Control Theory and Viability Methods for the Sustainable Management of Natural Resources

Michel De Lara
Cermics, École des Ponts ParisTech
Université Paris-Est
France

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The interface between nature and society raises management issues

Mathematical control theory is a powerful framework to deal with natural resources management issues

Problems. There are many natural resources management problems which may be grasped within mathematical control theory

- climate change mitigation, management of energies, etc.
- fisheries management, epidemics control, etc.

Methods. Theory provides concepts, tools and methods

- viability kernel, viable controls
- dynamic programming
- monotonicity

Answers. Practical answers can be obtained

- ecosystem viable yields
- precautionary rules
- tradeoffs display between economic and ecological sustainability thresholds and risk

Outline of the presentation

1. Natural resources management issues and viability
   - Examples of decision models
   - Discrete–time viability
   - Are the ICES fishing quotas recommendations "sustainable"?
   - Ecosystem viable yields (anchovy–hake application)
2. Risk management and stochastic viability
   - Uncertain systems and policies
   - Viable scenarios and viability probability
   - Dam management under environmental/tourism constraint
   - Bycatches in a nephrops-hake fishery
3. Contribution to quantitative sustainable management

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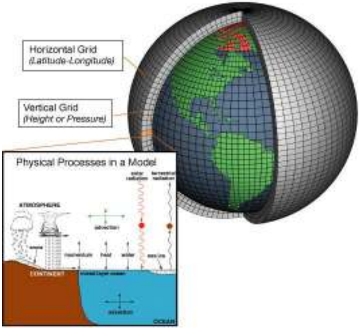
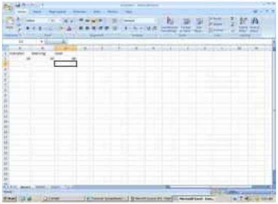
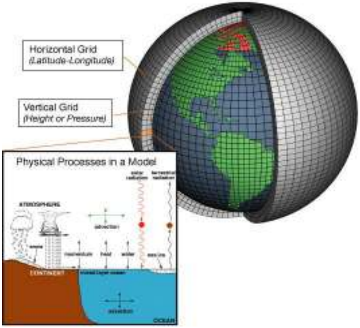
3. Contribution to quantitative sustainable management





We distinguish two polar classes of models: knowledge models *versus* decision models



Knowledge models:

1/1 000 000 → 1/1 000 → 1/1 maps

Office of Oceanic and Atmospheric Research (OAR) climate model

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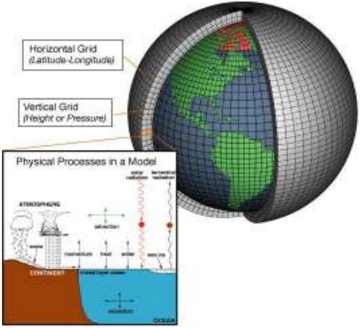

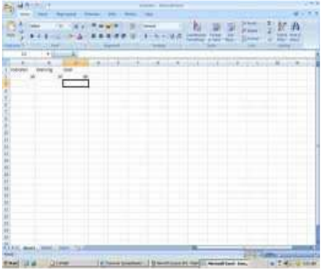

Knowledge models:
1/1 000 000 → 1/1 000 → 1/1 maps

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Action/decision models:
economic models are fables

William Nordhaus economic-climate model

A carbon cycle model "à la Nordhaus" is an example of decision model

- Time index t in years
- Economic production $Q(t)$

$$Q(t+1) = \overbrace{(1+g)}^{\text{economic growth}} \; Q(t)$$

- $\mathrm{CO_2}$ concentration $M(t)$

$$M(t+1) = M(t) \overbrace{-\delta(M(t) - M_{-\infty})}^{\text{natural sinks}} + \underbrace{\alpha}_{\text{physics}} \overbrace{\mathtt{Emiss}\big(Q(t)\big)}^{\text{emissions}} \underbrace{\big(1 - a(t)\big)}_{\text{abatement}}$$

A concentration target is pursued to avoid danger

United Nations Framework Convention on Climate Change

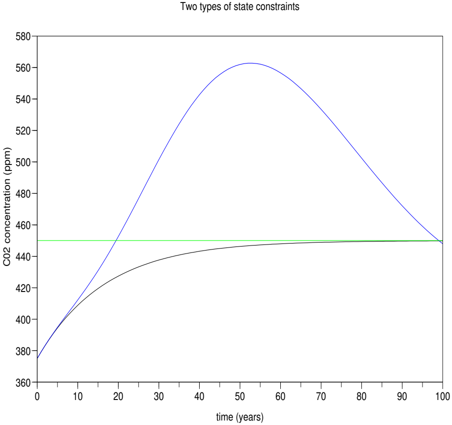
"to achieve, (...), stabilization of greenhouse gas concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system"

Limitation of concentrations of CO_2

- below a tolerable threshold $M^\sharp$ (say 350 ppm, 450 ppm)
- at a specified date $T > 0$ (say year 2050 or 2100)

$$\underbrace{M(T)}_{\text{concentration at horizon}} \leq \underbrace{M^\sharp}_{\text{threshold}}$$

Constraints capture different requirements



- The concentration has to remain below a tolerable level at the horizon T:

$$M(T) \leq M^{\sharp}$$

- More demanding: from the initial time t_0 up to the horizon T

$$M(t) \leq M^{\sharp} ,$$

$$t = t_0, \ldots, T$$

Constraints may be environmental, physical, economic

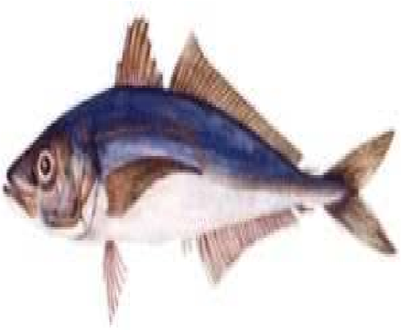
- The concentration has to remain below a tolerable level from initial time t_0 up to the horizon T

$$M(t) \leq M^{\sharp}, \quad t = t_0, \ldots, T$$

- Abatements $0 \leq a(t) \leq 1, \quad t = t_0, \ldots, T-1$
- Price ceiling on CO_2 $\rightarrow$ bounded abatement costs

$$\underbrace{\mathtt{Cost}\big(a(t), Q(t)\big)}_{\text{costs}} \leq c^{\sharp} \, (100 \text{ euros / tonne } \mathrm{CO}_2) \,, \quad t = t_0, \ldots, T-1$$

Populations may be described by abundances at ages



Jack Mackrel abundances (Chilean data)
in thousand of individuals

13651022 thousand of age < 1 (recruits)
7495888 thousand of age $\in [1, 2[$
6804151
4191318
4582943
2500338
1139182
523261
269328
166390
95606 thousand of age ≥ 11

Here are the ingredients of a harvested population age-class dynamical model

- Time $t \in \mathbb{N}$ measured in years
- Abundances at age $N = (N_a)_{a=1,\dots,A} \in \mathbb{X} = \mathbb{R}_+^A$
- $a \in \{1, \dots, A\}$ age class index
  - $A = 3$ for anchovy
  - $A = 8$ for hake
  - $A = 40$ for bacalao
- Control variable $\lambda \in \mathbb{U} = \mathbb{R}_+$ is fishing effort

Harvested population age-class dynamics

$$N_1(t+1) = \mathtt{S/R}\Big(\overbrace{\mathtt{SSB}\big(N(t)\big)}^{\text{spawning biomass}} \Big) \quad \text{recruitment}$$

$$N_2(t+1) = e^{-(M_1+\lambda(t)F_1)} N_1(t)$$

$$N_a(t+1) = e^{-\overbrace{(\underbrace{M_{a-1}}_{\text{natural}} + \underbrace{\lambda(t)F_{a-1}}_{\text{fishing}})}^{\text{mortality}}} N_{a-1}(t), \quad a = 2, \ldots, A-1$$

$$N_{A-1}(t+1) = e^{-(M_{A-2}+\lambda(t)F_{A-2})} N_{A-2}(t)$$

$$N_A(t+1) = e^{-(M_{A-1}+\lambda(t)F_{A-1})} N_{A-1}(t) + \underbrace{\pi e^{-(M_A+\lambda(t)F_A)}}_{\text{plus group}} N_A(t)$$

The ICES precautionary approach uses indicators and reference points to handle ecological objectives

International Council for the Exploration of the Sea precautionary approach

- keeping (or restoring) spawning stock biomass SSB indicator above a threshold reference point B_{lim}
- restricting fishing effort so that mean fishing mortality F indicator is below a threshold reference point F_{lim}

Definition	**Notation**	**Anchovy**	**Hake**
F limit RP	F_{lim}	/	0.35
SSB limit RP (t)	B_{lim}	21 000	100 000

The ICES uses two indicators and two reference points

- Spawning stock biomass

$$\mathtt{SSB}(N) = \sum_{a=1}^{A} \underbrace{\gamma_a}_{\text{proportion}} \overbrace{\mu_a}^{\text{mass}} \underbrace{N_a}_{\text{abundance}}$$

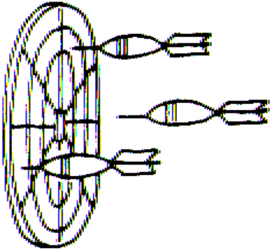
with reference point $\mathtt{SSB}(N) \geq B_{\lim}$

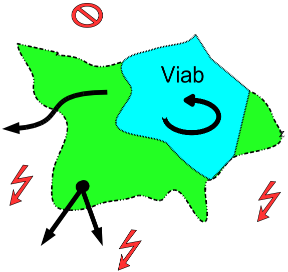
- Mean fishing mortality over age range from a_r to A_r

$$F(\lambda) := \frac{\lambda}{A_r - a_r + 1} \sum_{a=a_r}^{a=A_r} F_a$$

with reference point $F(\lambda) \leq F_{\lim}$

Outline of the presentation





A control system relates input and output variables

Output variables

soup quality
water vapor
temperature
(internal state)

Input variables

Control: wood logs
Uncertainty: wood humidity, metal conductivity

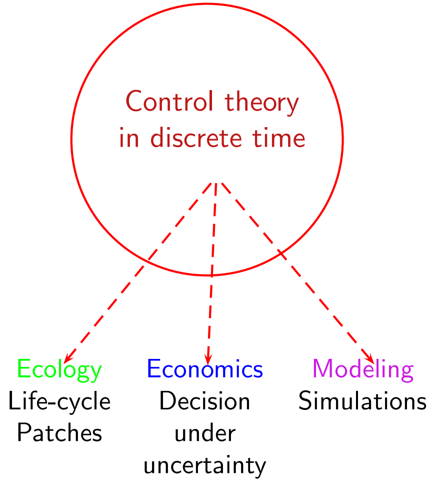
Discrete-time nonlinear state-control systems are special input-output systems

A specific output is distinguished, and is labeled state, when the system may be written

$$\begin{cases} x(t+1) = \mathtt{Dyn}(t, x(t), u(t)), & t \in \mathbb{T} = \{t_0, t_0+1, \ldots, T-1\} \\ x(t_0) & \text{given} \end{cases}$$

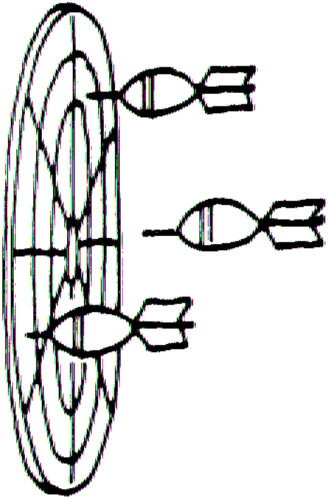
- the time $t \in \overline{\mathbb{T}} = \{t_0, t_0+1, \ldots, T-1, T\} \subset \mathbb{N}$ is discrete (*the time period* $[t, t+1[$ *may be a year, a month, etc.*) with initial time t_0 and horizon T ($T < +\infty$ or $T = +\infty$)
- the state variable $x(t)$ belongs to the finite dimensional state space $\mathbb{X} = \mathbb{R}^{n_\mathbb{X}}$ (*biomasses, abundances, capital, etc.*)
- the control variable $u(t)$ is an element of the *control set* $\mathbb{U} = \mathbb{R}^{n_\mathbb{U}}$ (*catches, harvesting effort, investment, etc.*)
- the dynamics $\mathtt{Dyn}$ maps $\mathbb{T} \times \mathbb{X} \times \mathbb{U}$ into $\mathbb{X}$ (*age-class model, population dynamics, economic model, etc.*)

Our main tool will be control theory in discrete time



- Problem
  - Find controls/decisions driving a dynamical system
  - To achieve various goals
- Three main ingredients
  - Controlled dynamics
  - Constraints
  - Criterion to optimize

How can we mathematically express the objectives pursued?



- The objectives can concern the input or the output variables
- More precisely, for a state-control system, the objectives will be expressed as constraints, and we shall distinguish

  control constraints (rather easy)
  state constraints (rather difficult)
- Viability theory deals with state constraints

Constraints may be explicit on the control

Examples of control constraints

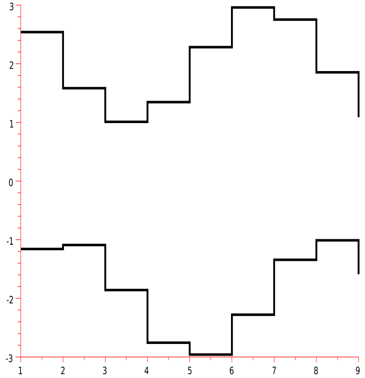
- Irreversibility constraints, physical bounds
  $0 \leq a(t) \leq 1 \quad , \quad 0 \leq h(t) \leq B(t)$
- Tolerable costs $c\big(a(t), Q(t)\big) \leq c^\sharp$
- Precautionary thresholds $F(\lambda(t)) \leq F_{\text{lim}}$

Control constraints / admissible decisions

$$\underbrace{u(t)}_{\text{control}} \in \underbrace{\mathbb{B}\big(t, x(t)\big)}_{\text{admissible set}} \quad , \quad t = t_0, \ldots, T-1$$

Easy because control variables $u(t)$ are precisely those variables whose values the decision-maker can fix at any time within given bounds

Constraints may concern the state



State constraints / admissible states

$$\underbrace{x(t)}_{\text{state}} \in \underbrace{\mathbb{A}(t)}_{\text{admissible set}} \quad , \quad t = t_0, \ldots, T$$

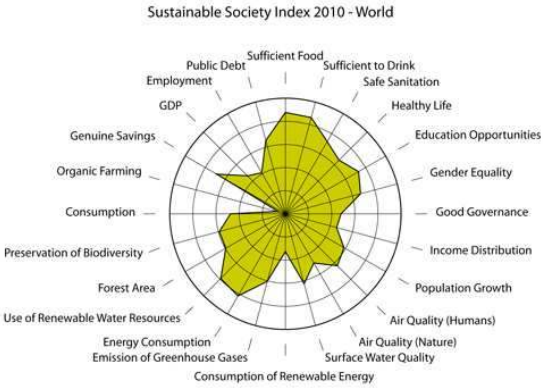
Examples

- CO_2 concentration $M(t) \leq M^{\sharp}$
- spawning stock biomass $\mathrm{SSB}\big(N(t)\big) \geq B_{\mathrm{lim}}$
- tipping points

State constraints are mathematically difficult because of “inertia”

$$x(t) = \underbrace{\text{function}}_{\text{iterated dynamics}} \underbrace{\big(u(t-1), \ldots, u(t_0)\big)}_{\text{past controls}}$$

Some economists recommend issues to be expressed in their own units, without aggregation

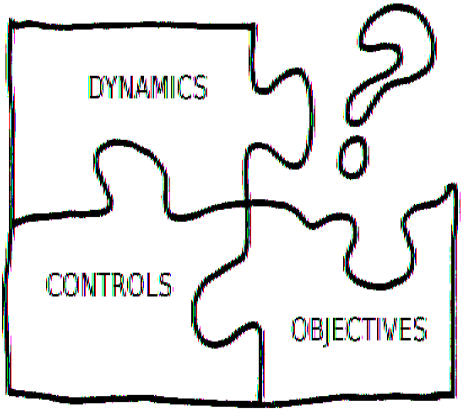


The "Stiglitz-Sen-Fitoussi" Commission (2009) déconseille de privilégier un indicateur synthétique unique car, quel que soit l'indicateur envisagé, l'agrégation de données disparates ne va pas de soi

When dealing with economic and environmental objectives, this disaggregated approach is coined co-viability

- Co-viability when
  - environmental constraints: conservation, viability
  - economic constraints: production, efficiency
- C. Béné, L. Doyen, and D. Gabay.
  A viability analysis for a bio-economic model.
  Ecological Economics, 36:385–396, 2001

Can we solve the compatibility puzzle between dynamics and objectives by means of appropriate controls?



- Given a dynamics, mathematically expressing the causal impact of controls on the state
- Imposing objectives bearing on output variables (states, controls)
- Is it possible to find a control path which achieves the objectives for all times?

How to anticipate the crisis?

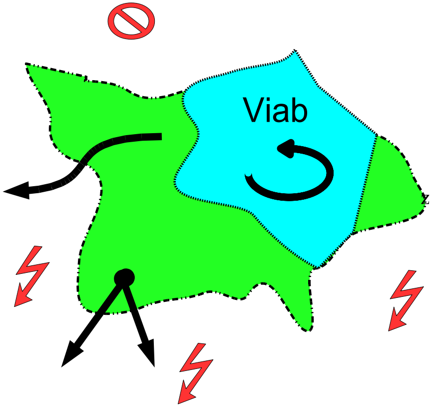
- An initial state is not viable if whatever the sequence of decisions a crisis occurs
- There exists a time when one of the state or control constraints is violated by the trajectories

The compatibility puzzle can be solved when the viability kernel is not empty

Viable initial states form the viability kernel (Jean-Pierre Aubin)

$$
\mathbb{V}\mathrm{iab}(t_0) := \left\{ \begin{array}{c} \text{initial} \\ \text{states} \\ x \in \mathbb{X} \end{array} \;\middle|\; \begin{array}{l} \text{there exist decisions } u(\cdot) = \\ \big(u(t_0), u(t_0+1), \dots, u(T-1)\big) \\ \text{and states } x(\cdot) = \\ \big(x(t_0), x(t_0+1), \dots, x(T)\big) \\ \text{starting from } x(t_0) = x \text{ at time } t_0 \\ \text{satisfying for any time } t \in \{t_0, \dots, T-1\} \\ x(t+1) = \mathtt{Dyn}\big(t, x(t), u(t)\big) \qquad \textit{dynamics} \\ u(t) \in \mathbb{B}(t, x(t)) \qquad \textit{control constraints} \\ x(t) \in \mathbb{A}(t) \qquad \textit{state constraints} \\ \text{and } x(T) \in \mathbb{A}(T) \qquad \textit{target constraints} \end{array} \right\}
$$

The viability kernel is included in the state constraint set



- The largest set is the state constraint set $\mathbb{A}$
- It includes the smaller blue viability kernel $\mathbb{V}\mathrm{iab}(t_0)$
- The green set measures the incompatibility between dynamics and constraints: good start, but inevitable crisis!

What is a solution to the viability problem?

- The viability kernel definition appeals to open-loop control, that is, a time-dependent sequence (planning, scheduling)

$$u : \underbrace{t \in \mathbb{T}}_{\text{time}} \mapsto \underbrace{u(t) \in \mathbb{U}}_{\text{control}}$$

- Another notion of solution is a decision rule, that is, a mapping $\texttt{Pol} : \mathbb{T} \times \mathbb{X} \to \mathbb{U}$ which assigns a control

$$\texttt{Pol} : \underbrace{(t, x) \in \mathbb{T} \times \mathbb{X}}_{\text{(time, state)}} \mapsto u = \underbrace{\texttt{Pol}(t, x) \in \mathbb{U}}_{\text{control}}$$

to any time t and state x

Viable decision rule

A viable decision rule Pol is a policy that drives the system within the constraints

There exists a dynamic programming equation relating viability kernels and displaying viable decision rules

Dynamic programming equation

$$\mathbb{V}\mathrm{iab}(T) = \mathbb{A}(T)$$

$$\begin{aligned}\mathbb{V}\mathrm{iab}(t) = \; & \{ \text{admissible states } x \in \mathbb{A}(t) \mid \\ & \text{there exists an admissible control } u \in \mathbb{B}(t,x) \\ & \text{such that the future state } \mathtt{Dyn}(t,x,u) \\ & \text{belongs to the next viability kernel } \mathbb{V}\mathrm{iab}(t+1) \; \}\end{aligned}$$

Monotonicity assumptions on dynamics and constraints can help identify viable decision rules

Monotonicity assumptions

- Dynamics Dyn is monotonous:
  - the more abundant today, the more tomorrow
  - the more harvested today, the less abundance tomorrow (monospecific models and technical interactions)
- Constraints/objectives are monotonous functions

Results

- Lower and upper approximations of the viability kernel
- Precautionary viable decision rules

Outline of the presentation

Is the ICES precautionary approach sustainable?

- The precautionary approach (PA) may be sketched as follows
  - the condition $\texttt{SSB}(N) \geq B_{\mathrm{lim}}$ is checked
  - if valid, the following usual advice is given

$$\underbrace{\lambda_{UA} \overbrace{(N)}^{\text{abundance}}}_{\text{effort}} = \max\{\lambda \in \mathbb{R}_+ \mid \overbrace{\texttt{SSB}(\texttt{Dyn}(N,\lambda))}^{\text{next year spawning biomass}} \geq B_{\mathrm{lim}} \text{ and } \underbrace{F(\lambda)}_{\text{fishing mortality}} \leq F_{\mathrm{lim}}\}$$

- Is it possible to apply the ICES precautionary rule every year?
- If so, can we remain within precautionary bounds as follows?

$$\texttt{SSB}(N(t)) \geq B_{\mathrm{lim}} \text{ and } F(\lambda(t)) \leq F_{\mathrm{lim}}\,, \quad \forall t = t_0, t_0+1, \ldots$$

The ICES precautionary rule is sustainable or not, depending on the model

- Bay of Biscay anchovy

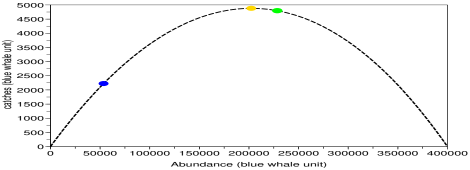
S/R Relationship	Constant	Constant	Constant (2002)	Constant (2004)	Linear	Ricker
Condition	$R_{\text{mean}} \geq \underline{R}$	$R_{\text{gm}} \geq \underline{R}$	$R_{\text{min}} \geq \underline{R}$	$R_{\text{min}} \geq \underline{R}$	$\gamma_1 \mu_1 r \geq 1$	
Left hand side	14 016 $\times 10^6$	7 109 $\times 10^6$	3 964 $\times 10^6$	696 $\times 10^6$	0.84	0
Right hand side	1 312 $\times 10^6$	1 312 $\times 10^6$	1 312 $\times 10^6$	1 312 $\times 10^6$	1	21 000
Sustainable	yes	yes	yes	no	no	no

- For species with late maturation, like hake, ICES precautionary approach is never sustainable!

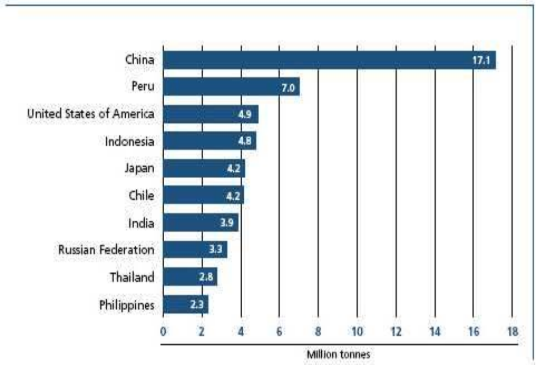
Outline of the presentation

Despite calls to an "ecosystem approach", practice remains monospecific

- The World Summit on Sustainable Development (Johannesburg, 2002) encouraged the application of the "ecosystem approach" by 2010
- but. . . following the World Summit on Sustainable Development (Johannesburg, 2002), the signatory States undertook to restore and exploit their stocks at maximum sustainable yield (MSY)
- The MSY is a concept which relies upon a monospecific dynamic model $\dot{B} = f(B) - qEB$ where B is biomass, and E fishing effort



Perú is second for marine and inland capture fisheries



The northern Humboldt current system off Perú represents less than 0.1% of the world ocean surface but presently sustains about 10% of the world fish catch

We consider two species targeted by two fleets in a biomass ecosystem dynamic

For simplicity, we consider a two–dimensional state model

$$\overbrace{y(t+1)}^{\text{future biomass}} = y(t)\overbrace{R_y\big(y(t), z(t), \underbrace{u_y(t)}_{\text{effort}}\big)}^{\text{growth factor}}$$

$$z(t+1) = z(t)R_z\big(y(t), z(t), \overbrace{u_z(t)}^{\text{control}}\big)$$

- State vector $(y(t), z(t))$ represents biomasses
- Control vector $(u_y(t), u_z(t))$ is fishing effort of each species
- The catches are $u_y(t)y(t)$ and $u_z(t)z(t)$ (measured in biomass)

Our objectives are twofold: conservation and production

The viability kernel is the set of initial species biomasses $\big(y(t_0), z(t_0)\big)$ from which appropriate effort controls $\big(u_y(t), u_z(t)\big)$, $t = t_0, t_0+1, \dots$ produce a trajectory of biomasses $\big(y(t), z(t)\big)$, $t = t_0, t_0+1, \dots$ such that the following goals are satisfied

- preservation (minimal biomass thresholds)

$$\underline{\text{stocks}}: \qquad y(t) \geq S_y^\flat \,, \quad z(t) \geq S_z^\flat$$

- economic/social requirements (minimal catch thresholds)

$$\underline{\text{catches}}: \qquad u_y(t)y(t) \geq C_y^\flat \,, \quad u_z(t)z(t) \geq C_z^\flat$$

An explicit expression for the viability kernel exists under weak assumptions

Proposition

- *If the growth factors R_y and R_z are decreasing in the fishing effort*
- *and if the thresholds $S_y^\flat, S_z^\flat, C_y^\flat, C_z^\flat$ are such that the following growth factors are greater than one*

$$R_y(S_y^\flat, S_z^\flat, \frac{C_y^\flat}{S_y^\flat}) \geq 1 \text{ and } R_z(S_y^\flat, S_z^\flat, \frac{C_z^\flat}{S_z^\flat}) \geq 1 ,$$

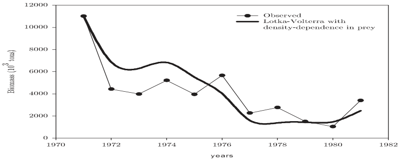
the viability kernel is given by

$$\left\{ (y,z) \mid y \geq S_y^\flat,\ z \geq S_z^\flat,\ yR_y(y,z,\frac{C_y^\flat}{y}) \geq S_y^\flat,\ zR_z(y,z,\frac{C_z^\flat}{z}) \geq S_z^\flat \right\}$$

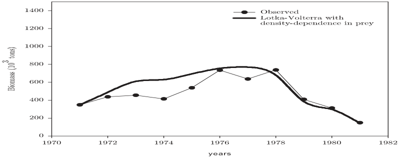
Hake–anchovy Peruvian fisheries data and model

Hake–anchovy Peruvian fisheries data between 1971 and 1985, in thousands of tonnes (10^3 tons)

- anchoveta_stocks=
  [11019 4432 3982 5220 3954 5667 2272 2770 1506 1044 3407 ... 1678 40 900 3944]
- merluza_stocks=
  [347 437 455 414 538 735 636 738 408 312 148 100 99 124 ... 194]
- anchoveta_captures=
  [9184 3493 1313 3053 2673 3211 626 464 1000 223 288 ... 1240 118 2 648]
- merluza_captures=
  [26 13 133 109 85 93 107 303 93 159 69 26 6 12 26]



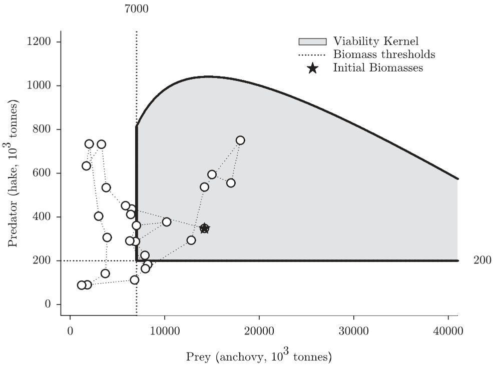
(a) Anchovy



(b) Hake

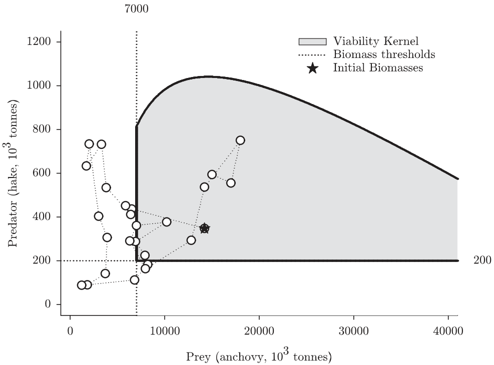
Figure: Comparison of observed and simulated biomasses of anchovy and hake using a Lotka–Volterra model with density-dependence in the prey. Model parameters are $R = 2.24$, $L = 0.98$, $\kappa = 64\,672 \times 10^3$ t ($K = 35\,800 \times 10^3$ t), $\alpha = 1.230 \times 10^{-6}\ t^{-1}$, $\beta = 4.326 \times 10^{-8}\ t^{-1}$.

For given thresholds, we can draw the viability kernel



- Minimal biomass thresholds
  - $S_y^\flat = 7\ 000\ kt$ (anchovy)
  - $S_z^\flat = 200\ kt$ (hake)
- Minimal catches thresholds
  - $C_y^\flat = 2\ 000\ kt$ (anchovy)
  - $C_z^\flat = 5\ kt$ (hake)

For given thresholds, we can draw the viability kernel



- Minimal biomass thresholds
  - $S_y^\flat = 7\ 000\ kt$ (anchovy)
  - $S_z^\flat = 200\ kt$ (hake)
- Minimal catches thresholds
  - $C_y^\flat = 2\ 000\ kt$ (anchovy)
  - $C_z^\flat = 5\ kt$ (hake)

For a given initial state, we can also look for thresholds such that this state belongs to the associated viability kernel

We can use the viability kernel the other way round, to design ecosystem viable yields

1. Considering that first are given
   minimal biomass conservation thresholds $S_y^\flat \geq 0 \ , \quad S_z^\flat \geq 0$
2. with initial biomasses $y(t_0) \geq S_y^\flat$ and $z(t_0) \geq S_z^\flat$,

We can use the viability kernel the other way round, to design ecosystem viable yields

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   minimal biomass conservation thresholds $S_y^\flat \geq 0$, $S_z^\flat \geq 0$
2. with initial biomasses $y(t_0) \geq S_y^\flat$ and $z(t_0) \geq S_z^\flat$,
   the following catch levels can be sustainably maintained

$$C_y^{\flat,\star} = \min \Big\{ C_y \geq 0 \mid \quad R_y(S_y^\flat, S_z^\flat, \frac{C_y}{S_y^\flat}) \geq 1 \text{ and } y(t_0) R_y(y(t_0), z(t_0), \frac{C_y}{y(t_0)}) \geq S_y^\flat \Big\}$$

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$$C_y^{\flat,\star} = \min\{C_y \geq 0 \mid \quad R_y(S_y^\flat, S_z^\flat, \frac{C_y}{S_y^\flat}) \geq 1 \text{ and } y(t_0)R_y(y(t_0), z(t_0), \frac{C_y}{y(t_0)}) \geq S_y^\flat\}$$

$$C_z^{\flat,\star} = \min\{C_z \geq 0 \mid \quad R_z(S_y^\flat, S_z^\flat, \frac{C_z}{S_z^\flat}) \geq 1 \text{ and } z(t_0)R_z(y(t_0), z(t_0), \frac{C_z}{z(t_0)}) \geq S_z^\flat\}$$

We can compare ecosystem viable yields to Perú official quotas

	Viable yields (kt)		Perú official quotas (kt)	
	Model 1	Model 2	2006	2007
Anchovy	5 152	5 399		

We can compare ecosystem viable yields to Perú official quotas

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Hake	49	56, 8	55	35

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	Viable yields (kt)		Perú official quotas (kt)	
	Model 1	Model 2	2006	2007
Anchovy	5 152	5 399	4 250	5 300
Hake	49	$56,8$	55	35

- Quotas are maximal bounds on catches
- Ecosystem viable yields (EVY) are minimal guaranteed yields
- EVY are obtained by "puzzling" viable effort rules: one can harvest more than the predator EVY to let the prey increase
- *Instituto del Mar del Perú* showed interest for this transparent method

Summary

- Examples of natural resources management problems where objectives are formulated as constraints
- Mathematical control theory framework
- Application of viability theory
- How do we move from deterministic dynamics and constraints to the uncertainty situation?

Outline of the presentation

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   - Examples of decision models
   - Discrete–time viability
   - Are the ICES fishing quotas recommendations "sustainable"?
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2. Risk management and stochastic viability
   - Uncertain systems and policies
   - Viable scenarios and viability probability
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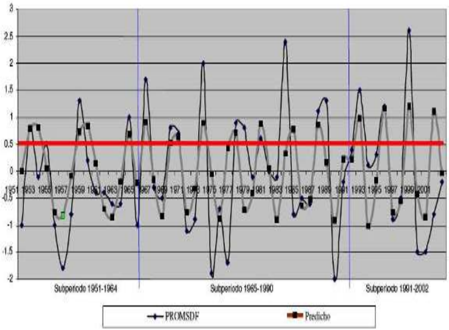
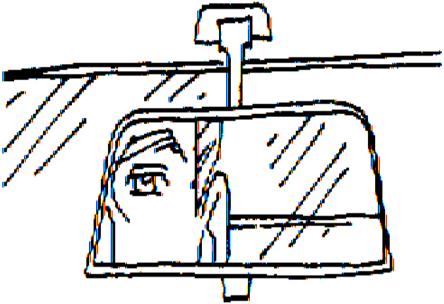
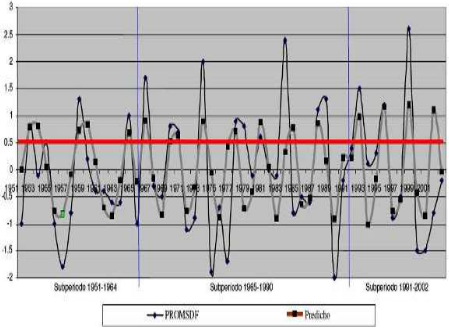
   - Bycatches in a nephrops-hake fishery
3. Contribution to quantitative sustainable management





Uncertainty is prevalent in natural resources management



- Environmental uncertainties (*El Niño*)
- Habitats changes, mortality, natality
- Scientific uncertainties (structure of trophic networks, ecosystem services)

Mitigation for climate change

- Economic production $Q(t)$

$$Q(t+1) = \Big(1 + \overbrace{g\big(w_e(t)\big)}^{\text{economic growth}}\Big) Q(t)$$

- CO_2 concentration $M(t)$

$$M(t+1) = M(t) - \delta(M(t) - M_{-\infty}) + \underbrace{\alpha\big(w_p(t)\big)}_{\text{physics}} \overbrace{\texttt{Emiss}\big(Q(t), w_z(t)\big)}^{\text{technologies}} \big(1 - a(t)\big)$$

- Vector of uncertainties $w(t) = (w_e(t), w_p(t), w_z(t))$ on
  - economic growth
  - technologies
  - climate dynamics

Age-class fishery model

$$N_1(t+1) = \mathtt{S/R}\Big(\mathtt{SSB}(N(t)), \underbrace{w(t)}_{\text{birth mortality, etc.}}\Big) \quad \text{recruitment}$$

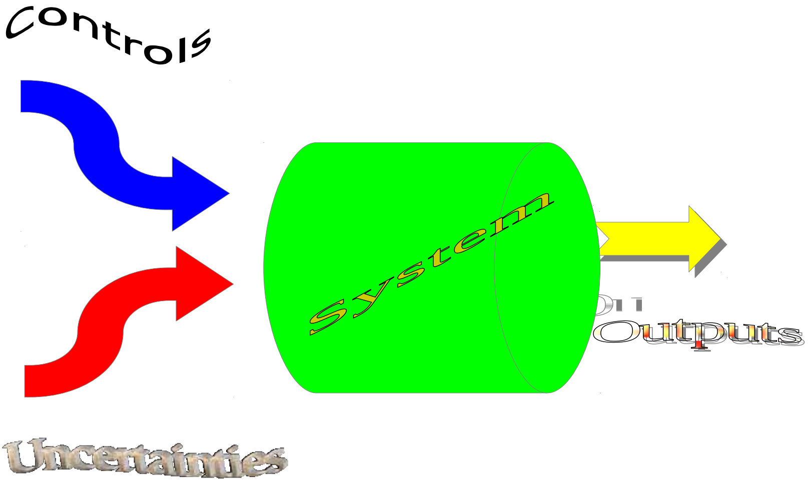
$$N_2(t+1) = e^{-(M_1+\lambda(t)F_1)} N_1(t)$$

$$\vdots = \vdots$$

$$N_a(t+1) = e^{-(\overbrace{M_{a-1}}^{\text{mortality}}+\lambda(t)F_{a-1})} N_{a-1}(t), \quad a = 2, \ldots, A-1$$

$$N_A(t+1) = e^{-(M_{A-1}+\lambda(t)F_{A-1})} N_{A-1}(t) + \pi e^{-(M_A+\lambda(t)F_A)} N_A(t)$$

Uncertainty variables are new input variables



Uncertainty variables are new input variables in a discrete-time nonlinear state-control system

A specific output is distinguished, and is labeled state, when the system may be written

$$x(t+1) = \mathtt{Dyn}\big(t, x(t), u(t), w(t)\big) , \quad t \in \mathbb{T} = \{t_0, t_0+1, \ldots, T-1\}$$

- time $t \in \overline{\mathbb{T}} = \{t_0, t_0+1, \ldots, T-1, T\} \subset \mathbb{N}$ (the time period $[t, t+1[$ may be a year, a month, etc.)
- state $x(t) \in \mathbb{X} := \mathbb{R}^n$, (biomasses, abundances, etc.)
- control $u(t) \in \mathbb{U} := \mathbb{R}^p$, (catches or harvesting effort)
- uncertainty $w(t) \in \mathbb{W} := \mathbb{R}^q$, (recruitment or mortality uncertainties, climate fluctuations or trends, etc.)
- dynamics $\mathtt{Dyn}$ maps $\mathbb{T} \times \mathbb{X} \times \mathbb{U} \times \mathbb{W}$ into $\mathbb{X}$ (biomass model, age-class model, economic model)

Distinguishing between uncertain and control variables frames how we "see" a system

At a certain stage of development men seem to have imagined that the means of averting the threatened calamity were in their own hands, and that they could hasten or retard the flight of the seasons by magic art. Accordingly they performed ceremonies and recited spells to make the rain to fall, the sun to shine, animals to multiply, and the fruits of the earth to grow.

The Myth of Adonis
The Golden Bough
Sir James George Frazer, 1922

Solutions are no longer control paths, as was the case in the deterministic setting

- Stationary (open-loop): stationary sequences

$$u : \underbrace{t \in \mathbb{T}}_{\text{time}} \mapsto \underbrace{u(t) \equiv u_{\mathrm{E}} \in \mathbb{U}}_{\text{control}}$$

Example: maximum sustainable yield

- Open-loop: time-dependent sequences (planning, scheduling)

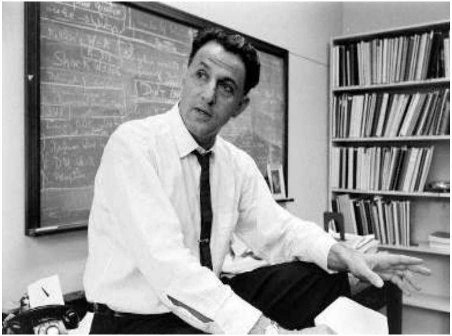
$$u : \underbrace{t \in \mathbb{T}}_{\text{time}} \mapsto \underbrace{u(t) \in \mathbb{U}}_{\text{control}}$$

Example: Pontryagin approach to optimal control

Solutions are no longer control paths, but are policies

From planning ⌚ to contingent planning ⌚×

Again the intriguing thought: A solution is not merely a set of functions of time, or a set of numbers, but a rule telling the decisionmaker what to do; a policy. (Richard Bellman)



Richard Ernest Bellman (August 26, 1920 – March 19, 1984) was an applied mathematician, celebrated for his invention of dynamic programming in 1953, and important contributions in other fields of mathematics.

Wikipedia

An example of policy embodied in computer code

```
if     state==0, do control=8
elseif state==1, do control=5.4
else do control=-15
```

> *On 4 June 1996, the maiden flight of the Ariane 5 launcher ended in a failure. (...) The attitude of the launcher and its movements in space are measured by an Inertial Reference System (SRI). (...) The data from the SRI are transmitted through the databus to the On-Board Computer (OBC), which executes the flight program (...)*
> *The Operand Error occurred due to an unexpected high value of an internal alignment function result called BH, Horizontal Bias, related to the horizontal velocity sensed by the platform*

The concept of policy as a contingent planning

- Richard Bellman autobiography, Eye of the Hurricane

  However, the thought was finally forced upon me that the desired solution in a control process was a policy: 'Do thus-and-thus if you find yourself in this portion of state space with this amount of time left.'

- Closed-loop: state feedback (decision rule)

$$\mathtt{Pol} : \underbrace{(t,x) \in \mathbb{T} \times \mathbb{X}}_{\text{(time, state)}} \mapsto u = \underbrace{\mathtt{Pol}(t,x) \in \mathbb{U}}_{\text{control}}$$

ICES precautionary approach

$$\lambda_{UA}(N) = \max\{\lambda \in \mathbb{R}_+ \mid \mathtt{SSB}(\mathtt{Dyn}(N,\lambda)) \geq B_{\lim} \text{ and } F(\lambda) \leq F_{\lim}\}$$

Going from planning to contingent planning, we have considerably enlarged the set of solutions

- Stationary (open-loop): stationary sequences

$$u : t \in \mathbb{T} \mapsto u(t) \equiv u_{\mathrm{E}} \;, \quad u_{\mathrm{E}} \in \mathbb{U}$$

Once the control space $\mathbb{U}$ is discretized in $n_{\mathbb{U}}$ elements,
the solution space cardinality is $n_{\mathbb{U}}$

- Open-loop: time-dependent sequences (planning, scheduling)

$$u : t \in \mathbb{T} \mapsto u(t) \;, \quad u(\cdot) \in \mathbb{U}^{\mathbb{T}}$$

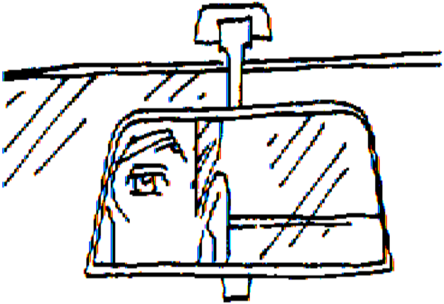
With $n_{\mathbb{T}}$ time periods, the solution space cardinality is $n_{\mathbb{U}}^{n_{\mathbb{T}}}$

- Closed-loop: time and state-dependent sequences

$$\mathtt{Pol} : (t,x) \in \mathbb{T} \times \mathbb{X} \mapsto u = \mathtt{Pol}(t,x) \in \mathbb{U} \;, \quad \mathtt{Pol} \in \mathbb{U}^{\mathbb{T} \times \mathbb{X}}$$

Once the state space $\mathbb{X}$ is discretized in $n_{\mathbb{X}}$ elements,
the solution space cardinality is $n_{\mathbb{U}}^{n_{\mathbb{T}} \times n_{\mathbb{X}}}$

"The blind cat does not catch mice"



- A decision rule depends on online information
- State feedback decision rules are natural solutions given by dynamic programming methods
- Adaptive decision rules
  - Appropriate for managing uncertain systems
  - More robust

How clouded the crystal ball looks beforehand

What is worth noting about the foregoing development is that I should have seen the application of dynamic programming to control theory several years before. I should have, but I didn't. It is very well to start a lecture by saying, 'Clearly, a control process can be regarded as a multistage decision process in which. . . ,' but it is a bit misleading.
Scientific developments can always be made logical and rational with sufficient hindsight. It is amazing, however, how clouded the crystal ball looks beforehand. We all wear such intellectual blinders and make such inexplicable blunders that it is amazing that any progress is made at all.

Richard Bellman autobiography, Eye of the Hurricane

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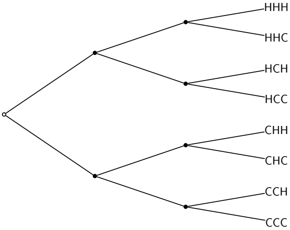
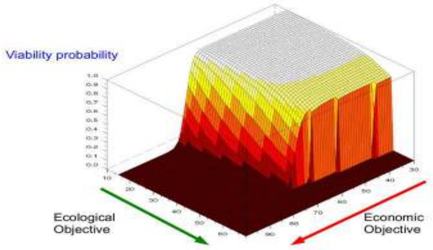
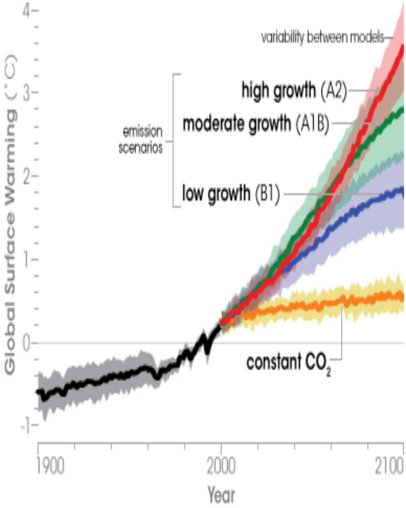




Scenarios in the modelers scientific community



- In practice, what modelers call a "scenario" is a mixture of
  - a sequence of uncertain variables (also called a pathway, a chronicle)
  - a policy `Pol`
  - and even a static or dynamical model
- In what follows

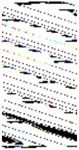
scenario = pathway = chronicle

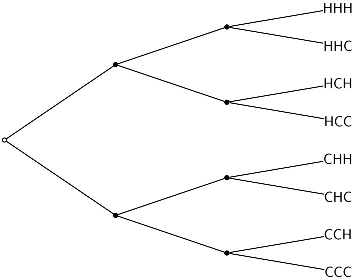
A scenario is a temporal sequence of uncertainties

Scenarios

A scenario (pathway, chronicle) is a sequence of uncertainties

$$w(\cdot) := \big(w(t_0), \dots, w(T-1)\big) \in \Omega := \mathbb{W}^{T-t_0}$$





A scenario is said to be viable for a given policy if the trajectories satisfy the constraints

Viable scenario

A scenario $w(\cdot) \in \Omega$ is said to be a viable under decision rule $\texttt{Pol}$ if the state and control trajectories $x(\cdot)$ and $u(\cdot)$ generated by

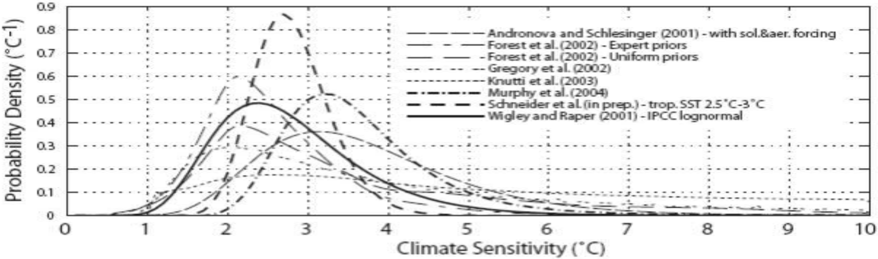
$$\begin{aligned} x(t+1) &= \texttt{Dyn}\big(t, x(t), \texttt{Pol}\big(t, x(t)\big), w(t)\big) , \quad t = t_0, \ldots, T-1 \\ u(t) &= \texttt{Pol}\big(t, x(t)\big) \end{aligned}$$

satisfy the state and control constraints

$$u(t) \in \mathbb{B}(t, x(t)) \text{ and } x(t) \in \mathbb{A}(t) , \quad \forall t = t_0, \ldots, T$$

- The set of viable scenarios is denoted by $\Omega_{\texttt{Pol}, t_0, x_0}$
- The larger set of viable scenarios, the better

The set Ω of scenarios can be equipped with a probability $\mathbb{P}$ (though this is a delicate issue!)



In practice, one often assumes that the components $\big(w(t_0), \ldots, w(T-1)\big)$ form an independent and identically distributed sequence of random variables, or form a Markov chain, or a time series

The viability probability is the probability of satisfying constraints under a decision rule

The viability probability associated with the initial time t_0, the initial state x_0 and the decision rule Pol is

$$\mathbb{P}\left[\Omega_{\mathtt{Pol},t_0,x_0}\right] =$$

$$\text{Probability}\left\{\begin{array}{l}\text{scenarios along which} \\ \text{the state } x(\cdot) \text{ and control } u(\cdot) \text{ trajectories} \\ \text{generated by dynamics } \mathtt{Dyn} \text{ and decision rule } \mathtt{Pol} \\ \text{starting from initial state } x_0 \text{ at initial time } t_0 \\ \text{satisfy the constraints} \\ \text{from initial time } t_0 \text{ to horizon } T\end{array}\right\}$$

The maximal viability probability is the upper bound for the probability of satisfying constraints

- The maximal viability probability is

$$\sup_{\texttt{Pol}} \mathbb{P}\left[\Omega_{\texttt{Pol},t_0,x_0}\right]$$

- An optimal viable decision rule $\texttt{Pol}^\star$ maximizes the probability of viable scenarios

$$\mathbb{P}\left[\Omega_{\texttt{Pol}^\star,t_0,x_0}\right] \geq \mathbb{P}\left[\Omega_{\texttt{Pol},t_0,x_0}\right]$$

  - dynamic programming
  - monotonicity

- In a sense, the decision rule $\texttt{Pol}^\star$ makes the set of viable scenarios the "largest" possible

The dynamic programming equation, or Bellman equation, is a backward equation satisfied by the stochastic viability value function

Proposition

If the primitive random variables $\big(w(t_0), w(t_0+1), \ldots, w(T-1)\big)$ *are independent under the probability* $\mathbb{P}$*, the stochastic viability value function*

$$V(t,x) := \sup_{\mathtt{Pol}} \mathbb{P}\left[\Omega_{\mathtt{Pol},t,x}\right]$$

satisfies the following backward induction, where t *runs from* $T-1$ *down to* t_0

$$\begin{aligned} V(T,x) &= \mathbf{1}_{\mathbb{A}(T)}(x) \\ V(t,x) &= \mathbf{1}_{\mathbb{A}(t)}(x) \max_{u \in \mathbb{B}(t,x)} \mathbb{E}_{w(t)}\Big[V\Big(t+1, \mathtt{Dyn}\big(t,x,u,w(t)\big)\Big)\Big] \end{aligned}$$

The stochastic viable dynamic programming equation yields stochastic viable policies

For any time t and state x, the viable controls are

$$\mathbb{B}^{\text{viab}}(t,x) := \underset{u \in \mathbb{B}(t,x)}{\operatorname{argmax}} \left(\mathbf{1}_{\mathbb{A}(t)}(x) \mathbb{E}_{w(t)} \Big[V\Big(t+1, \mathtt{Dyn}(t,x,u,w(t))\Big) \Big] \right)$$

Proposition

Then, any (measurable) policy $\mathtt{Pol}^\star$ *such that* $\mathtt{Pol}^\star(t,x) \in \mathbb{B}^{viab}(t,x)$ *is an optimal viable policy which achieves the maximal viability probability*

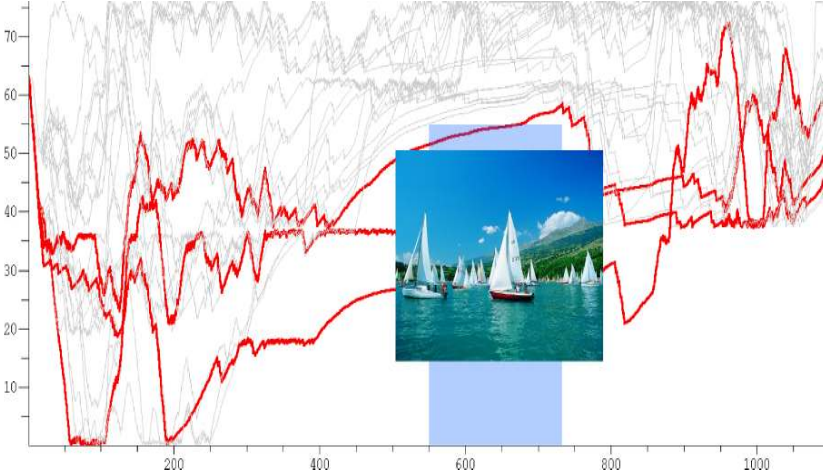
$$V(t_0, x_0) = \max_{\mathtt{Pol}} \mathbb{P}\left[\Omega_{\mathtt{Pol},t_0,x_0}\right]$$

Outline of the presentation

Dam management under "tourism" constraint

- Maximizing the revenue from turbinated water
- under a tourism constraint of having enough water
  in July and August

Stock trajectories



A single dam nonlinear dynamical model in decision-hazard

We can model the dynamics of the water volume in a dam by

$$\underbrace{S(t+1)}_{\text{future volume}} = \min\{S^{\sharp}, \underbrace{S(t)}_{\text{volume}} - \underbrace{q(t)}_{\text{turbined}} + \underbrace{a(t)}_{\text{inflow volume}} \}$$

- $S(t)$ volume (stock) of water at the beginning of period $[t, t+1[$
- $a(t)$ inflow water volume (rain, etc.) during $[t, t+1[$
- decision-hazard: $a(t)$ is not available at the beginning of period $[t, t+1[$
- $q(t)$ turbined outflow volume during $[t, t+1[$
  - decided at the beginning of period $[t, t+1[$
  - supposed to depend on $S(t)$ but not on $a(t)$
  - chosen such that $0 \leq q(t) \leq S(t)$

The traditional economic problem is maximizing the expected payoff

- Suppose that a probability $\mathbb{P}$ is given on the set $\Omega = \mathbb{R}^{T-t_0}$ of water inflows scenarios $\big(a(t_0), \ldots, a(T-1)\big)$
- Suppose that turbined water is sold at price $p(t)$, a price related to the price at which energy can be sold at time t
- Suppose that, at the horizon T, the final volume $S(T)$ has a value given by $\texttt{UtilFin}\big(S(T)\big)$
- The traditional economic problem is to maximize the intertemporal payoff (without discounting if the horizon is short)

$$\max \mathbb{E}\left[\sum_{t=t_0}^{T-1} \overbrace{p(t)q(t)}^{\text{turbined water payoff}} + \overbrace{\texttt{UtilFin}\big(S(T)\big)}^{\text{final volume utility}}\right]$$

Dam management under "tourism" constraint

- Traditional cost minimization/payoff maximization

$$\max \mathbb{E}\left[\sum_{t=t_0}^{T-1} \overbrace{p(t)q(t)}^{\text{turbined water payoff}} + \overbrace{\texttt{UtilFin}(S(T))}^{\text{final volume utility}}\right]$$

- For "tourism" reasons:

$$\text{volume } S(t) \geq S^{\flat} , \quad \forall t \in \{ \text{July, August} \}$$

- In what sense should we consider this inequality which involves the random variables $S(t)$ for $t \in \{ \text{July, August} \}$?

Robust / almost sure / probability constraint

- Robust constraints: for all the scenarios in a subset $\Omega' \subset \Omega$

$$S(t) \geq S^\flat \,, \quad \forall t \in \{ \text{July, August} \}$$

- Almost sure constraints

$$\text{Probability} \left\{ \begin{array}{l} \text{water inflow scenarios along which} \\ \text{the volumes } S(t) \text{ are above the} \\ \text{threshold } S^\flat \text{ for periods } t \text{ in summer} \end{array} \right\} = 1$$

- Probability constraints, with "confidence" level $p \in [0,1]$

$$\text{Probability} \left\{ \begin{array}{l} \text{water inflow scenarios along which} \\ \text{the volumes } S(t) \text{ are above the} \\ \text{threshold } S^\flat \text{ for periods } t \text{ in summer} \end{array} \right\} \geq p$$

Environmental/tourism issue leads to dam management under probability constraint

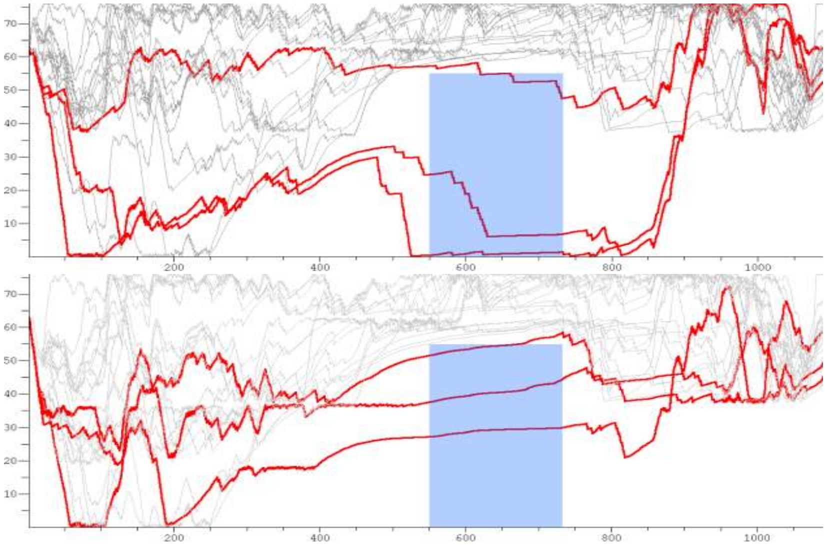
- The traditional economic problem is $\max \mathbb{E}\left[P(T)\right]$ where the payoff/utility criterion is

$$P(T) = \sum_{t=t_0}^{T-1} \overbrace{p(t)q(t)}^{\text{turbined water payoff}} + \overbrace{\texttt{UtilFin}\big(S(T)\big)}^{\text{final volume utility}}$$

- and a failure tolerance is accepted

$$\text{Probability} \left\{ \begin{array}{l} \text{water inflow scenarios along which} \\ \text{the volumes } S(t) \geq S^{\flat} \\ \text{for periods } t \text{ in July and August} \end{array} \right\} \geq 99\%$$

Stock trajectories

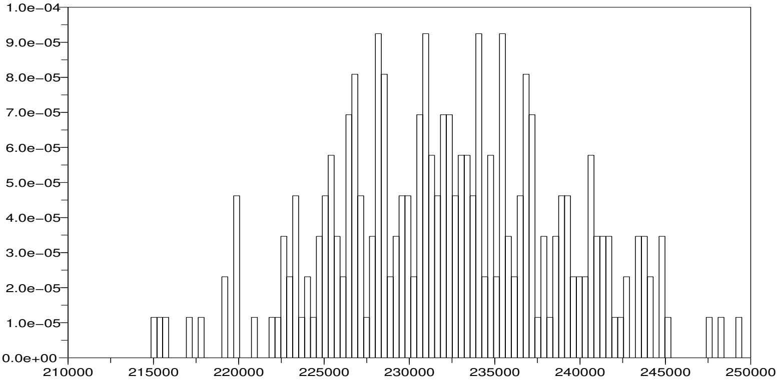


An improvement compared to standard procedures

OPTIMAL POLICIES	OPTIMIZATION		SIMULATION		
	Iterations	Time	Gain	Respect	Well behaviour
Standard	15	10 mn	**ref**	**0,9**	**no**
Convenient	**10**	**160 mn**	-3.20%	**0,9**	**yes**
Heuristic	10	160 mn	-3.25%	**0,9**	**yes**

However, though the mean is optimal, the payoff effectively realized can be far from it

Histogram of the optimal payoff with a final value of water



This is why we propose an alternate stochastic viability formulation to treat symmetrically and to guarantee both environmental and economic objectives

- Given two thresholds to be guaranteed
  - a volume $S^\flat$ (measured in cubic hectometers hm^3)
  - a payoff $P^\flat$ (measured in numeraire \$)
- we look after policies achieving the maximal viability probability $\Pi(S^\flat, P^\flat) =$

$$\max \text{Probability} \left\{ \begin{array}{l} \text{water inflow scenarios along which} \\ \text{the volumes } S(t) \geq S^\flat \,, \quad \forall t \in \{ \text{July, August} \} \\ \text{and the final payoff } \; P(T) \geq P^\flat \end{array} \right\}$$

- The maximal viability probability $\Pi(S^\flat, P^\flat)$ is the maximal probability to guarantee to be above the thresholds $S^\flat$ and $P^\flat$

A stochastic viability formulation

State, control and dynamic

- The state is the couple $x(t) = \big(S(t), P(t)\big)$
- The control $u(t) = q(t)$ is the turbined water
- The dynamics is

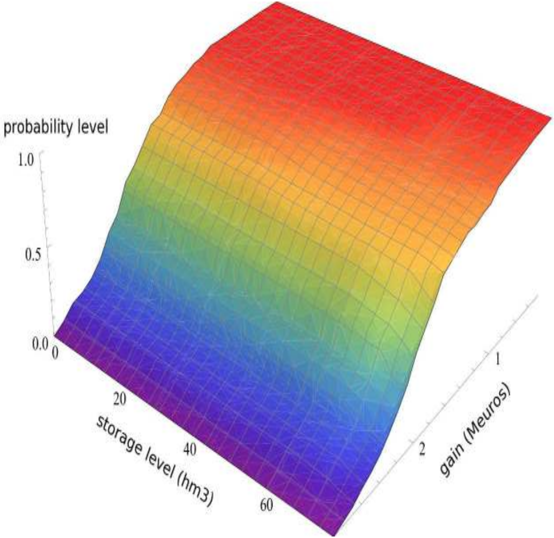
$$
\begin{aligned}
\underbrace{S(t+1)}_{\text{future volume}} &= \min\{S^\sharp, \underbrace{S(t)}_{\text{volume}} - \underbrace{q(t)}_{\text{turbined}} + \underbrace{a(t)}_{\text{inflow volume}}\}\,, \\
& \qquad t = t_0, \ldots, T-1 \\
\underbrace{P(t+1)}_{\text{future payoff}} &= \underbrace{P(t)}_{\text{payoff}} + \underbrace{p(t)q(t)}_{\text{turbined water payoff}}\,, \quad t = t_0, \ldots, T-2 \\
P(T) &= P(T-1) + \underbrace{\texttt{UtilFin}\big(S(T)\big)}_{\text{final volume utility}}
\end{aligned}
$$

Dynamic programming equation

```
 for bb=1:nb_BB
  SS_min=Thresholds_BB(bb)* summer ;
    for ee=1:nb_EE
    PP_min=Thresholds_EE(ee);
    //
    VALUE=list();
//    FEEDBACK=list();
    VALUE(horizon)=ones(nb_SS,1)*bool2s( state_PP>=PP_min ) ;

    shift=[(horizon-1):(-1):1];
    for tt=shift
      VVdot=VALUE(tt+1);
      VV=zeros(VVdot);
      for ss=1:nb_SS
SS=ss-1;
if SS>=SS_min(tt)
  for pp=1:nb_PP
    PP=pp-1;
    locext=[];
    for cc=1:ss
      // control constraint
      UU=cc-1;
locint=0;
ppdot=(min(PPmax-1,PP+UU))+1;
ssdot0=(min(SSmax,SS-UU))+1;
ssdot1=(min(SSmax,SS-UU+1))+1;
locint=locint+proba0(tt)*VVdot(ssdot0,ppdot)+...
       proba1(tt)*VVdot(ssdot1,ppdot);;
locext=[locext, locint ];
    end
    // of the control loop
```

Maximal viability probability $\Pi(S^\flat, P^\flat)$ as a function of guaranteed thresholds $S^\flat$ and $P^\flat$

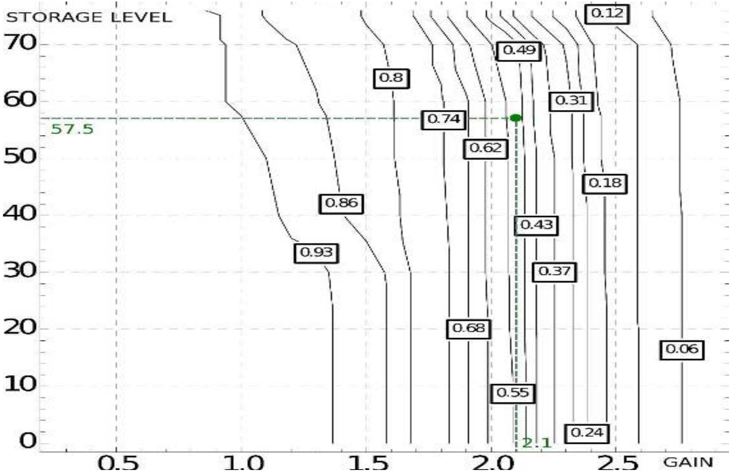


For example, the probability to guarantee

- a final payoff above $P^\flat = 1$ Meuros
- and a volume above $S^\flat = 40$ hm^3 in July and August

is about 90%

Iso-values for the maximal viability probability as a function of guaranteed thresholds $S^\flat$ and $P^\flat$



Outline of the presentation

1. Natural resources management issues and viability
   - Examples of decision models
   - Discrete–time viability
   - Are the ICES fishing quotas recommendations "sustainable"?
   - Ecosystem viable yields (anchovy–hake application)
2. Risk management and stochastic viability
   - Uncertain systems and policies
   - Viable scenarios and viability probability
   - Dam management under environmental/tourism constraint
   - Bycatches in a nephrops-hake fishery
3. Contribution to quantitative sustainable management

Hake and nephrops in technical interaction

$$N^h_1(t+1) = w^h(t) \text{ uncertain hake recruitment}$$
$$N^n_1(t+1) = w^n(t) \text{ uncertain nephrops recruitment}$$

$$N^h_a(t+1) = N^h_{a-1}(t)\left(1 - M^h_{a-1} - \overbrace{u(t)F^{nh}_{a-1}}^{\text{hake bycatch}} - F^{hh}_{a-1}\right)$$

$$N^n_a(t+1) = N^n_{a-1}(t)\left(1 - M^n_{a-1} - \overbrace{u(t)F^{nn}_{a-1}}^{\text{nephrops fishing mortality}}\right)$$

$$\begin{aligned} N^h_A(t+1) = {} & N^h_{A-1}(t)\left(1 - M^h_{A-1} - u(t)F^{nh}_{A-1} - F^{hh}_{A-1}\right) \\ & + N^h_A(t)\left(1 - M^h_A - u(t)F^{nh}_A - F^{hh}_A\right) \\ N^n_A(t+1) = {} & N^n_{A-1}(t)\left(1 - M^n_{A-1} - u(t)F^{nn}_{A-1}\right) \\ & + N^n_A(t)\left(1 - M^n_A - u(t)F^{nn}_A\right) \end{aligned}$$

The relative effort of the nephrops fleet has to be controlled to ensure both nephrops fleet profitability and hake preservation

- Economic objective: nephrops fishery is economically viable if the gross return is greater than a threshold

$$\underbrace{P\big(N^n(t), u(t)\big)}_{\text{payoff}} \geq P^\flat$$

- Ecological objective: fishery is ecologically viable if its impact by bycatch on the hake biology is compatible with sufficient recruitment of mature hakes

$$\underbrace{N_4^h(t)}_{\text{fourth age-class}} \geq (N_4^h)^\flat$$

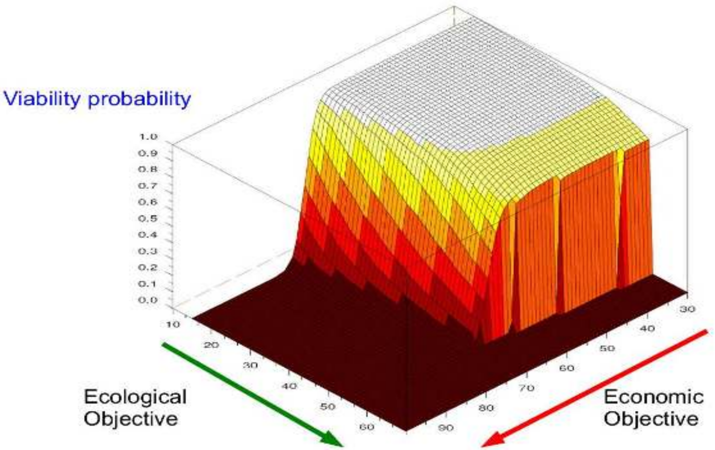
An optimal viable policy can be calculated thanks to monotonicity properties

- Due to monotonicity properties
  - of the dynamics, increasing in the state variable and decreasing in the control
  - of the constraints, increasing in the state variable and decreasing in the control
- we can prove that

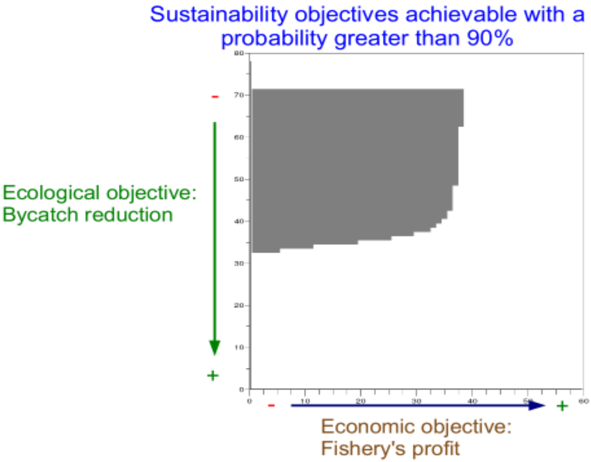
$$\mathtt{Pol}^{\star}(t,N) = \inf\{u \in [0,u^{\sharp}] \mid P(N^{n},u) \geq P^{\flat}\}$$

is an optimal viable policy

Maximal viability probability function of $P^\flat$ and $(N_4^h)^\flat$

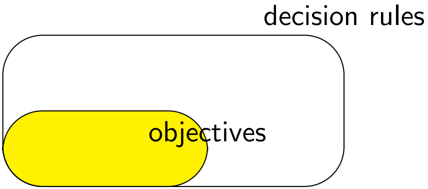


Iso-values for the maximal viability probability as a function of guaranteed thresholds $P^\flat$ and $(N_4^h)^\flat$



Outline of the presentation

Resource managers often make a confusion between objectives and decision rules



In practice, we observe that resource managers generally

- design decision rules
- which directly incorporate objectives
- with confusion between objectives and decision rules

Mismatch can be avoided by making a clear distinction between objectives and decision rules

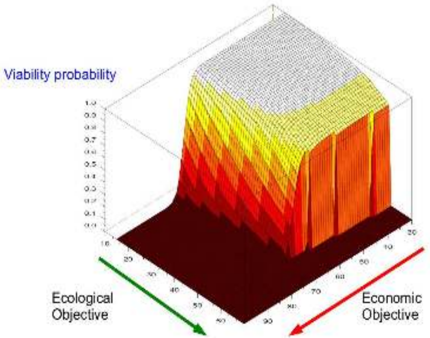
- Control theory makes a clear distinction between objectives and decision rules

  objectives $\Rightarrow$ adapted decision rules

- More specifically, viability theory puts emphasis on consistency between dynamics and objectives

  objectives + dynamics $\Rightarrow$ decision rules

Contribution to quantitative sustainable management



- Conceptual framework for quantitative sustainable management
- Managing ecological and economic conflicting objectives
- Ecosystem viable yields as a contribution to the "ecosystem approach"
- Displaying tradeoffs between ecology and economy sustainability thresholds and risk

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"Nul n'est mieux servi que par soi-même"
"Self-promotion, nobody will do it for you" ;-)

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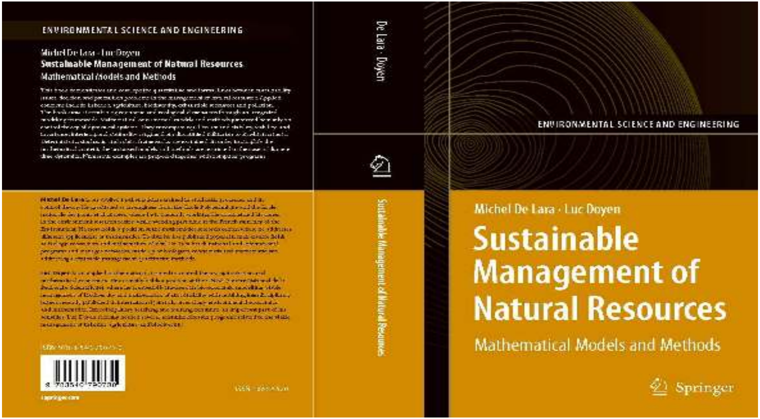

Some related initiatives

- From June 25 to July 6, 2012, CEA / EDF / INRIA Summer School on Stochastic Optimization
  http://www-hpc.cea.fr/SummerSchools2012-SO.htm
- From January 7 to April 5 in 2013, Institut Henri Poincaré (IHP) trimester Mathematics of Bio-Economics as a contribution to Mathematics of Planet Earth - MPE2013
  http://www.ihp.fr/en/ceb/mabies
- More on my web page
  http://http://cermics.enpc.fr/˜delara/

THANK YOU!