Comparison of Sensitivity Analysis methods applied on a groundwater flow and mass transport model

The case of Andra Meuse/Haute-Marne site

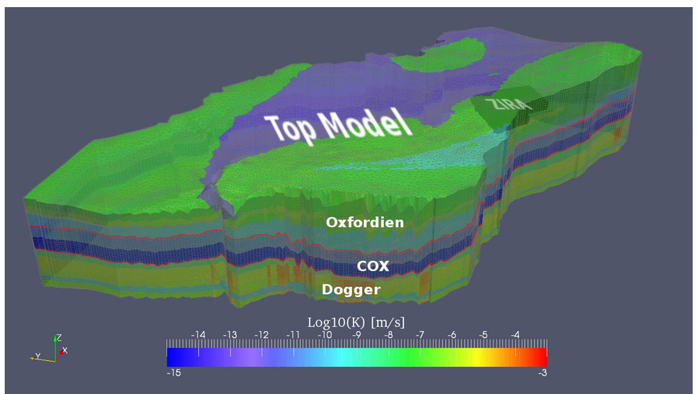
G. Deman[1], J. Kerrou[1], H. Benabderrahmane[2], P. Perrochet[1]

[1] *CHYN - Centre of Hydrogeology and Geothermics, University of Neuchâtel - Emile-Argand 11, CH-2000 Neuchâtel - Switzerland*
[2] *Andra - Agence Nationale pour la gestion des Dechets Radioactifs - 1/7 rue Jean Monnet, 92298 Châtenay-Malabry - France*

contact: gregory.deman@unine.ch

FLOW MODEL AND LIFETIME EXPECTANCY CALCULATION

The Model



Model Statistics : Finite Element - 16 Millions Elements - 16 Layers
Problem Definition : Steady State Flow & Mass Transport
Simulation time : 1 Million years
Observed Response : Lifetime Expectancy from ZIRA Zone

The Problem

Assess Model's Response sensitivity to correlated Hydraulic conductivities (K) and porosities (n) of 14 hydrogeological layers

- Apply multiplicative factors to correlated inputs : **sample correlated factors**
- Inputs *n* & log(*K*) correlated within each layer : ***2 by 2*** **correlations C = 0.8**
- Sampling based upon **Gaussian distribution** centred on 1

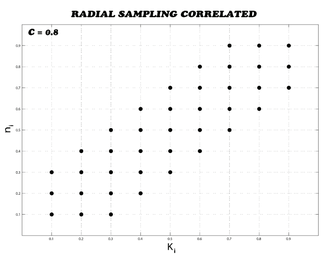
Multiplicative Factors	
n_mult	*log(K_mult)*
[0.85 ; 1.15]	[-1 ; 1]

The Parameters

	Layer name	ID	Porosity "n" [-]	Hydraulic Conductivity "K" [m/s]
Oxfordian	Top of upper Sequanian	14	0.1585	4.66 E-07
	Base of upper Sequanian	13	0.1615	1.16 E-06
	Lower Sequanian	12	0.1756	4.14 E-08
	Upper Rauracian	11	0.1867	2.96 E-07
	Lower Rauracian	10	0.2068	1.05 E-07
	Upper Argovian	9	0.2103	4.64 E-11
COx	Lower Argovian	8	0.1705	8.00 E-13
	Argilleous Callovian	7	0.1681	1.92 E-15
Dogger	Dalle nacrée	6	0.2043	3.90 E-08
	Mid-upper Bathonian	5	0.2043	7.81 E-06
	Lower Bathonian	4	0.1999	3.90 E-06
	Marnes de Longwy	3	0.1878	4.64 E-11
	Upper Bajocian	2	0.1878	1.95 E-06
	Lower Bajocian	1	0.2089	5.79 E-08

EXPERIMENTAL DESIGNS

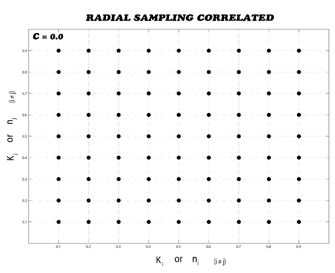
Correlated Radial Sampling
(Campolongo et al., 2011)



Within Layers

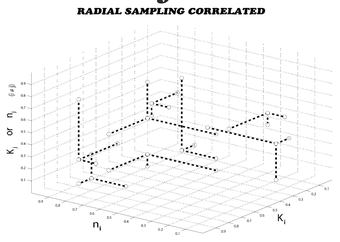
Domain of existence

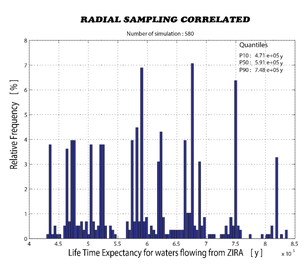
Draw trajectories



Between Layers

Simulator Calculation





Simulator Calculation

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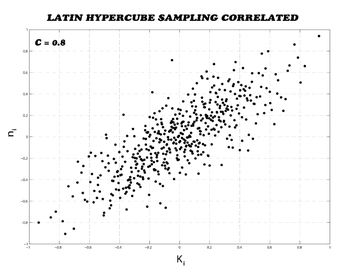
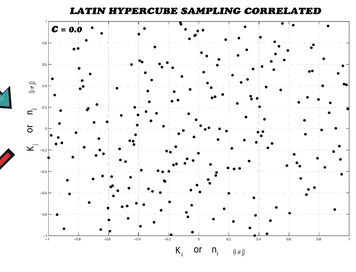
Correlated Latin Hypercube Design
(Iman & Conover, 1982)



Within Layers

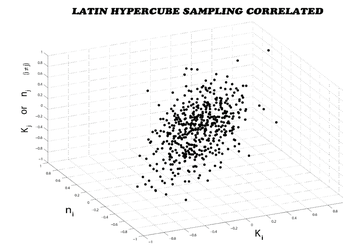
Latin Hypercube Samples

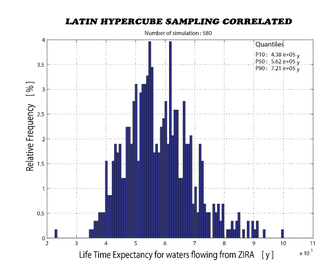
Aggregate samples



Between Layers

Simulator Calculation





Simulator Calculation

use of code ***GroundWater*** (Cornaton, 2007)

Result = Mean value of the Lifetime Expectancy (time required for a water molecule to reach an outlet limit of the system) calculated over the nodes representing the ZIRA zone (within the COx formations)

Steady-state Lifetime Expectancy

$$\nabla . qE + \nabla . \theta \mathbf{D} \nabla E + \theta = 0$$

Where E is the mean lifetime expectancy
$\mathbf{D}$ is the macro-dispersion tensor
θ is the porosity

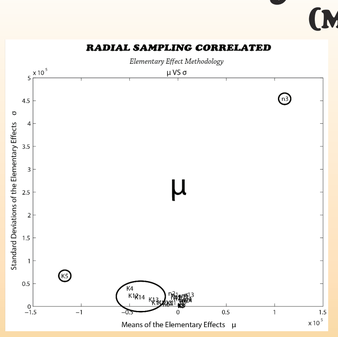
And, with the fluid flux

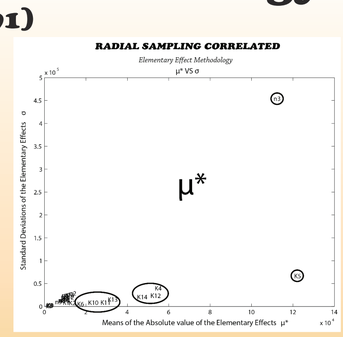
$$\mathbf{q} = -k_r \mathbf{K} \nabla H$$

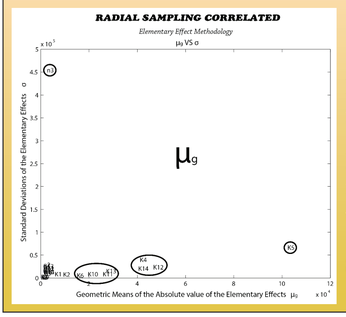
Where H is the hydraulic head
$\mathbf{K}$ is the saturated tensor of hydraulic conductivity
k_r is the relative permeability of the medium with respect to the degree of water saturation

SENSITIVITY ANALYSIS

Elementary Effects Methodology
(Morris, 1991)







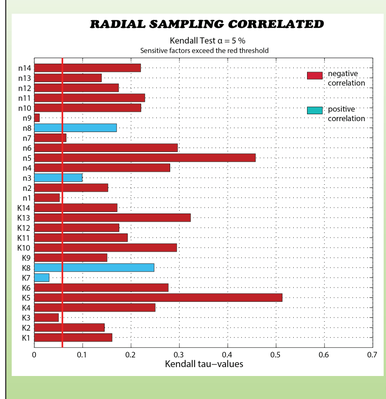
Results

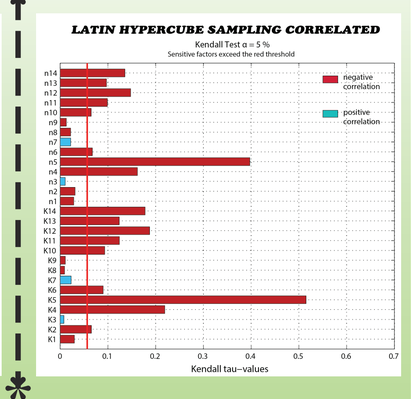
Purely **negative** effect of K_5 with **little interaction and/or 2cd order** effect

Purely **positive** effect of n_3 with **strong interactions and/or 2cd order** effect leading to **wide spreading of response value**

Slight purely negative effect of K_4, K_6, $K_{10, ..., 14}$

Kendall Rank Correlation Coefficient

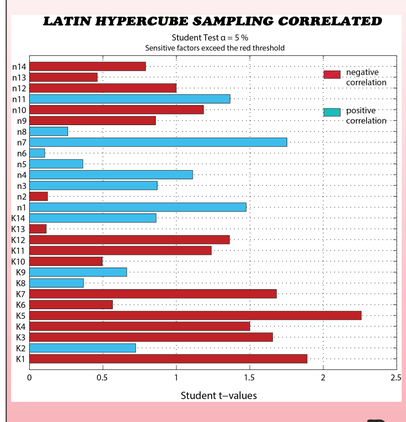


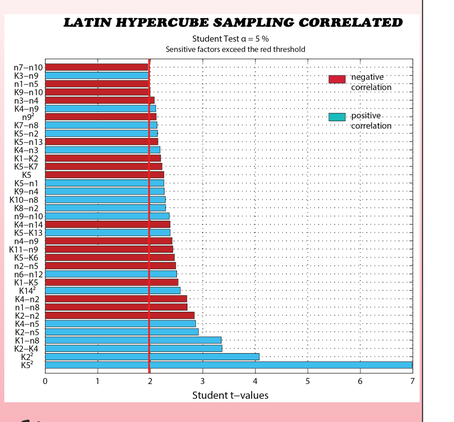


Results

Strong **negative** effect of K_5 & n_5
Slight **positive** effect of n_3
Slight **negative** effect of K_4 & n_4, K_6 & n_6, $K_{10, ..., 14}$ & $n_{10, ..., 14}$

Polynomial Regression + Student t-test





Results

Negative 1st order effect of K_1, $K_{3, ..., 5}$, K_7, K_{11}, K_{12}, n_{10}, n_{12} and n_{14}
Positive 1st order effect of K_2, K_9, K_{14}, n_1, n_3, n_4, n_7 and n_{11}
Strong **positive 2cd order** effect of K_5, K_2 and K_{14}
Many **two-ways interactions**

DISCUSSION

Methods

Correlated sampling techniques **avoid unrealistic parameters combinations** that could possibly give misleading Sensitivity Analysis results

	Elementary Effects Methodology (Morris, 1991)	Kendall RCC	Student t-test on polynomial regression
Advantages	Gives reliable qualitative assessment of the sensitivity of the response to model's parameters	Allows the selection of a subset of unimportant parameters	Allows the detection of high-order and interactions effects Allows the selection of a subset of unimportant parameters
Disadvantages	Does not allow the selection of a subset of unimportant parameters	Does not detect high-order effects or interactions	Uses an approximation of the true response

Results

Layer **Mid-upper Bathonian** (n°5) strongly **reduces the transit time** of solute throughout the model because of its **high Hydraulic conductivity** (K_5).
Layers **Lower Bathonian** (n°3), **Dalle Nacrée** (n°6), **Rauracian** (n°10 - 11) and **Sequanian** (n°12 - 13 - 14) also **reduce the Lifetime Expectancy** to a lesser degree because of their **high Hydraulic conductivities**.
Porosities have little direct effects on the response ; except for layer **Marnes de Longwy** (n_3) where **strong interactions** or **high-order effects** take place.

References

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Morris, M. D., *"Factorial sampling plans for preliminary computational experiments."*, Technometrics **33**: 161-174 (1991).
Iman, R. L. and Conover, W. J., *"A Distribution- Free Approach to Inducing Rank Correlation Among Input Variables."* Communications in Statistics-Simulation and Computation **11**: 311-334 (1982).

Acknowledgment

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