Approximate inverse problem solving with joint generative models and set estimation in latent space

E. Maalouf
University of Neuchâtel

Supervisor(s): Prof. David Ginsbourger (IDIAP & University of Bern), Prof. Kilian Stoffel (University of Neuchâtel)

Ph.D. expected duration: Sept. 2018 - Sep. 2021

Adress: Information Management Institute; A.-L. Breguet, 2; 2000 Neuchâtel; Switzerland

Email: eliane.maalouf@unine.ch

Abstract:

Generative Neural Networks (GNN) are being increasingly used in the context of inverse problem solving (e.g., in imaging [5][3] and in geoscience [4][6]). Probabilistic inversion in such a framework was recently proposed by [4] where a Generative Adversarial Network is trained to map from a low-dimensional latent space, endowed with a simple prior, into the targeted input space, allowing to sample unconditionally from this space using a low-dimensional parametrization. Samples from the posterior on the latent vectors are then generated using Markov Chain Monte Carlo. We leverage recent advances in set estimation with the aim to uncover the whole range of possible inverse solutions associated with a data response while keeping a moderate computational budget and complying with likelihood-free approaches. We denote by $\mathbf{x}$ the targeted input, by $\mathbf{y}$ the response of interest, and by F the deterministic mapping linking $\mathbf{y}$ to $\mathbf{x}$ (possibly up to some measurement error, or "noise"). The vector $\mathbf{x}$ is typically of very large dimension and $\mathbf{y}$ of smaller dimension. We further assume that both $\mathbf{x}$ and $\mathbf{y}$ can be expressed as a function of some latent vector $\mathbf{z}$ (of moderate dimension compared to $\mathbf{x}$), say $(\mathbf{x}, \mathbf{y}) = g(\mathbf{z}) = (g_1(\mathbf{z}), g_2(\mathbf{z}))$. In GNN modelling, it is customary to postulate a probability distribution for $\mathbf{z}$. We denote this distribution by $\mu_{\mathbf{z}}$ and use the notation $\mathbf{Z}$ to speak of a random vector possessing this distribution. We similarly denote by $(\mathbf{X}, \mathbf{Y}) = g(\mathbf{Z}) = (g_1(\mathbf{Z}), g_2(\mathbf{Z}))$ and by $\mu_{(\mathbf{X},\mathbf{Y})}$ the induced "push-forward" probability distribution of $(\mathbf{X}, \mathbf{Y})$. A consequence of the underlying latent space assumption is that, under this formalism, uncovering the conditional distribution of $\mathbf{X}$ knowing $\mathbf{Y}$ amounts to uncovering the distribution of $g_1(\mathbf{Z})$ knowing $g_2(\mathbf{Z})$, which follows in turn from the conditional distribution of $\mathbf{Z}$ knowing $g_2(\mathbf{Z})$. Yet the latter is not straightforward to derive for several reasons:

- g_2 is not known a priori and must me estimated from data;
- $g_2(\mathbf{Z}) = \mathbf{Y}$ is generally observed in noise, and the noise distribution is not necessarily known;
- Deriving conditional distributions typically requires heavy computational procedures.

Our approach starts with a sample from the joint distribution $\mu_{(\mathbf{X},\mathbf{Y})}$ and we rely on existing approaches for fitting a GNN model based on it, resulting in some estimated $\hat{g} = (\hat{g}_1, \hat{g}_2)$. In the example of the figures below we used couples (X, Y) such that $Y = X^2$ and $X \sim \mathcal{U}(-3, 3)$. We want to retrieve the set of values of X that correspond to the condition $y \approx 7$. We used vectors $\mathbf{Z} = (Z_1, Z_2)$ with each $Z_i \sim \mathcal{U}(-5, 5)$. Our approach then consists in using statistical techniques to estimate the set of values of the latent vector $\mathbf{Z}$, Γ_Z, such that $\hat{g_2}(\mathbf{Z})$ is close to the observed y. Particularly, we rely on recent algorithms in Bayesian set estimation based on Gaussian Process models [2][1] and demonstrate their potential to gain insights on regions of the latent space that are compatible with observed data (cf. top line of Figure 1). This leads to new approaches for inverse problem solving and related uncertainty quantification tasks (cf. bottom line of Figure 1). We will evaluate our methodology on linear, non-linear, toy and real inverse problems.

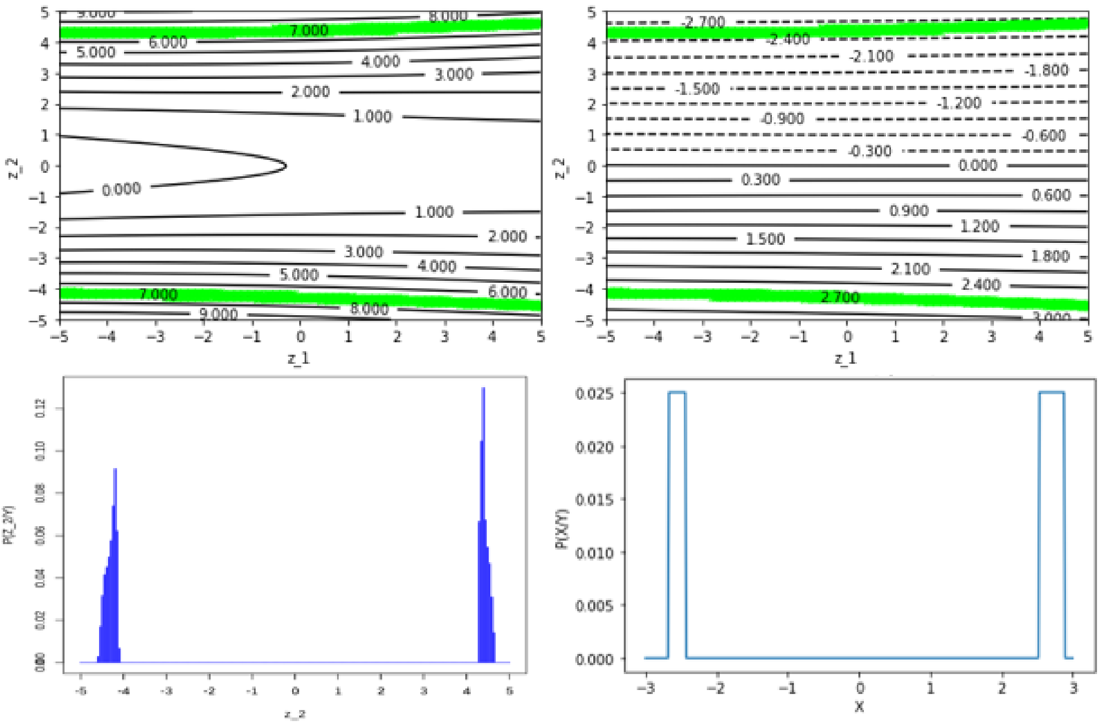

Figure 1: Top left: Γ_Z (in green) with Y contours; Top right: Γ_Z (in green) with X contours; Bottom left: posterior $P(Z_2/y \approx 7)$; Bottom right: posterior $P(X/y \approx 7)$

References

[1] Dario Filippo Azzimonti. *Contributions to Bayesian set estimation relying on random field priors.* PhD thesis, Philosophisch-naturwissenschaftliche Fakultät der Universität Bern, 2016.

[2] Clément Chevalier. *Fast uncertainty reduction strategies relying on Gaussian process models.* PhD thesis, 2013.

[3] Kyong Hwan Jin, Michael T McCann, Emmanuel Froustey, and Michael Unser. Deep convolutional neural network for inverse problems in imaging. *IEEE Transactions on Image Processing*, 26(9):4509–4522, 2017.

[4] Eric Laloy, Romain Hérault, Diederik Jacques, and Niklas Linde. Training-image based geostatistical inversion using a spatial generative adversarial neural network. *Water Resources Research*, 54(1):381–406, 2018.

[5] Morteza Mardani, Enhao Gong, Joseph Y Cheng, Shreyas S Vasanawala, Greg Zaharchuk, Lei Xing, and John M Pauly. Deep generative adversarial neural networks for compressive sensing mri. *IEEE transactions on medical imaging*, 38(1):167–179, 2018.

[6] Lukas Mosser, Olivier Dubrule, and Martin J Blunt. Stochastic seismic waveform inversion using generative adversarial networks as a geological prior. *Mathematical Geosciences*, 52(1):53–79, 2020.

Short biography – Eliane Maalouf earned a telecommunications engineering diploma from the University of Saint-Joseph (Lebanon) and a MSc in Statistics from the University of Neuchâtel. Her research interests lie in machine learning, uncertainty quantification and statistical modelling.