Sensitivity Analysis for Functional Inputs in a Sheet Metal Forming Process

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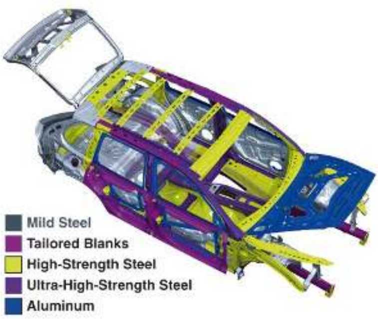
Sheet metal forming

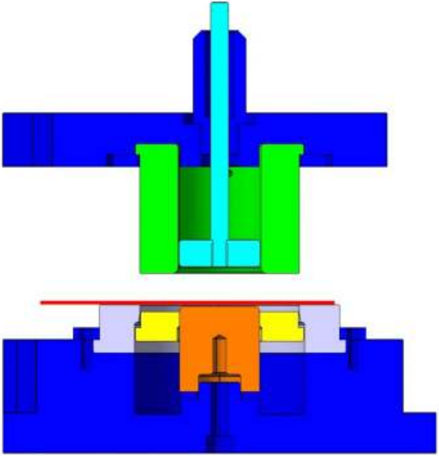
Standard industrial process, e.g. in the forming of car parts



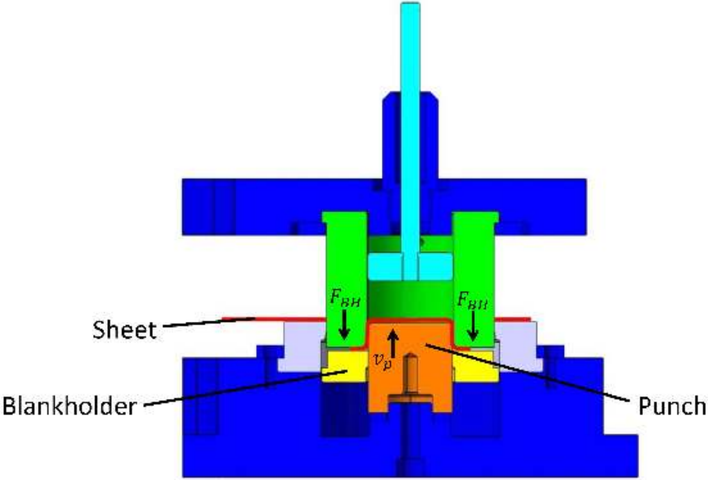
Formed parts must satisfy high requirements

Source: American Iron and Steel Institute via `france-metallurgie.com`

Forming process



Source: Tekkaya et al. (2012)

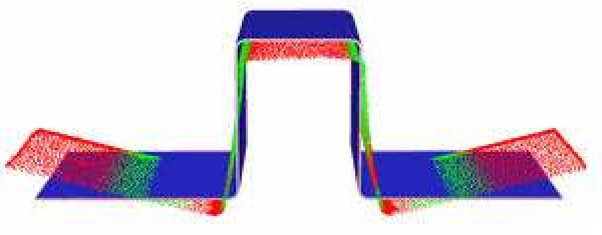

Sheet
Blankholder
Punch
F_{BH}
F_{BH}
v_p

Forming press at TU Dortmund University

Shape examples

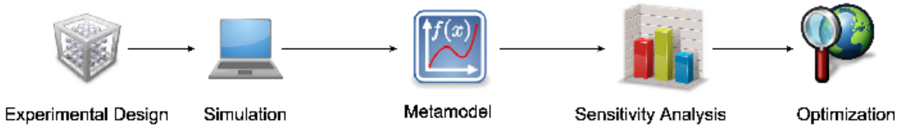
Occuring problems

- Springback
- Tearing
- Wrinkling



Finite Element simulations, e.g. by LS-DYNA

Standard approach: prediction, sensitivity analysis, optimal springback compensation

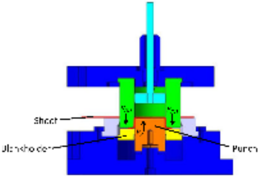


Now

- Technical improvements to machines

→ Temporal change of blankholder force and friction possible

- Adaptions to simulation tool (partition, starting, stopping via Linux shell script) → simulations with functional input possible



change parameters during forming

Goals

- Knowledge on temporal influence of the parameters
  → sensitivity analysis
- Improvements in forming, springback optimization

Preview

Runs with constant friction compared to varying friction

time interval	$[0, 0.25[$	$[0.25, 0.5[$	$[0.5, 0.75[$	$[0.75, 1]$	springback
low	0.05	0.05	0.05	0.05	**6.10550**
high	0.2	0.2	0.2	0.2	**2.13370**
varying	0.05	0.05	0.2	0.2	**1.39423**

Friction with same overall mean, but different functional behaviour

time interval	$[0, 0.25[$	$[0.25, 0.5[$	$[0.5, 0.75[$	$[0.75, 1]$	springback
run 1	0.2	0.2	0.05	0.05	**14.66389**
run 2	0.05	0.05	0.2	0.2	**1.44208**
run 3	0.2	0.05	0.05	0.2	**0.75362**
run 4	0.2	0.05	0.2	0.05	**6.01258**

$$\overline{x} = 0.125$$

Mathematical formulation

Analyse experiment with

- $Y \in \mathbb{R}$ scalar response
- $x_i \in [0,1]$, $i = 1, \dots, d_s$ scalar input variables
- $g_j : [0,1] \mapsto [0,1]$, $j = 1, \dots, d_f$ functional input variables

Connected by a black-box function $f : \mathbb{R}^{d_s} \times \mathcal{F}^{d_f}_{[0,1]} \mapsto \mathbb{R}$

$$Y = f\left(x_1, \dots, x_{d_s}, g_1, \dots, g_{d_f}\right)$$

- All input variables can be controlled!
- Function evaluations very time consuming

Overview

Short review: functional data analysis

Ramsay, Silverman (2005): Functional linear model with functional input and scalar response

$$Y_i = \alpha + \sum_{j=1}^{p} \int_0^1 x_{ij}(s)\beta_j(s)ds + \varepsilon_i, \quad i = 1, \ldots, N$$

with

- Y response
- $x_1(\cdot), \ldots, x_p(\cdot)$ input functions
- α unknown constant effect
- $\beta_1(\cdot), \ldots, \beta_p(\cdot)$ unknown regression fucntions
- ε_i error term with $\mathrm{E}(\varepsilon_i) = 0$, $\mathrm{cov}(\varepsilon_i, \varepsilon_k) = 0$, $i \neq k$

- Express regression function $\beta_j(s)$ by well-chosen basis functions $\theta_{jk}(s)$:

$$\beta_j(s) = \sum_{k=1}^{K_\beta} b_{jk}\theta_{jk}(s) = \theta_j(s)^T b_j,$$

where now b is the unknown parameter vector

- Approximate the usually given $x_j(s)$ by appropriate basis functions $\psi_{jk}(s)$

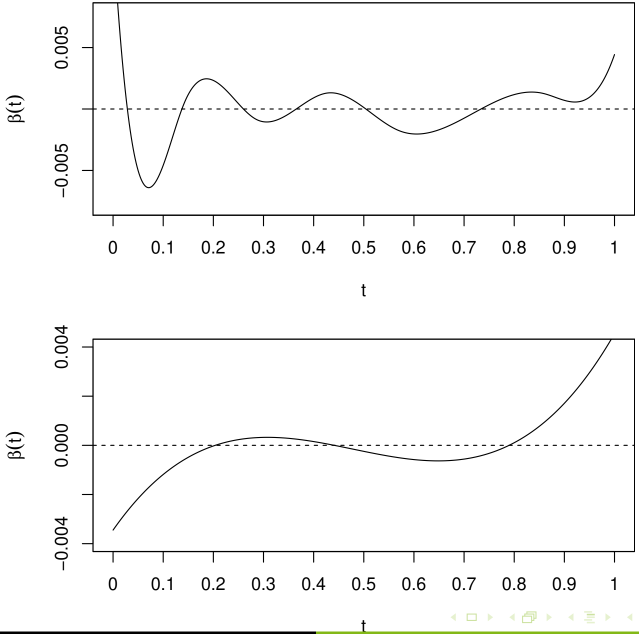
$$x_{ij}(s) = \sum_{k=1}^{K_j} C_{ijk}\psi_{jk}(s) \Rightarrow x_j(s) = C_j\psi_j(s),$$

with $C_j \in \mathbb{R}^{N\times K_j}$ coefficient matrix

The model can then be simplified to

$$\begin{aligned} Y &= \boldsymbol{\alpha} + \int_0^1 x(s)\beta(s)ds + \varepsilon \\ &= \boldsymbol{\alpha} + \underbrace{\int_0^1 \boldsymbol{C}\psi(s)\theta(s)^T}_{Z} \boldsymbol{b}\, ds + \varepsilon \\ &= \boldsymbol{\alpha} + \qquad Z \qquad \boldsymbol{b} + \varepsilon \end{aligned}$$

→ Then vector $\boldsymbol{b}$ can be estimated by least squares


0.005
−0.005
β(t)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t
0.004
0.000
−0.004
β(t)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t

Interpretation of the coefficient function

- Dimension basis hopefully large enough to capture the patterns, but small enough to regularize the fit
- β determines the effect of g on Y:
  No effect at t when $\beta(t) = 0$, greater effect when $|\beta(t)|$ large

Drawbacks

- β-curves with wiggles
- Not easy to interpret in terms of zero, constant, linear behaviour
- Only exactly zero at a few locations even if there is no influence for large regions

Source: James and Wang (2009)

Iooss and Ribatet (2009): Sensitivity analysis with functional input

- Situation
  - Model with scalar parameters and functional input as stochastic process simulation
- Aim
  - Measure the uncertainty of a functional input in one index
- Three methods
  - Macroparameter method
  - Trigger method
  - Joint modelling approach

Mühlenstädt (2012): Spacefilling design and Kriging model for scalar and function input

- Functional norm

$$d_2(g, \tilde{g}) = \sqrt{\int_0^1 (g(t) - \tilde{g}(t))^2 \, dt}$$

- Maximin criterion for spacefilling design
- Kriging model

$$Y \sim \mathbb{N}(\mu, \sigma^2 R)$$

$$R_{i,j} = \prod_{k=1}^{d_s} k_{\theta_k}(x_{k,i} - x_{k,j}) \prod_{k=1}^{d_f} k_{\theta_k}(d_2(g_{k,i}, g_{k,j}))$$

- Kriging parameter as first impression on important inputs

Overview

Motivation for a new method

- Design for functional inputs, jointly with scalar inputs
- Different sensitivities for different regions of time instead of only one per functional input

$\rightarrow$ Function of sensitivities

Functional sensitivity analysis by sequential splitting

Assumptions: for each functional input g_i, $i = 1, \ldots, d_f$

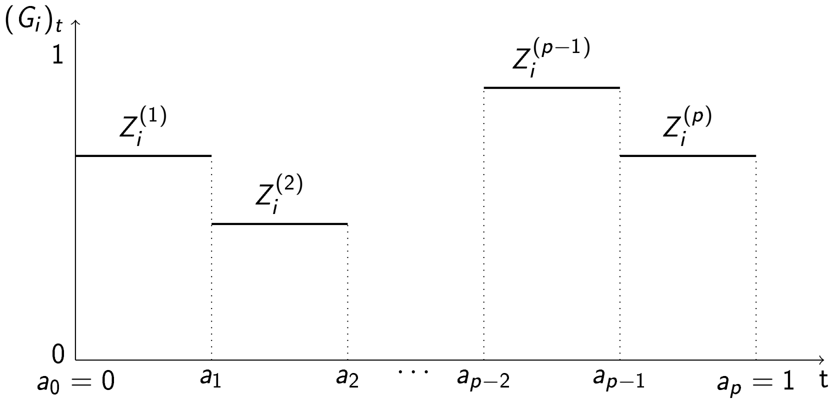
- Part the domain of the function $[0,1]$ at $p+1$ knots

$$\boldsymbol{a} = \{a_0 = 0, a_1, a_2, \ldots, a_{p-1}, a_p = 1\}$$

- Model the $g_i = (G_i)_t$ as a piecewise constant random process

$$(G_i)_t = Z_i^{(1)} 1_{[0,a_1[}(t) + \cdots + Z_i^{(p)} 1_{[a_{p-1},1]}(t)$$

with $Z_i^{(j)} \in [0,1]$ independent random variables for $j = 1, \ldots, p$ and $i = 1, \ldots, d_f$


$(G_i)_t$
1
0
$Z_i^{(1)}$
$Z_i^{(2)}$
$Z_i^{(p-1)}$
$Z_i^{(p)}$
$a_0 = 0$
a_1
a_2
$\cdots$
a_{p-2}
a_{p-1}
$a_p = 1$
t

Transformation from functional to linear space

$$\begin{aligned} Y &= f\left(x_1, \ldots, x_{d_s}, g_1, \ldots, g_{d_f}\right) \\ &= \tilde{f}\left(X_1, \ldots, X_{d_s}, Z_1^{(1)}, \ldots, Z_{d_f}^{(p)}\right) \end{aligned}$$

Sensitivity analysis on Y 'reduces' to sensitivity analysis on $\tilde{f}$

Some possible methods for the sensitivity analysis

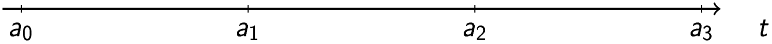
Method	Sampling design
Linear regression	DoE
Morris indices	Morris design
Sobol indices	(Quasi-) Monte Carlo (on meta model)

$\longrightarrow$ Sensitivity index for each interval

Source: Saltelli et al. (2000)

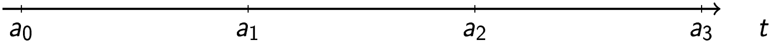
Sequential splitting

1. Step

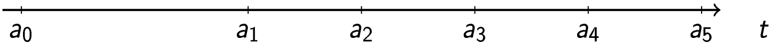


Sequential splitting

1. Step

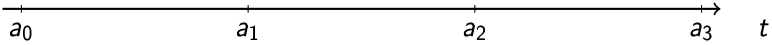


2. Step



Sequential splitting

1. Step



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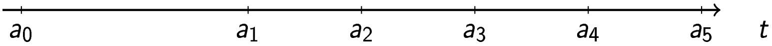
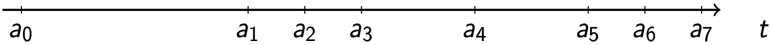


3. Step



Sequential splitting

1. Choose a low number ($p = 2, p = 3$) of intervals
2. Perform sensitivity analysis on $\tilde{f}$
3. For each interval k^* that has a meaningful influence, include

$$\boldsymbol{a}^* = \frac{a_{k^*} - a_{k^*-1}}{2}$$

   to the vector of knots $\boldsymbol{a}$
4. For each unimportant interval $\tilde{k}$, set $Z_{\tilde{k}} = c_{\tilde{k}}$, with $c_{\tilde{k}}$ a suitable average constant value for the interval so that it is not varied any more in the next steps

⋮ Repeat steps 2-4 until the functions are sufficiently recovered

Overview

Investigation with regression coefficients as sensitivity method

Assumptions:

- One dimensional case $f : \mathcal{F}_{[0,1]} \mapsto \mathbb{R}$

$$Y = f(g) = \tilde{f}\left(Z^{(1)}, \ldots, Z^{(p)}\right)$$

- Linear regression coefficients as sensitivity indices for the main effects of Z_i

$$(\hat{\beta}_1, \ldots, \hat{\beta}_p)' = (\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{Z}'y \quad \text{with } (\boldsymbol{Z}_{ij}) = z^{(j)} \text{ of run } i$$

  and similarly $\hat{\beta}_{12}, \hat{\beta_{13}}, \ldots$ coefficients for interaction effects

Investigation with regression coefficients as sensitivity method

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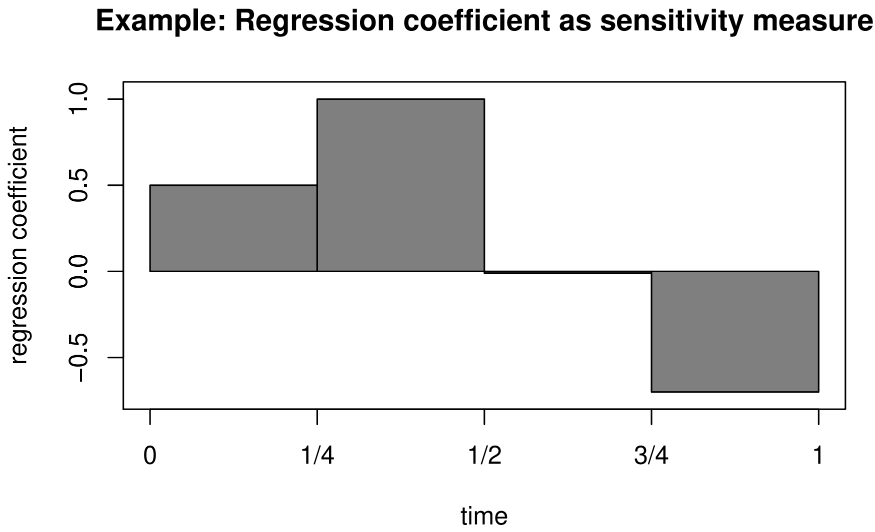
→ Low number of runs

→ Interpretation and significance tests possible

→ Only linear effects and linear interactions detectable


Example: Regression coefficient as sensitivity measure
regression coefficient
−0.5
0.0
0.5
1.0
0
1/4
1/2
3/4
1
time

Normalization

Definition

Let β_i be the regression coefficient of the corresponding variable Z_i, the value of function g at $[a_{i-1}, a_i[$, and β_{ij} the interaction coefficient of Z_i and Z_j. Then define

$$H_i = \frac{\beta_i}{a_i - a_{i-1}} \quad \text{and} \quad H_{ij} = \frac{\beta_{ij}}{(a_i - a_{i-1})(a_j - a_{j-1})}$$

the **normalized regression index** of Z_i and the **normalized interaction regression index** of Z_i and Z_j resp.

$\rightarrow$ Regression coefficient per unit of time

$\rightarrow$ Independence of the current partition

Aim

Investigate the effect of certain functional shapes of f on the indices $H_1, \ldots, H_p$

Case 1: $f(g) = \int_0^1 g(t)\, dt$

Case 2: $f(g) = \int_0^1 w(t) g(t)\, dt$

Case 3: $f(g) = \int_0^{\frac{1}{2}} g(t) dt \times \int_{\frac{1}{2}}^1 g(t)\, dt$

Case 4: $f(g) = \int_0^1 \zeta(g(t))\, dt$

$\rightarrow H_i?, H_{ij}?$

Case 1

Constant linear functional influence

$$f(g) = \int_0^1 g(t)\,dt$$

$$\tilde{f}(Z^{(1)}, \ldots, Z^{(p)}) = \int_0^1 Z^{(1)} \mathbb{1}_{[a_0, a_1[}(t) + \cdots + Z^{(p)} \mathbb{1}_{[a_{p-1}, a_p]}(t)\,dt$$
$$= Z^{(1)}(a_1 - a_0) + \cdots + Z^{(p)}(a_p - a_{p-1})$$

→ $\tilde{f}$ is a linear function with factors $a_i - a_{i-1}$

→ $\beta_i = a_i - a_{i-1}, \quad i = 1, \ldots, p$

Case 1

Constant linear functional influence

$$f(g) = \int_0^1 g(t)\,dt$$

$$\begin{aligned}\tilde{f}(Z^{(1)},\dots,Z^{(p)}) &= \int_0^1 Z^{(1)}\mathbb{1}_{[a_0,a_1[}(t) + \dots + Z^{(p)}\mathbb{1}_{[a_{p-1},a_p]}(t)\,dt \\ &= Z^{(1)}(a_1 - a_0) + \dots + Z^{(p)}(a_p - a_{p-1})\end{aligned}$$

→ $\tilde{f}$ is a linear function with factors $a_i - a_{i-1}$

→ $\beta_i = a_i - a_{i-1}, \quad i = 1,\dots,p$

$$\boxed{H_1 = \dots = H_p = 1, \quad H_{ij} = 0, \quad i \neq j = 1,\dots,p}$$

Case 2

Linear functional weights $w : [0, 1] \mapsto \mathbb{R}$ on t

$$f(g) = \int_0^1 w(t) g(t)\, dt$$

$$\tilde{f}(Z^{(1)}, \dots, Z^{(p)}) = \int_0^1 w(t) Z^{(1)} \mathbb{1}_{[a_0, a_1[}(t) + \dots + w(t) Z^{(p)} \mathbb{1}_{[a_{p-1}, a_p]}(t)\, dt$$

$$= Z^{(1)} \int_{a_0}^{a_1} w(t)\, dt + \dots + Z^{(p)} \int_{a_{p-1}}^{a_p} w(t)\, dt$$

Case 2

Linear functional weights $w : [0,1] \mapsto \mathbb{R}$ on t

$$f(g) = \int_0^1 w(t) g(t)\, dt$$

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$$= Z^{(1)} \int_{a_0}^{a_1} w(t)\, dt + \cdots + Z^{(p)} \int_{a_{p-1}}^{a_p} w(t)\, dt$$

$$\boxed{H_i = \frac{\int_{a_{i-1}}^{a_i} w(t)}{a_i - a_{i-1}} \quad i = 1, \ldots, p} \qquad \lim_{\delta \to 0} \frac{\int_{a_i}^{a_i + \delta} w(t)\, dt}{\delta} = w(a_i)$$

$$\boxed{H_{ij} = 0, \quad i \neq j = 1, \ldots, p}$$

Case 3

Linear interactions between functional regions

$$f(g) = \int_0^{\frac{1}{2}} g(t)dt \times \int_{\frac{1}{2}}^1 g(t)\,dt \quad \text{(say } a_{i^*} = 1/2 \text{ exists:)}$$

$$\begin{aligned}
\tilde{f}(Z^{(1)}, \dots, Z^{(p)}) &= \int_0^{\frac{1}{2}} Z^{(1)} \mathbb{1}_{[a_0, a_1[}(t) + \cdots + Z^{(p)} \mathbb{1}_{[a_{p-1}, a_p]}(t)\,dt \times \\
&\times \int_{\frac{1}{2}}^1 Z^{(1)} \mathbb{1}_{[a_0, a_1[}(t) + \cdots + Z^{(p)} \mathbb{1}_{[a_{p-1}, a_p]}(t)\,dt \\
&= \left(Z^{(1)}(a_1 - a_0) + \dots Z^{i^*}(1/2 - a_{i^*-1}) \right) \times \\
&\times \left(Z^{i^*+1}(a_{i^*+1} - 1/2) + \cdots + Z^{(p)}(a_p - a_{p-1}) \right)
\end{aligned}$$

Case 3

Linear interactions between functional regions

$$f(g) = \int_0^{\frac{1}{2}} g(t)dt \times \int_{\frac{1}{2}}^1 g(t)\,dt \quad \text{(say } a_{i^*} = {}^1\!/_2 \text{ exists:)}$$

$$\begin{aligned}\tilde{f}(Z^{(1)}, \dots, Z^{(p)}) &= \int_0^{\frac{1}{2}} Z^{(1)}\mathbb{1}_{[a_0,a_1[}(t) + \dots + Z^{(p)}\mathbb{1}_{[a_{p-1},a_p]}(t)\,dt \times \\ &\times \int_{\frac{1}{2}}^1 Z^{(1)}\mathbb{1}_{[a_0,a_1[}(t) + \dots + Z^{(p)}\mathbb{1}_{[a_{p-1},a_p]}(t)\,dt \\ &= \left(Z^{(1)}(a_1 - a_0) + \dots Z^{i^*}({}^1\!/_2 - a_{i^*-1})\right) \times \\ &\times \left(Z^{i^*+1}(a_{i^*+1} - {}^1\!/_2) + \dots + Z^{(p)}(a_p - a_{p-1})\right)\end{aligned}$$

$$\boxed{H_i = 0, \quad H_{ij} = 1, \quad i \neq j = 1, \dots, p}$$

Case 4

Function $\zeta : \mathbb{R} \mapsto \mathbb{R}$ on g, not necessarily linear

$$f(g) = \int_0^1 \zeta(g(t))\, dt$$

$$\begin{aligned}
\tilde{f}(Z^{(1)}, \dots, Z^{(p)}) &= \int_0^1 \zeta(Z^{(1)} \mathbb{1}_{[a_0, a_1[}(t) + \dots + Z^{(p)} \mathbb{1}_{[a_{p-1}, a_p]}(t))\, dt \\
&= \int_{a_0}^{a_1} \zeta(Z^{(1)} \mathbb{1}_{[a_0, a_1[}(t) + \dots + Z^{(p)} \mathbb{1}_{[a_{p-1}, a_p]}(t))\, dt + \dots \\
&\quad + \int_{a_{p-1}}^{a_p} \zeta(Z^{(1)} \mathbb{1}_{[a_0, a_1[}(t) + \dots + Z^{(p)} \mathbb{1}_{[a_{p-1}, a_p]}(t))\, dt \\
&= \int_{a_0}^{a_1} \zeta(Z^{(1)})\, dt + \dots + \int_{a_{p-1}}^{a_p} \zeta(Z^{(p)})\, dt \\
&= \zeta(Z^{(1)})(a_1 - a_0) + \dots + \zeta(Z^{(p)})(a_p - a_{p-1})
\end{aligned}$$

Remark: Link to functional linear regression in case 2

- Reminder: Functional linear regression model with basis function representation for β

$$Y_i = \alpha + \int_0^1 x(s)\beta(s)ds + \varepsilon_i, \quad \beta(s) = \sum_{k=1}^{K_\beta} b_k \theta_k(s)$$

- Let basis functions θ_k be the B-spline basis of order 1

$$\beta(s) = \sum_{k=1}^{p} b_k \mathbb{1}_{[a_{i-1}, a_i]}(s)$$

Then

$$H_i = \frac{\int_{a_{i-1}}^{a_i} \beta(s)}{a_i - a_{i-1}} = b_i, \quad i = 1, \dots, p$$

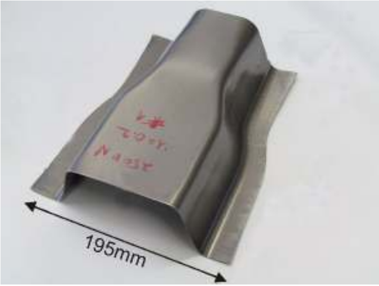
Different scopes

Functional linear regression	**Sequential splitting**
$\alpha + \int_0^1 x(s)\beta(s)ds + \varepsilon_i$	$\tilde{f}\left(Z^{(1)}, \ldots, Z^{(p)}\right)$
Modeling & prediction	Sensitivity analysis
Approximation of inputs	Design of inputs
Noisy observations	Deterministic
No interactions	Interactions possible
Only linear	Non-linearities considered

Overview

Application to sheet metal forming

Forming of an open demonstrator, inspired by a B-pillar



Two functional inputs

- Blankholder force
- Friction

Output

- Mean shape deviation

Sensitivity analysis

- Regression on factorial design on two levels

Sequential Screening

Step 1

- Two intervals each (variables $Z_1^{(1)}, Z_2^{(1)}, Z_1^{(2)}, Z_2^{(2)}$)

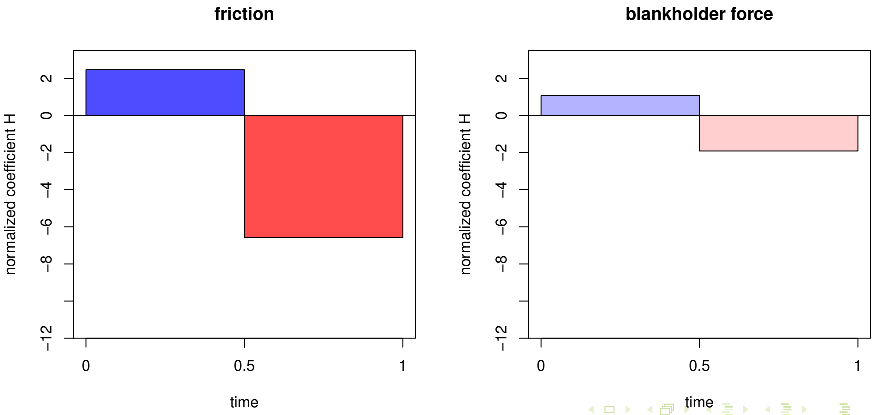
$\rightarrow$ Full factorial 2^4-design (16 runs)

Sequential Screening

Step 1

- Two intervals each (variables $Z_1^{(1)}, Z_2^{(1)}, Z_1^{(2)}, Z_2^{(2)}$)

$\longrightarrow$ Full factorial 2^4-design (16 runs)



Sequential Screening

Step 2

- Four intervals each

→ Fraction factorial 2^{8-3}-design (32 runs)

Sequential Screening

Step 2

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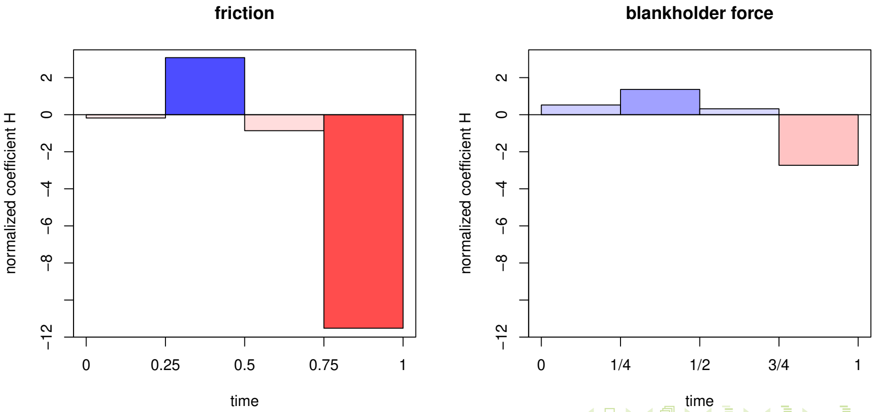
→ Fraction factorial 2^{8-3}-design (32 runs)



Sequential Screening

Step 3

- Four varied and four constant intervals each

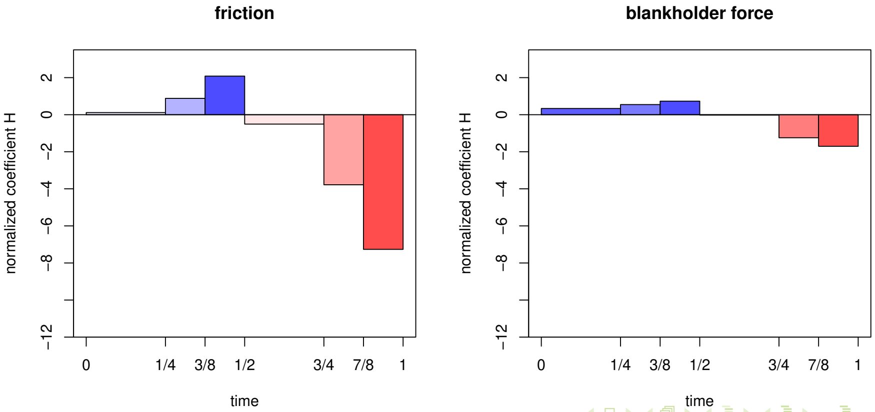
→ Fractional factorial 2^{8-3}-Design (32 runs)

Sequential Screening

Step 3

- Four varied and four constant intervals each

→ Fractional factorial 2^{8-3}-Design (32 runs)



Summary and Perspective

Sensitivity analysis for experiments with functional input

- Focus on sensitivity of regions
- Idea: Sequential splitting on B-spline of order 1-design
- Successful application to sheet metal forming process

Perspectives

- Consider dependency in Z-values
- Extension to spatial input

Literature

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Interaction Analysis

f21:f3	-0.525155	0.000920	***
f22:f41	-0.148317	0.276414	
f22:f42	-0.203745	0.139475	
f3:f41	0.339523	0.027332	*
f3:b1	-0.244428	0.089002	.
f3:b3	0.175992	0.213476	
f3:b41	0.162643	0.249052	
f41:f42	0.833154	1.80e-05	***
f41:b21	-0.098302	0.467288	
f42:b22	-0.124871	0.357598	
f42:b41	0.130391	0.337167	
b1:b21	0.357779	0.020771	*
b21:b22	0.249661	0.116857	