UASA of complex models: Coping with dynamic and static inputs

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Introduction

Building energy model

Uncertainties on:

- thermophysical materials
- weather data
- user behavior, ...

⇒ Uncertainties on energy consumption

- How are the uncertainties propagated through the model ?
- Which inputs are responsible for these uncertainties ?

⇒ Need to analyze the uncertainties to better understand the energy consumption

Contents

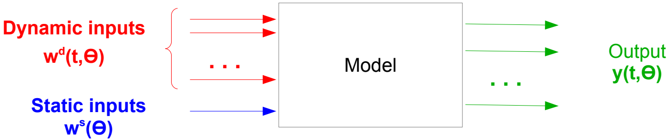
Contents

Model

Energy building model

$$\mathbf{y}(t,\theta) = g(\omega^{\mathbf{d}}(t,\theta), \omega^{\mathbf{s}}(\theta), t)$$

- $\omega^{\mathbf{d}}(t,\theta)$: uncertain weather data depending on t
- $\omega^{\mathbf{s}}(\theta)$: uncertain thermophysical properties of materials
- $\mathbf{y}$: energy consumption
- t: time (spatio/temporal variable)
- g: PDE



Model

Energy building model

$$\mathbf{y}(t,\theta) = g(\boldsymbol{\omega}^{\mathbf{d}}(t,\theta), \boldsymbol{\omega}^{\mathbf{s}}(\theta), t)$$

- Thermophysical properties $\omega_i^s(\theta)$ (static inputs)
  - Random variable
  - Marginal distribution
  - Random sampling methods
- Weather data $\omega_i^d(t,\theta)$ (dynamic inputs)
  - Random processes
  - Mean value $\bar{\omega}_i{}^d(t)$
  - Covariance function $C_i(t_1, t_2)$

Problem: how to generate consistent samples of dynamic inputs?

Contents

Generation of dynamic inputs samples

Weather data $\omega_i^d(t,\theta)$

- Random processes
- Mean value $\bar{\omega_i}^d(t)$
- Covariance function $C_i(t_1,t_2)$

Series expansion

Complete set of deterministic functions with corresponding random coefficients

Karhunen Loève decomposition

- Eigen-decomposition of the covariance function
- Orthogonal deterministic basis functions
- Uncorrelated random coefficients
- Optimal encapsulation of information contained in the random process into a set of discrete uncorrelated random variables

Generation of dynamic inputs samples

Weather data $\omega_i^d(t,\theta)$

- Random processes
- mean value $\bar{\omega}_i{}^d(t)$
- covariance function $C_i(t_1, t_2)$

Karhunen-Loève decomposition

$$\omega_i^d(t,\theta) \approx \bar{\omega}_i^d(t) + \sum_{k=1}^{M_i} \sqrt{\lambda_k}\xi_k(\theta)f_k(t)$$

- λ_k : eigenvalues of $C_i(t_1, t_2)$
- f_k : eigenfunctions of $C_i(t_1, t_2)$
- M_i : number of modes
- ξ_k : independent normally distributed random variables

Generation of dynamic inputs samples

Weather data $\omega_i^d(t,\theta)$

- Random processes
- mean value $\bar{\omega}_i{}^d(t)$
- covariance function $C_i(t_1, t_2)$

Karhunen-Loève decomposition

$$\omega_i^d(t,\theta) \approx \bar{\omega}_i^d(t) + \sum_{k=1}^{M_i} \sqrt{\lambda_k}\xi_k(\theta) f_k(t)$$

- λ_k and $f_k(t)$ solutions of Fredholm integral:

$$\int_{\mathcal{D}} C_i(t_1, t_2) f_k(t_1) \mathrm{d}t_1 = \lambda_k f_k(t_2)$$

- Resolution based on wavelet transform of $C_i(t_1, t_2)$ [Phoon]

Generation of dynamic inputs samples

Karhunen-Loève decomposition

$$\omega_i^d(t,\theta) \approx \bar{\omega}_i^d(t) + \sum_{k=1}^{M_i} \sqrt{\lambda_k}\xi_k(\theta) f_k(t)$$

- M_i modes containing 95% of $V(\omega_i^d(t,\theta))$
- Influence of the inputs $\{\omega_1^d(t_1,\theta),\ldots,\omega_1^d(t_f,\theta),\omega_2^d(t_1,\theta),\ldots,\omega_2^d(t_f,\theta),\ldots\}$
- Influence of the modes

Sensitivity analysis of the dynamic inputs

- N_d dynamic inputs $\omega_i^d(t,\theta)$
- M_i modes for each input $\omega_i^d(t,\theta)$
- In all: $n = \sum_{i=1}^{N_d} M_i$ modes to analyze
- SA of the inputs through $\{\underbrace{\xi_1, \cdots, \xi_n}_{\boldsymbol{\omega}^\mathbf{d}}, \boldsymbol{\omega}^s\}$
- Sensitivity indices

$$\{\underbrace{\xi_1, \cdots, \xi_{n_1}}_{M_1 \text{modes of } \omega_1^d}, \underbrace{\xi_{n_1+1}, \cdots, \xi_{n_2}}_{M_2 \text{modes of } \omega_2^d}, \cdots, \xi_n, \omega_1^s, \ldots, \omega_{N_s}^s\}$$

Sensitivity index of ω_i^d (grouped modes)

$$S_1 = \frac{V(E(y|\xi_1, \cdots, \xi_{n_1}))}{V(y)},$$

...

Sensitivity index of ω_i^s

$$S_l = \frac{V(E(y|\omega_1^s))}{V(y)},$$

...

Contents

Results

Initial problem

$$\mathbf{y}(\theta) = g(\boldsymbol{\omega}^{\mathbf{d}}(t, \theta), t)$$

6 dynamic inputs		2 outputs	
Temperature T	ω_1^d	Heat consumption grnd floor	y_1
Direct solar radiation D	ω_2^d	Heat consumption 1st floor	y_2
Diffuse solar radiation d	ω_3^d		
Wind speed V	ω_4^d		
Wind direction V_d	ω_5^d		
Relative humidity H	ω_6^d		

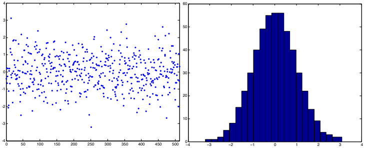
- Daily consumption summed over one month
- Data for representative month of january

Results

Procedure

1. Generation of the dynamic inputs
   1.1 Perform a 2D wavelet transform of $C_i(t_1, t_2)$, $i = 1, \ldots, 6$
   Fast Haar wavelet transform algorithm
   Here 3072 modes in all
   $\Rightarrow \lambda_k,\ f_k(t)$

   1.2 Generate the independent random variables ξ_k



   1.3 Generate the $N_d = 6$ dynamic inputs ω_i^d

$$\omega_i^d \approx \bar{\omega}_i^d(t) + \sum_{k=1}^{M_i} \sqrt{\lambda_k}\,\xi_k(\theta) f_k(t)$$

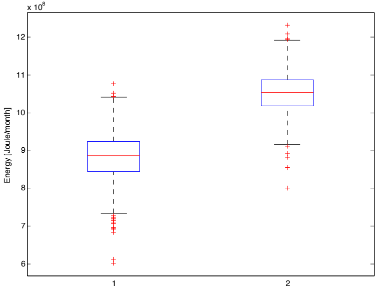
Results

Procedure

2 Sensitivity analysis

2.1 Simulate the model with the dynamic inputs

$$\mathbf{y}(\theta) = g(\boldsymbol{\omega}^{\mathbf{d}}(t, \theta), t)$$



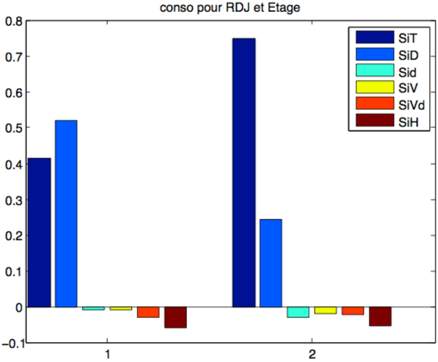
Ground floor 1st floor

- Higher consumption at the 1st floor
- More glass surface at the ground floor
- Solar gain more important at the ground floor
- Sensitivity indices to check this assumption

Results

2.2 Sensitivity indices of the grouped modes

$$S_i = \frac{V(E(y|\xi_i, \cdots, \xi_{n_i}))}{V(y)}$$



Ground floor 1st floor

- Direct solar radiation more influent at the ground floor
- Temperature more influent at the 1st floor
- Importance of solar gain during winter time

Contents

Conclusion

In prospect

- Study the influence of the individual modes
- Include the thermophysical properties of the materials $\boldsymbol{\omega}^{\mathbf{s}}$ (static inputs)
- Consider the dynamic output $\mathbf{y(t,\theta)}$ (thermal comfort)
- Work done in the context of the french project ASenDyn granted by the CNRS

Thank you for your attention

Questions ?