Wind speed modelling with stochastic processes for wind turbine production study

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Wind turbine production/degradation

- Different failure causes
- Several degradation sources
- Interest : Observable degradation indicator modelling for prediction (availability, maintenance, production)

Wind turbine failure

WT Components	Annual Failure Rate	Down Time per Failure (days)
Electrical system	0.57	1.53
Electronic control	0.43	1.59
sensors	0.25	1.41
Hydraulic system	0.23	1.36
Yaw system	0.18	2.70
Rotor hub	0.17	3.71
Mechanical brakes	0.13	2.89
Rotor blades	0.11	2.60
gearbox	0.10	6.21
generator	0.11	5.39
Support & housing	0.10	4.90
Drive train	0.05	5.71

Figure – © Faulstich, S. et al. (2011)

Degradation

- Crack propagation on blades (Fatigue,...)
- Control system degradation (pitch angle controller)

Blade crack growth :Physical (phenomenogical) model

Paris et Erdogan (1963), Forman (1967) et Kung et Ortiz (1990) :

$$\frac{dx_t}{dt} = \gamma(\Delta K)^n$$

$$\frac{dx_t}{dt} = \frac{\gamma(\Delta K)^n}{(1-R)K_c - \Delta K'}$$

$$\frac{dx_t}{dt} = \exp\left(\beta_0 + \beta_1 \ln(\Delta K) + \beta_2 ln(\Delta K)^2 + \beta_3 ln(\Delta K)^3\right)$$

where γ, n, $(\beta_i)_{0\leq i\leq 3}$ are constants related to the material. The constraint intensity factor $\Delta K = g(x_t)\Delta\sigma\sqrt{\pi x_t}$ where $\Delta\sigma$ is constraint amplitude and g is a function related to the geometry of the structure of x_t.

Control system : actif aerodynamic regulation system

Positioning of the blade according to the wind

$$\ddot{\beta} + 2\zeta\omega\dot{\beta} + \omega^2\beta = \omega^2\beta_r,$$

β the blade angle, β_r reference angle, ω natural frequency ζ depretitation rate.

No degradation

$\zeta = \zeta_0$ et $\omega = \omega_0$.

Actif aerodynamic regulation system

Degradation

- If $\omega < L$, L fixed value : failure
- High air-oil ratio reduces ω
- Degradation : air-oil evolution
- Degradation indicator : ω

- Available measurements for cracks, sensor data for air/oil ratio
- Control system simulator
- Statistical model/ Physical model/ hybrid model
- Choice : Statistical model considering the general trend of the physical model

Formalism

- X_t health indicator at time t
- $X = (\Omega, \mathcal{F}, \mathbb{P}, \{X_t, t \in T\})$, where $(\Omega, \mathcal{F}, \mathbb{P})$ is a probability space and $\{X_t, t \in T\}$ is a set of r.v. with values in $(E, \mathcal{E})$.
- $E = \mathcal{U} \cup \bar{\mathcal{U}}$, where $\mathcal{U}$ is the acceptable zone.

$$T_{\bar{\mathcal{U}}} = \inf\{t > 0, X_t \in \bar{\mathcal{U}}\}$$

Remaining Useful Life at time t :

$$\mathcal{RUL}_t = \inf\{s \geq t, \quad X_s \notin \mathcal{U}\} - t$$

Degradation model : Lévy subordinator

A Lévy process $\{X_t,\ t \in \mathbb{R}\}$ is a stochastic process $X_t : \Omega \mapsto \mathbb{R}^d$ satisfying

- $X_0 = 0 \ \ a.s.$
- stationary increments $X_t - X_s \sim X_{t-s} - X_0\ \forall s \leq t$
- independent increments $X_t - X_s \perp\!\!\!\perp X_{t-s} - X_0\ \forall s \leq t$
- In the non-discrete setting we will also require a mild regularity condition continuity in probability lim $\lim_{t\to 0} \mathbb{P}(|X_t - X_0| > \epsilon) = 0, \forall \epsilon > 0$

Candidates : Lévy subordinators (non-decreasing Lévy processes) such as Gamma process, inverse gaussian process.

For $\bar{\mathcal{U}} = [L, \infty[$, $L \in \mathbb{R}$

$$\mathbb{P}(T_L < t) = \mathbb{P}(X_t > L)$$

Degradation model : Gamma process

- Deterioration level at time t, X_t
- Deterioration process $(X_t)_{t\geq 0}$
- $X_t - X_s \sim \Gamma(\alpha(t) - \alpha(s), \beta)$
- $\alpha(t) = \int_{[0,t]} d\alpha(r)$ and $d\alpha$ is a positive measure defined on $\mathbb{R}^+$.
- $X_t - X_s$ has the following pdf :

$$f_{\alpha,\beta,s,t}(x) = \frac{x^{\alpha(t)-\alpha(s)-1} e^{-\frac{x}{\beta}}}{\Gamma(\alpha(t)-\alpha(s))\beta(t)^{\alpha(t)-\alpha(s)}} \mathbb{1}_{\{x\geq 0\}}.$$

Wind Impact

- Crack growth increases with long period of large variations and high wind speed
- The natural frequency decrease faster with some classes of wind
- No production in some wind speed intervals
- production/reliability related to wind speed
- Wind as covariates

$$\alpha(t) = \alpha(t, W_t)$$

W_t wind speed at time t.

- proportional hasard model (Cox) $\alpha(t, W_t) = \alpha_0 \exp(\beta W_t) t^a$, α_0 and a baseline parameters, $\beta \in \Theta$, Θ a compact set
- additive hasard model
- affine model

Wind/production

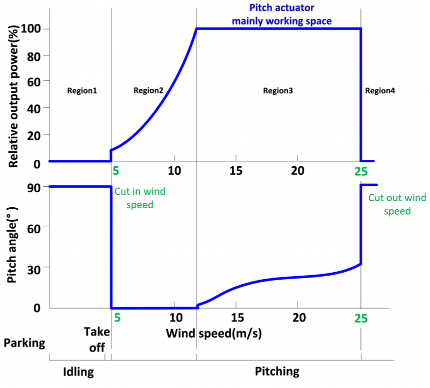


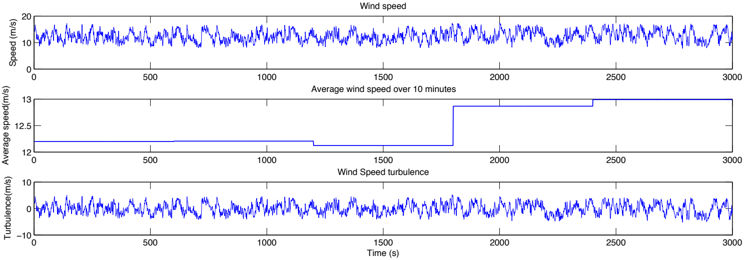
Figure – Production versus wind speed

Wind modelling

Wind speed or velocity modelling

- Navier Stokes equations (Costly)
- Statistical modelling (based on available data) : actual presentation
- Hybrid :work initiated

wind


Wind speed
Speed (m/s)
20
10
0
0
500
1000
1500
2000
2500
3000
Average wind speed over 10 minutes
Average speed(m/s)
13
12.5
12
Wind Speed turbulence
Turbulence(m/s)
10
0
−10
Time (s)

Times series

- Randomness tests :Wald-Wolfowitz runs test, difference-sign test,rank test.
- Stationarity test (Autocorrelation test) : Dickey-Fuller Test, Augmented Dickey-Fuller Test
- Ljung-Box test : white noise
- Residual normality : QQ plot, Shapiro-Wilk, Kolmogorov-Lilliefors, Anderson Darling
- Goodness of fit for increments : Kolmogorov-Lilliefors, Anderson-Darling, Cramer von Mises
- Information Criterion : BIC, AIC, AICC

Wind speed statistical modelling

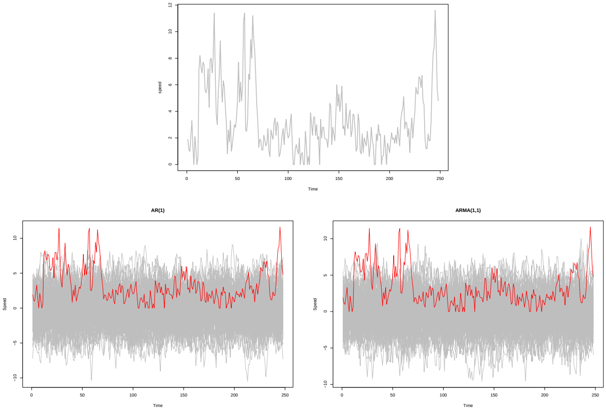


Figure – AR and ARMA model fitting

Wind speed statistical modelling

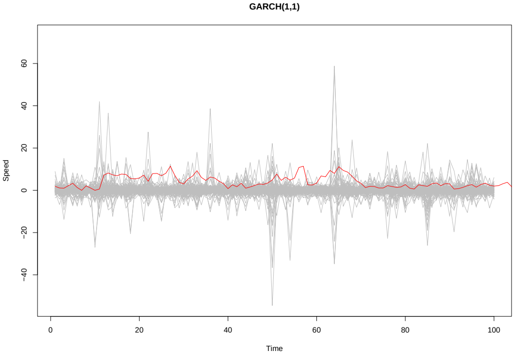


Figure – GARCH model fitting

Wind speed statistical modelling

The Ornstein-Uhlenbeck process is a stochastic process that satisfies the following stochastic differential equation :

$$dX_t = a(c - X_t)dt + bdW_t,\ t \in [0, T],\ \ X_0 = 0.$$

where $(W_t)_{t\geq 0}$ is a standard Brownian motion. The parameters $a >$ is the rate of mean reversion, $c > 0$ is the long-term mean of the process and $b > 0$ is the volatility or average magnitude, per square-root time, of the random fluctuations that are modelled as Brownian motions.

Estimation : maximum likelihood

Wind speed statistical modelling

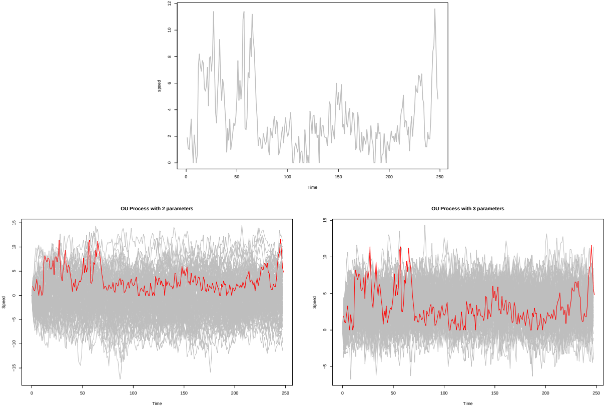
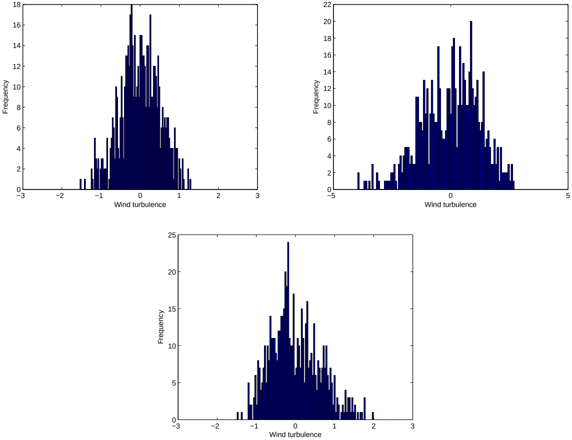
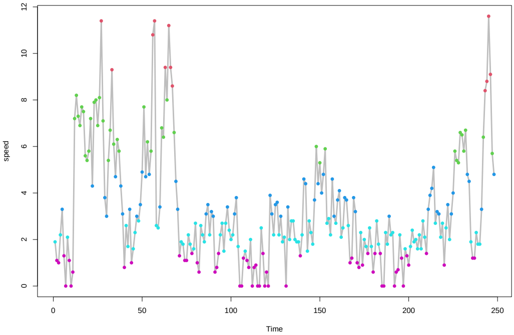


Figure – OU process fitting

Wind Distribution



Clustering :kmeans



Selection criteria : Given a set of observations $(x_1, x_2, \ldots, x_n)$, where each observation is a d-dimensional real vector, k-means clustering aims to partition the n observations into k $(\leq n)$ sets $S = \{S_1, S_2, \cdots, S_k\}$ so as to minimize the within-cluster sum of squares (WCSS) (i.e. variance). Considering μ_i the mean of points in S_i, formally, the objective is to find :

$$\underset{\mathbf{S}}{\arg\min} \sum_{i=1}^{k} \sum_{\mathbf{x} \in S_i} \|\mathbf{x} - \boldsymbol{\mu}_i\|^2 = \underset{\mathbf{S}}{\operatorname{argmin}} \sum_{i=1}^{k} |S_i| \operatorname{Var} S_i$$

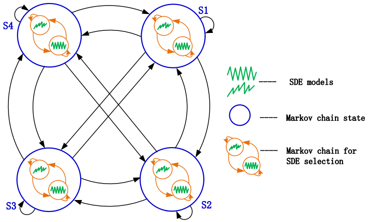
Switching OU process

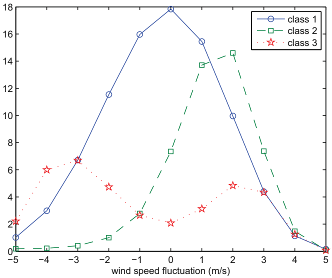
- The Ornstein-Uhlenbeck process parameters are randomly changing.
- A Markov chain is considered and in its state i the model is defined by

$$dX_t = a_i(c_i - X_t)dt + b_i dW_t,\ t \in [0, T],\ \ X_0 = 0,\ \ 1 \leq i \leq 4$$

- In each state the OU process has constant parameters.

Parameter Switching OU Process





Model selection

Model selection and validation

- non indentically distributed data
- small data size
- Goodness of fit test limited to specific models and data

Depth function

- $\mathcal{M}(T)$:the set of real valued functions on T.
- T_0 a compact subset of T.
- The sample paths of $\mathbf{X} = \{X(t), t \in T_0\}$ are in $\mathcal{M}(T_0)$.
  Definition :
  For a function $f \in \mathcal{M}(T_0)$ a data depth with respect to P is defined as

$$\begin{aligned} D_P : \mathcal{M}(T_0) &\to [0, 1] \\ f &\to D_P(f) \end{aligned}$$

- Properties : Affine invariance,Maximality at center,Monotonicity relative to deepest point,Vanishing at infinity

Depth function example : Tukey Depth

Regard a random vector $\mathbf{X}$ distributed as P.
In $\mathbb{R}^d$, the Tukey depth of a point $\mathbf{z} \in \mathbb{R}^d$ w.r.t. $\mathbf{X}$, further $D(\mathbf{z}|\mathbf{X})$, is defined as the smallest probability mass of a closed halfspace containing $\mathbf{z}$:

$$D(\mathbf{z}|P) = \inf\{P(H)|H \text{ closed half-space}, \ \ \mathbf{z} \in H\}$$

Depth function : two examples

$\mathbf{X_1}, \mathbf{X_2}, \cdots, \mathbf{X_n}$ i.i. copies of $\mathbf{X}$

- $D_n(\mathbf{X_i}(t))$ be the univariate depth of $\mathbf{X_i}$ at t Integrated Data Depth :

$$I_i = \int_0^1 D_n(\mathbf{X_i}(t))dt, \ \ 1 \leq i \leq n$$

- h-modal functional depth (the trajectory most densely surrounded by other trajectories of the process) :

$$MD(\mathbf{X_i}) = \sum_{k=1}^{n} K(\frac{\|\mathbf{X_i} - \mathbf{X_k})\|}{h})$$

where $\|\cdot\|$ is a norm in the functional space, $K : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is a kernel density function, and h is a bandwidth

The Area of the Convex Hull of Sampled Curves

The convex hull of a set of points S in n dimensions is the intersection of all convex sets containing S. For N points $p_1, \cdots, p_N$, the convex hull is then given by the expression :

$$\left\{ \sum_{j=1}^{N} \lambda_j p_j, \; \sum_{j=1}^{N} \lambda_j = 1, \quad \lambda_j > 0, \forall j \right\}$$

- $conv(A)$: the convex hull of a subset $A \in \mathcal{M}(T_0)$
- λ :the Lebesgue measure on $\mathbb{R}^2$.
- $\cup_{i=1}^{n} graph(\{f_i\})$: $graph\{(f_1, f_2, \ldots, f_n)\}$ the set defined by a collection of $n \geq 1$ functions $(\{f_1, f_2, \ldots, f_n)\} \in \mathcal{M}(T_0)$.

The Area of the Convex Hull of Sampled Curves

$\boldsymbol{X_1}, \boldsymbol{X_2}, \ldots$ are i.i. copies of $\boldsymbol{X}$.
Definition(Staerman et al.2020) : Let $J \geq 1$ be a fixed integer, the ACH depth of degree J with respect to P is defined by

$$D_{J,P} : \mathcal{M}(T_0) \to [0,1]$$
$$f \to \mathbb{E}\left[\frac{\lambda(conv(graph(\{\boldsymbol{X_1}, \ldots, \boldsymbol{X_J}\})))}{\lambda(conv(graph(\{\boldsymbol{X_1}, \ldots, \boldsymbol{X_J}\} \cup \{f\})))}\right]$$

The Area of the Convex Hull of Sampled Curves

Its average version $\bar{D}_{J,P}$ is defined by :

$$\bar{D}_{J,P}(h) = \frac{1}{J}\sum_{i=1}^{J} \frac{\lambda(conv(graph(\{\boldsymbol{X_1}^{(i)},\ldots,\boldsymbol{X_J}^{(i)}\})))}{\lambda(conv(graph(\{\boldsymbol{X_1}^{(i)},\ldots,\boldsymbol{X_J}^{(i)}\} \cup \{f\})))}.$$

For $n \geq J$ and $\forall h \in \mathcal{M}(T_0)$

$$D_{J,n}(h) = \frac{1}{\binom{n}{J}} \sum_{1 \leq i_1 < \ldots < i_J \leq n} \frac{\lambda(conv(graph(\{\boldsymbol{X_{i_1}},\ldots,\boldsymbol{X_{i_J}}\})))}{\lambda(conv(graph(\{\boldsymbol{X_{i_1}},\ldots,\boldsymbol{X_{i_J}}\},h)))}$$

is an unbiased statistical estimation of $D_{J,P}(.)$, (Lee1990).
The empirical average version is given by $\forall h \in \mathcal{M}(T_0)$,

$$\bar{D}_{J,n}(h) = \frac{1}{J}\sum_{i=1}^{J} D_{i,n}(h).$$

Model selection

- Depth function for candidate models
- Deepest model could be selected
- Asymptotic distribution of depth function are available or could be approximated

Application to crack data

Application on real crack data

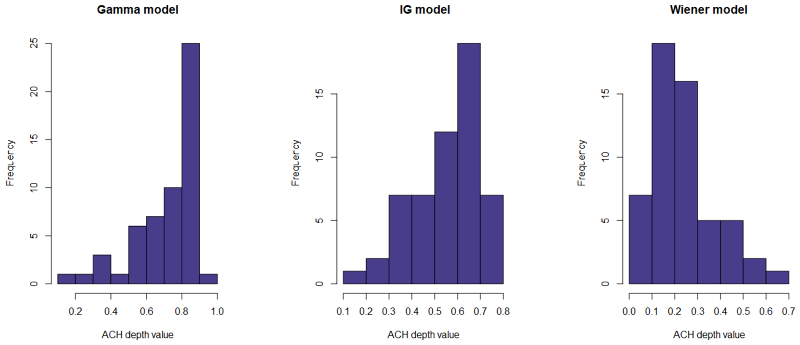


Figure – Histograms of the ACH depth values

Application

Model	0%	25%	50%	75%	100%
Gamma	0.1952	0.6444	0.7918	0.8491	0.9135
IG	0.1672	0.4765	0.5933	0.6516	0.7893
Wiener	0.0322	0.1301	0.2099	0.2920	0.6543

Table – Quantiles of the ACH depth values

Model	0%	25%	50%	75%	100%
Gamma	1.18	73.35	202.48	697.001	3580.22
IG	20.13	144.51	346.41	781.20	92335.88
Wiener	14.73	86.45	247.31	487.83	3770.12

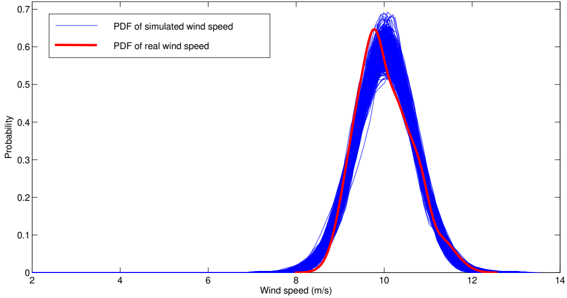
Table – Quantiles of the Mean Square Errors

Wind Model selection

Table 1: Depth values

	Mode	RP	FM	ACH
AR(1)	0.5370	0.1370	0.3358	0.6965
ARMA(1,1)	0.5087	0.1696	0.3188	0.6927
GARCH(1,1)	0.3657	0.0642	0.2354	0.8853
OU(3)	0.8925	0.5974	0.5051	0.8602
OU(2)	0.8571	0.4933	0.4679	0.8628

Prediction



PDF of simulated wind speed
PDF of real wind speed
Probability
Wind speed (m/s)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
2
4
6
8
10
12
14

QUESTIONS