Summary MASCOT-NUM 2011 : A Bayesian solution to characterizing uncertainty in inverse problems

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In the situation where some data of a $p-$dimensional variable $Y = H(X, d)$ are observable with known error, H is some time-consuming computer code (related to a physical model) and d is a known set of influential parameters (typically environnemental), the probabilistic inversion problem we consider is estimating the distribution of the $q-$ dimensional random variable $X \in \Omega$, for which no observation is usually available. As an example in risk and reliability assessment, Y can represent a river waterheight, H is a hydraulic code, d indexes the river discharge and X is a set of friction coefficients summarizing the river features, which randomly evolve in normal (non-flooding) conditions. More generally, such inversion problems are often encountered in treatment of uncertainties in decision-helping computer codes, for which X is a probabilistic input.

Some expert knowledge about features of X can be elicited from past studies on close examples, which invites to consider a Bayesian estimation of X, including a data augmentation step. A bayesian framework permits us to take into account of the available expert knowledge in the situation of relatively few data. In the cases we consider, X is given a multinormal distribution for which a multinormal-Wishart prior is elicited, and a MCMC algorithm is carried out to approach the posterior predictive distribution of interest and results are obtained for environmental industrial applications.

When H is time-consuming, a limited computing budget has to be managed. The high number of iterations of this algorithm implies to replace H by a kriging emulator $\hat{H}$ based on a design of experiment (*DOE*), namely a limited set of points of Ω over which H is run. We then focus on ≪ a smart choice ≫of this *DOE*, which can be made statically or dynamically along the algorithmic steps under a reasonable quality criterion, so called ≪ *adaptive kriging method* ≫. We describe the needed algorithmic modifications to complete sequentially the *DOE* and a robust criterion of

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the quality of design has been established. The performance of the MCMC algorithm has been widely improved by using this new adaptive method. The adaptive point of view is crucial for the following work. Finally we show on the hydraulic example that the obtained approximate posterior distribution remains satisfactory with respect to the true one.

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Biography Shuai FU, who started her Ph. D research from November 2009, is advised by Gilles Celeux, within team SELECT (Model Selection and Statistical Learning), INRIA Saclay as well as department MRI (Industrial Risk Management), EDF R&D, about the subject "Inverse problem in uncertainty analysis". Before the Ph.D research, she completed her master's study in Financial Mathematics (DEA EL Karoui) at University of Paris 6, following her dual bachelor's degrees in Applicated Mathematics at University of Lille 1 and Harbin Institute of Technology in China.