Adaptive Stochastic Metamodels for Truss Optimization

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My background

I pursued my academic studies at the French Institute for Advanced Mechanics (FIAM) in Clermont-Ferrand (France) from 2005 to 2009, where I obtained my mechanical engineering diploma in July 2009. I studied there structural engineering and enlarged my interests to reliability based design of structures. Through my last studies project supervised by Professor Lemaire in collaboration with Phimeca Engineering, I worked on efficient (stochastic) methods for the calculation of fragility curves. This gave me the opportunity to start a PhD Thesis at the Free University of Brussels in October 2009 which is entitled " Stochastic Finite Elements for Multicriteria Optimization under Uncertainty ". The aim of this thesis is to build uncertainties treatment in a multicriteria optimization process for an industrial use (i.e. at low computational cost). This thesis is realised in the laboratory BATir, supervised by Rajan Filomeno Coelho and in collaboration with the laboratory Roberval from the (Technological University of Compiègne, France).

My abstract

Multiobjective optimization has met a growing interest the last decades. In mechanical engineering, these methods become more and more used thanks to their ability to solve "real world" optimization problems in which objectives functions are usually conflicting. A reasonable way to solve these Multiobjective Optimization Problems (MOP) is to investigate a set of compromise solutions (called the *Pareto Set* see definition below) which satisfy the objectives and the constraints at an acceptable level without being dominated by any other solutions. To fathom this notion few definitions are needed:

- *Weak Pareto dominance:* A vector $\mathbf{u}$ weakly dominates a vector $\mathbf{v}$ in a Pareto sense if and only if $\forall i \in \{1, ..., m\}, u_i \leq v_i$. We write then $\mathbf{u} \succeq \mathbf{v}$.
- *Pareto Set:* It is the set P^* of all non dominated objective functions values $P^* = \{\mathbf{x}^* \in \Omega \mid \nexists \mathbf{x} \in \Omega; \mathbf{f}(\mathbf{x}) \succeq \mathbf{f}(\mathbf{x}^*)\}$
- *Pareto Front:* It is the representation in the objectives functions space of the Pareto Set.

Evolutionary algorithms are well suited to address MOPs because they intrinsically manipulate a population solutions from which, it is then possible to extract the Pareto Set. In the literature, Genetic Algorithms (GA) are widely used and particularly adapted to treat multiobjective optimization problems. Moreover, a great attention has also been devoted to adapt evolutionary algorithms in order to evaluate the robustness of each solutions produced in relation to deterministic parameters. However, one aspect has to be taken into account more carefully: the *handling of uncertainties*. An original formulation to tackle with uncertainties in a robust multiobjective optimization context is proposed here (Eq 1):

$$\min_{\mathbf{x},\boldsymbol{\eta}} \quad \boldsymbol{\zeta} = [\zeta_1, \cdots, \zeta_m]^\top \left\{ \begin{array}{ll} \text{subjected to} & \\ \quad P_{\text{non-dominance}} & \equiv P[\mathbf{f}(\mathbf{x},\boldsymbol{\theta}) \succ \boldsymbol{\zeta}] \geq \alpha^{\mathbf{f}} \\ \quad P_{\text{reliability}} & \equiv P[\mathbf{g}(\mathbf{x}) \leq 0] \geq \alpha^{\mathbf{g}} \\ \text{and with } \zeta_i = E[f_i(\mathbf{x},\boldsymbol{\theta})] + \eta_i \sigma[f_i(\mathbf{x},\boldsymbol{\theta})] & \end{array} \right. \tag{1}$$

where $\mathbf{f}$ is the vector of objectives functions; $\mathbf{g}$ is the vector of constraints; $\mathbf{x}$ and $\boldsymbol{\theta}$ are the variables vectors of the problem, respectively, deterministic and random variables; $E[\ldots]$ and $\mu[\ldots]$ the mean and the standard deviation operators, ζ_i are the quantile approximations of the objective function f_i, and η_i a control on the dispersion of the objective function f_i around its mean.

Practically, the probabilities, that appeared in the formulation (Eq 1), have to be computed at each design point for each iteration of the GA. Their estimation at low computational costs is the main limitation of this formulation. To alleviate this issue, non-intrusive stochastic surrogates based on the polynomial chaos expansion are proposed. This is aimed at decreasing the number of calls to the mechanical model along the optimization process. Those surrogates are

computed in two steps:

1- First, for a given sampling of the deterministic variables $\mathbf{x} = \{x^{(1)}, \ldots, x^{(N)}\}$, a decomposition of the random responses f_i, g_i are performed through the Polynomial Chaos Expansion (PCE).

$$f_i(\mathbf{x}, \boldsymbol{\theta}) = \gamma_0(\mathbf{x}) + \sum_{j=1}^{J} \Psi_j(\boldsymbol{\theta}) \gamma_j^{f_i}(\mathbf{x}) \tag{2}$$

where $\boldsymbol{\theta} = \{\theta_1, \cdots, \theta_m\}$, $J = \frac{(m+p)!}{m!p!}$, where p is the order of the polynomial chaos, Ψ_j, $j = 1, \ldots, J$ are the polynomial basis on which are expanded the stochastic objective and constraint functions, $\gamma_j^{f_i}$ $j = 1, \ldots, J$ are the coefficient of the PCE for one particular stochastic objective function f_i obtained by regression.

2- Then another issue is to compute the limited set of chaos coefficients. Some approaches (regression and projection approach) have been developed [Berveiller, 2005], [Blatman & Sudret, 2008] to compute these coefficients by performing calls to the deterministic model (Non-intrusive approaches). Here a method inspired from the non-intrusive regression method is proposed for problems with mixed deterministic and random variables. Practically, the coefficients of the PCE are defined by using a predefined number Q of collocation points for each deterministic design point. Thus , for each design point $\mathbf{x}$, Q collocation points $\boldsymbol{\Theta} = \{\boldsymbol{\theta}^1, \ldots, \boldsymbol{\theta}^Q\}$ are defined. By calling directly the mechanical model (*high fidelity simulations*) the so called "high fidelity" PCE coefficients $\boldsymbol{\gamma}$ are obtained by solving the following set of equations (Eq 3) and stored in the Design Of Experiment (DOE). A surface response is then built thanks to this DOE. Then, the PCE of each stochastic objective function can be built thanks to these metamodels. At the end of the multicriteria optimization process to the PCE based Pareto front (Fig 1) is obtained.

$$\begin{Bmatrix} f(\mathbf{x}, \boldsymbol{\theta}^1) \\ \vdots \\ f(\mathbf{x}, \boldsymbol{\theta}^Q) \end{Bmatrix} = \begin{bmatrix} \Psi_1(\boldsymbol{\theta}^{(1)}) & \cdots & \Psi_j(\boldsymbol{\theta}^{(1)}) \\ \vdots & & \vdots \\ \Psi_1(\boldsymbol{\theta}^{(Q)}) & \cdots & \Psi_j(\boldsymbol{\theta}^{(Q)}) \end{bmatrix} \begin{Bmatrix} \gamma_1(\mathbf{x}) \\ \vdots \\ \gamma_j(\mathbf{x}) \end{Bmatrix} \tag{3}$$

$$\text{or more concisely:} \qquad \mathbf{f}(\mathbf{x}, \boldsymbol{\Theta}) = \boldsymbol{\Psi}(\boldsymbol{\Theta})\boldsymbol{\gamma}$$

The methods shortly described above is applied first to classical truss structures. The PCE based Pareto Front is here represented in the case where only two stochastic objectives stochastic functions were considered with non-correlated Gaussian variables. Each point $\boldsymbol{\zeta} = [\zeta_1, \zeta_2]^\top$ of a non deterministic Pareto Front can be associated with a probability of α^f that the chosen solution dominates the point.

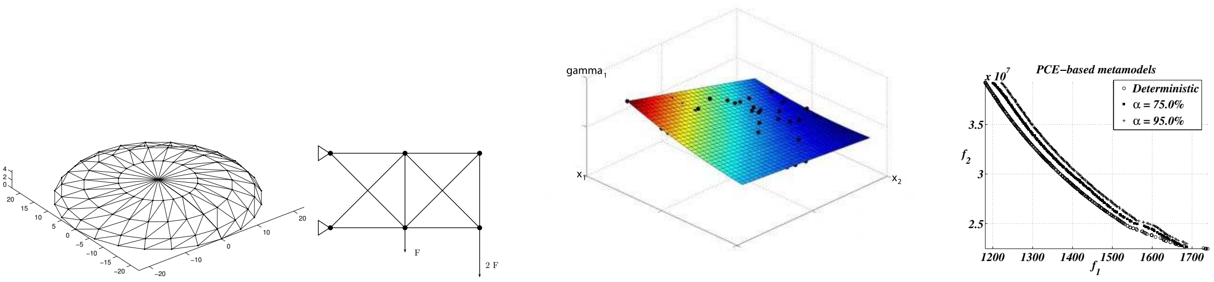

Figure 1: Example of Structures used and Stochastic Metamodels obtained to establish a PCE based Pareto Front

This method is based on stochastic metamodels which can be performed by few well known methods like Polynomial Response Surfaces, Kriging, Support Vector Machine, Moving Least Squares, Radial Basis Functions, Artificial Neural Networks, Evolutionary Response Surfaces [Filomeno Coelho & Breitkopf, 2009], which allow for a given set of deterministic variables to establish the approximate surface responses of the chaos coefficients. Nevertheless, as the precision of the stochastic response of the structure is a key point which directly depends on the calculation of the PCE coefficients, a further move has to be performed to improve the accuracy of the metamodels used. Efficient adaptive surrogates are developed to enrich the DOE by computing the high fidelity model. Further researches are to be carried out on this topic.

References

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