Adapted line sampling and neural networks for evaluating the info-gap robustness of reliability estimates for penstocks

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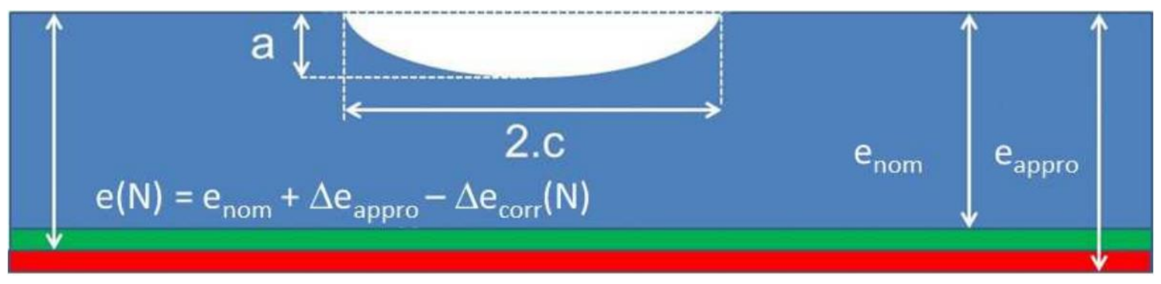
1. Industrial context

- Assessing the **reliability** of systems for risk-sensitive industries such as power generation is crucial to ensure their safety. Such assessment is subject to **uncertainty** which is commonly distinguished into two types:
  - **Aleatory:** inherent property of the physical quantity, considered irreducible
  - **Epistemic:** result of a lack of knowledge, potentially reducible by gathering more information
- Evaluating the **robustness** of reliability quantities (e.g., probabilities of failure) aims at checking that the reliability quantity of interest fulfills a safety requirement despite the presence of epistemic uncertainty.

❖ Motivation:
How to efficiently quantify the robustness of reliability quantities ?

❖ Challenge:
Propose a methodology based on a performant failure probability estimator and smart algorithms for reducing the computational burden induced by the info-gap method.

2. Reliability of penstocks [1]



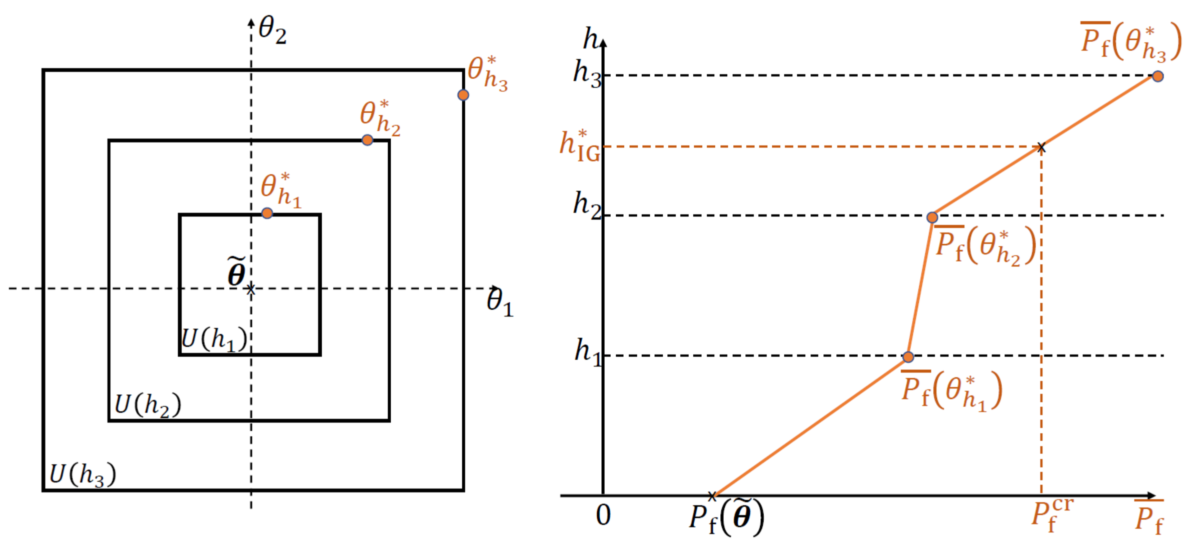
- The failure probability of penstocks under brittle failure at year $N+1$ is expressed as:

$$P_f = \frac{\Pr(\{G_{N+1} \leq 0\} \cap \{G_N > 0\} \cap \{G_{HPT} > 0\})}{\Pr(\{G_{HPT} > 0\})}$$

- **Three equivalent events:**
  - $E_1 = \{\max(G_{N+1}, -G_N, -G_{HPT}) \leq 0\}$
  - $E_2 = \{G_{N+1}.G_N \leq 0\} \cap \{G_{HPT} > 0\}$
  - $E_3 = \{G_{N+1} \leq 0\} \cap \{G_N > 0\} \cap \{G_{HPT} > 0\}$

X_i	Dist.	p1	p2	p3
$X_1 = R_m$	LN	480	24	-
$X_2 = \Delta e_{app}$	N	θ_1	0.25	-
$X_3 = \Delta e_{cor}$	N	θ_2	0.4	-
$X_4 = \varepsilon$	N	0	16.8	-
$X_5 = a$	U	0	θ_3	-
$X_6 = K_{IC}$	WM	θ_4	4	20

3. Info-gap [2]



- Robustness of P_f w.r.t. the uncertain distribution parameters $\boldsymbol{\theta}$:

$$h_{IG}^* = \max_{h \geq 0}\left\{\max_{\boldsymbol{\theta} \in U(\tilde{\boldsymbol{\theta}},h)} P_f(\boldsymbol{\theta}) \leq P_f^{cr}\right\}$$

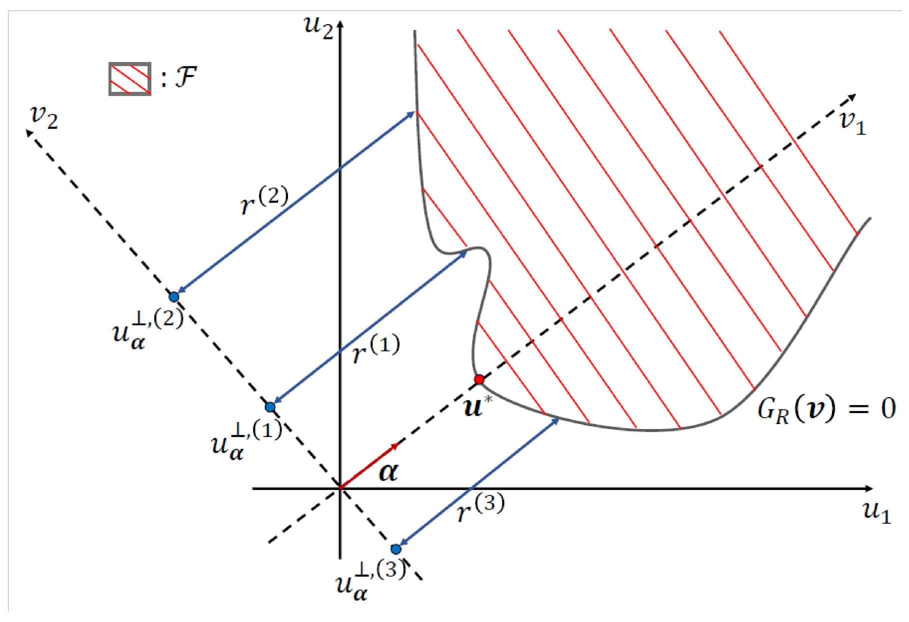
4. Generalities on line sampling [3]

- Isoprobabilistic mapping to the standard space:

$$\boldsymbol{U} = T(\boldsymbol{X})$$

- Isoprobabilitic rotation driven by $\boldsymbol{\alpha}$:

$$\boldsymbol{V} = \boldsymbol{R}\boldsymbol{U}$$



$$P_f = \int_{\mathbb{R}^{n-1}} \int_{G^\perp \leq 0} \varphi_{V_1(v_1)}\, dv_1 \varphi_{U_\alpha^\perp}(\boldsymbol{u}_\alpha^\perp) d\boldsymbol{u}_\alpha^\perp = \int_{\mathbb{R}^{n-1}} \Phi(-r(\boldsymbol{u}_\alpha^\perp))\varphi_{U_\alpha^\perp}(\boldsymbol{u}_\alpha^\perp) d\boldsymbol{u}_\alpha^\perp$$

$$P_f = \mathbb{E}_{U_\alpha^\perp}[\Phi(-r(\boldsymbol{u}_\alpha^\perp))] \approx \frac{1}{n_{LS}}\sum_{i=1}^{n_{LS}} \Phi\left(-r\left(\boldsymbol{u}_\alpha^{\perp,(i)}\right)\right) = \frac{1}{n_{LS}}\sum_{i=1}^{n_{LS}} p_f^{(i)}$$

5. Adapted LS for the reliability of penstocks

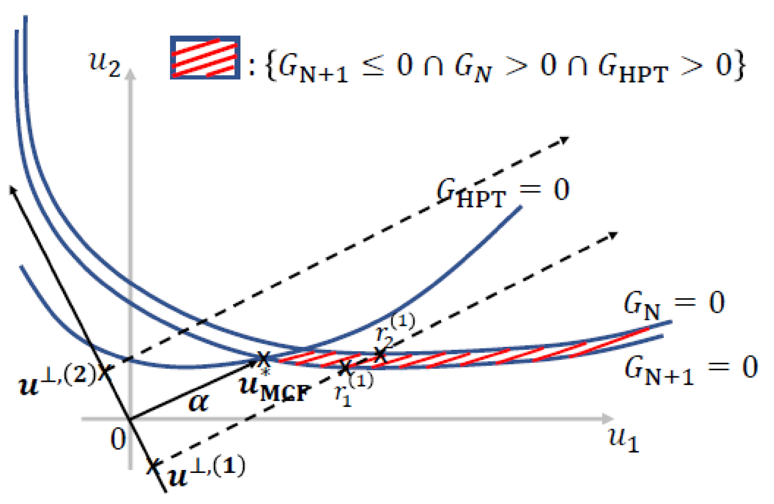
⚠ No unicity of the roots::

- **Case 1:** there is no root

$$p_f^{(i)} = 0$$

- **Case 2:** there are two roots

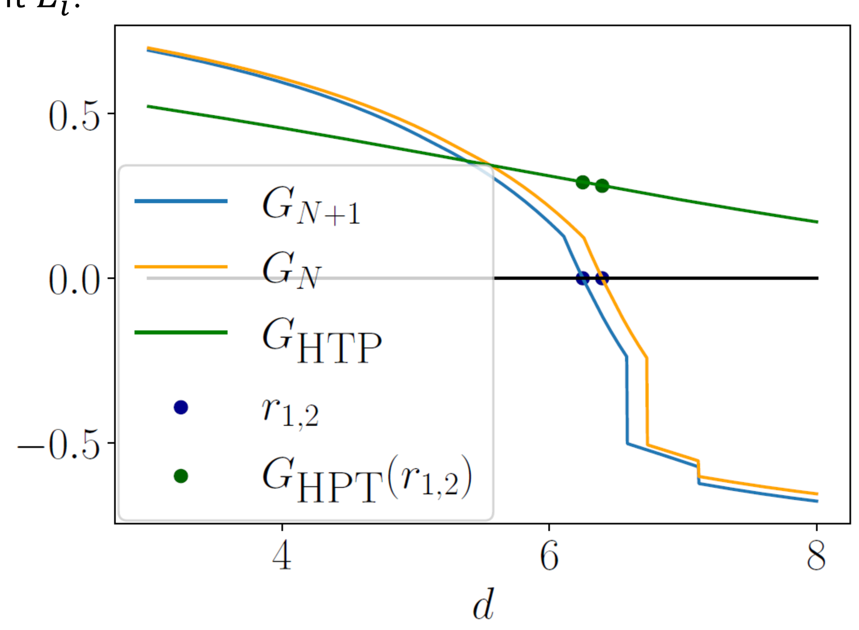
$$p_f^{(i)} = \Phi\left(-r_1^{(i)}\right) - \Phi\left(-r_2^{(i)}\right)$$



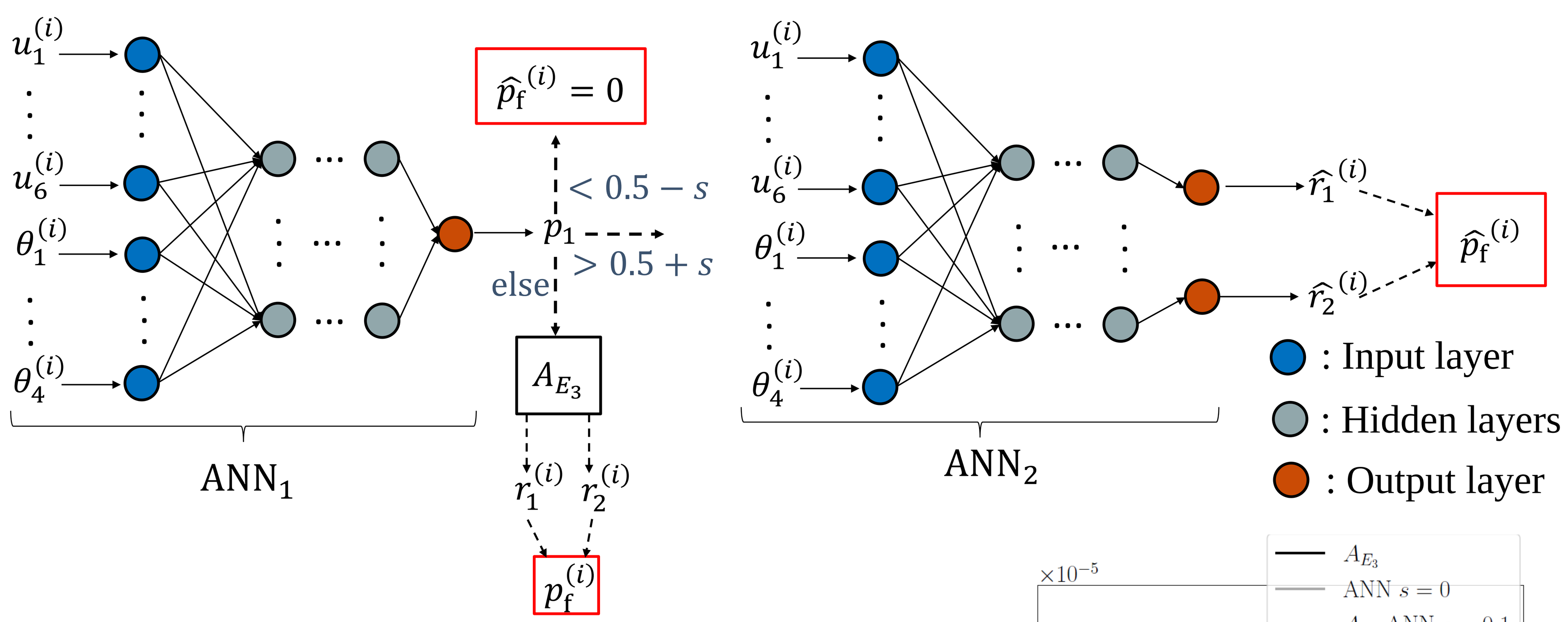
✓ Construction of one algorithm A_{E_i} for each event E_i:

A_{E_3}

Find r_1 s.t. $G_{N+1}^\perp(r_1) = 0$
If $G_{HPT}^\perp(r_1) > 0$ **then**
  Find r_2 s.t. $G_N^\perp(r_2) = 0$
  If $G_{HPT}^\perp(r_2) < 0$ **then**
    Find r_2 s.t. $G_{HPT}^\perp(r_2) = 0$
Else:
  No roots

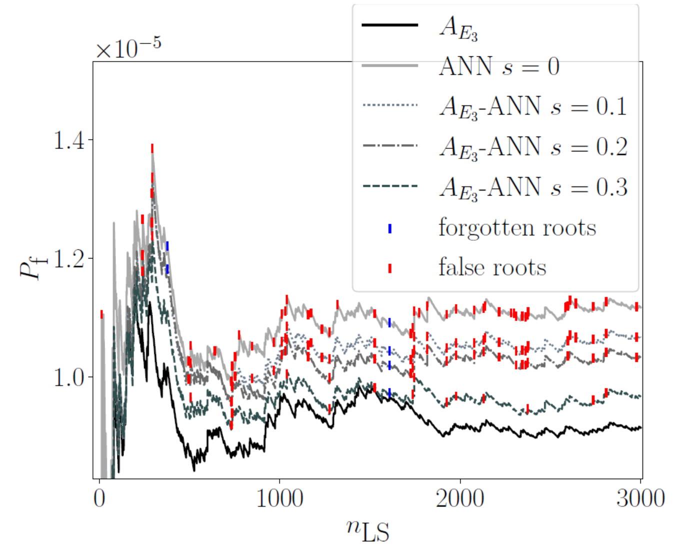


6. Combination of LS and Neural Networks



✓ Combination of two neural networks in the augmented space $(\boldsymbol{U}, \boldsymbol{\Theta})^T$.

- ANN_1: classification neural network → predicts the existence or not of both roots.
- ANN_2: regression neural network → predicts the values of both roots when they exist.
- s: value that decides if the output probability p_1 of ANN_1 is to be trusted or not.

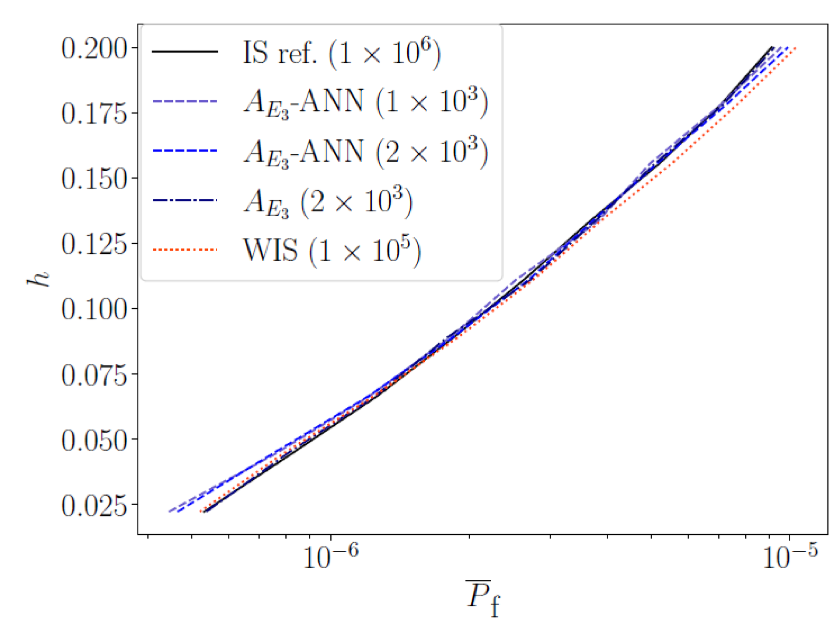


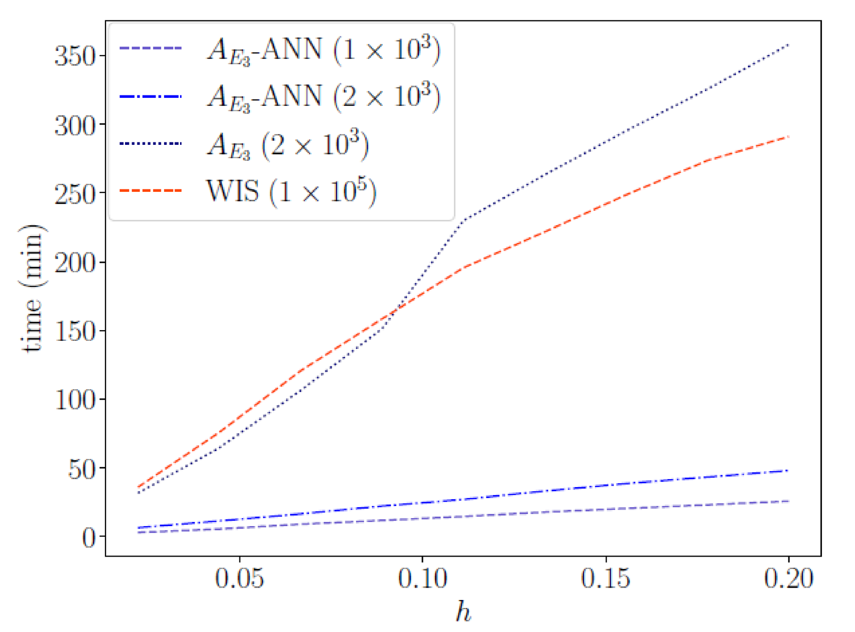
7. Results

- Robustness analysis

$$h_{IG}^* = \max_{h \geq 0}\left\{\max_{\boldsymbol{\theta} \in U(\tilde{\boldsymbol{\theta}},h)} P_f(\boldsymbol{\theta}) \leq P_f^{cr}\right\}; \quad I_{\theta_i}(h) = \begin{cases} [\tilde{\theta}_i(1-h), \tilde{\theta}_i(1+h)], & \text{if } \tilde{\theta}_i > 0 \\ [1-h, 1+h], & \text{if } \tilde{\theta}_i = 0 \end{cases}$$

- Search of $\overline{P_f}(h_j)$ for 10 value of $h_j \in [0, 0.2]$ with ANN_1 and ANN_2 trained on 3×10^4 samples.





8. Conclusions and perspectives

- ✓ Line sampling was successfully adapted on a complex limit-state function.
- ✓ Two artificial neural networks were combined to directly predict the LS roots in the info-gap augmented space with the possibility to control the error of the first ANN.
- ❖ Most applications cannot afford as many training samples. There is a need to try other surrogate models in the augmented space.
- ❖ Sensitivity analysis could considerably help the optimization process.

References

[1] E. Ardillon, Bryla, P., and A. Dumas. Reliability-based optimization of quantiles for diagnoses of hydropower penstock pipes. Proceedings of the 14th International Conference on Structural Safety and Reliability, Shanghai, 2022.

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[3] P. Koutsourelakis, H. Pradlwarter, G. Schueller, Reliability of structures in high dimensions, part I: algorithms and application, Probabilistic Engineering Mechanics 19 (2004) 409–417.