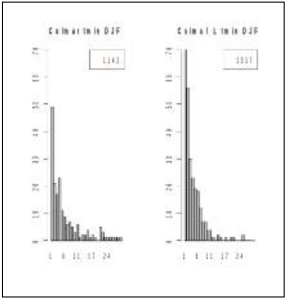


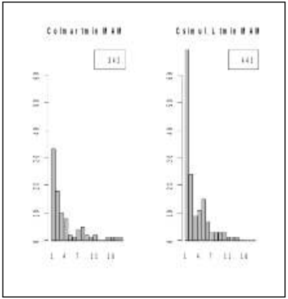


Precipitation

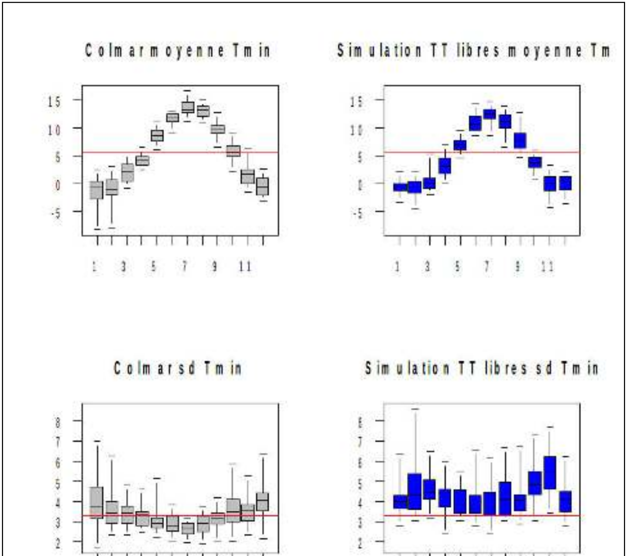


Persistence of $Tn < 0$





Variability



In conclusion

WACS-gen is a flexible weather generator which

- overcomes existing limitations of previous WG
  - beyond dry/wet days
  - skewed densities able to capture natural asymetries of climatic variables
  - good variability of monthly averages
- But still problems
  - non robust estimators for skewness parameter
  - too many free parameters?

[Flecher, Naveau, Allard, Brisson (2010) A stochastic daily weather generator for skewed data, *Water Resources Research*, **46**, W07519]