# A quality measure for comparing different feature deviations to perform sensitivity analysis in tolerancing

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## Framework

### 1. Tolerances in mechanical engineering design

**Axiom of manufacturing imprecision:** All manufacturing and assembly processes are inherently imprecise and produce parts and products that vary. [1]

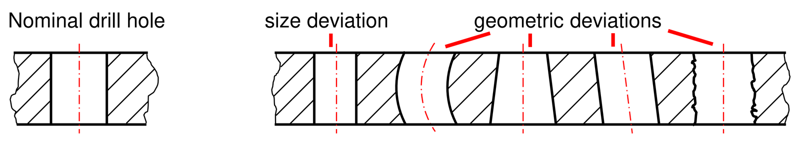

Figure 1: Different deviating drill holes

Tolerances restrict the geometric variations of parts with respect to the nominal geometry.

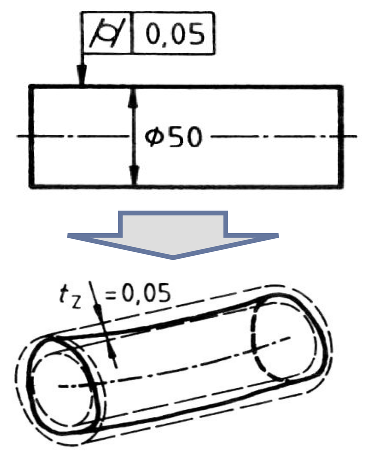

Figure 2: Cylindricity tolerance and the associated tolerance zone [2]

Tolerance analysis explores the relations between the assigned part-tolerances and the products assemblability and functionality.

### 2. Tolerance analysis in early design stages

To shorten the product design process, product analyses have to be performed as early as possible to avoid late product modifications, which are cost- and time-consuming. A simulation model for early design stages

- should have a high performance
- could handle partially incomplete geometry
- doesn't has to recognize detailed manufacturing information

### 3. Features

Features are generic shapes with which engineers associate certain properties or attributes and knowledge useful in reasoning about the product. [3]

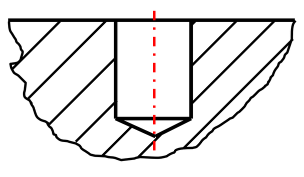
Figure 3: The feature drill hole

A feature can be a flange, a drill hole, a ball bearing, a screw, etc.. A feature usually is defined by multiple conditions and parameters!

### 4. Tolerance Analysis Simulation Model

The proposed simulation model bases on Homogenous Transformations $T \in SE(2)/SE(3)$. The simulation model consists of:

**Model Input – Tolerances:** Manufacturing deviations are represented by transformations $T^{\delta}_{d}$ which are restricted by tolerances (figure 4).

**Assembly Operation:** Assemble the components is represented by transformations $T$ which are guided by control points to avoid collisions (figure 5).

**Clearance Deviations:** Assembled features have position deviations $T^{\delta}_{c}$ inside the clearance domain which also are restricted by control points (figure 6).

**Model Output:** The resulting relative position of two features or a resulting assembly clearance. Functional requirements often are formulated as geometric tolerance with associated functional domain [4] (figure 7).

**Characteristic:** In the proposed model, several parameters are restricted together by tolerances – the parameters are dependent (figure 4, 6 and 7 (r)).

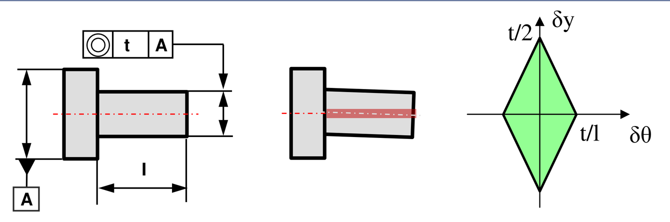

Figure 4: Pin with coaxiality tolerance (l), deviating front cylinder (m) and associated parameters: the deviation domain for $T^{\delta}_{d}$ (r) [4]

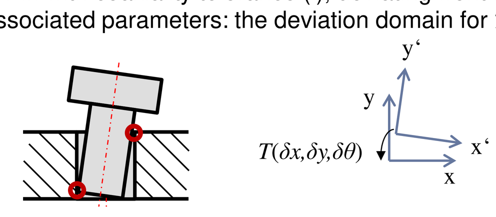

Figure 5: Pin-hole assembly with contact control points (l), represented by $T$ (r)

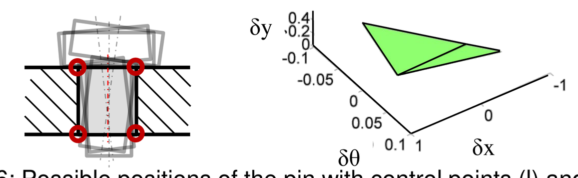
Figure 6: Possible positions of the pin with control points (l) and associated parameters: the clearance domain for $T^{\delta}_{c}$ (r) [4]

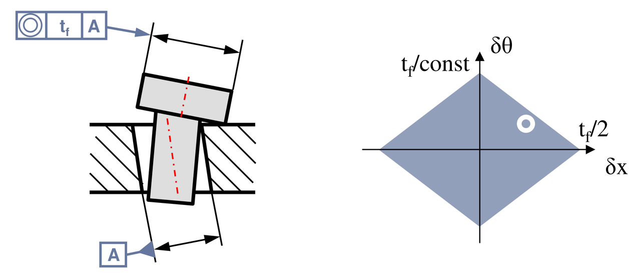

Figure 7: Important here is the relative position of the two axes. The functional requirement is formulated as coaxiality tolerance (l), with associated functional domain (r)

### 5. Sensitivity Analysis in Tolerancing

Commonly one-dimensional sensitivity analysis considering *size deviation* is performed. The methods are restricted to independent input parameters as well as independent model outputs.

**Problems in sensitivity analysis of geometric tolerances**

- The deviation quality is for general deviation domains trivially not clear
- Only the deviation domain size of a geometric tolerance can be changed, not the form (figure 8)

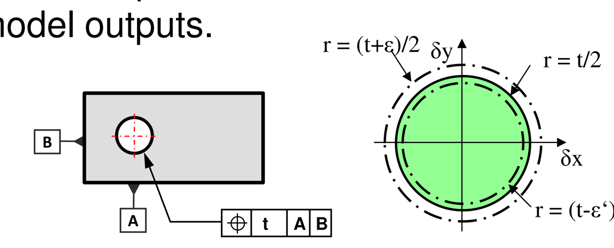

Figure 8: Position tolerance of a drill hole: The tolerance quantity t only restricts the radius of the tolerance zone, the angular position of a deviation is not controlled [5]

## Sensitivity Analysis of Geometric Tolerances

### 6. Deviation quality measure

For the deviation domain $D$

$$D=\{(\delta \underline{x},\delta \underline{\theta}) \in \mathbb{R}^6 \mid \textit{for all } u \in f\text{: } \|T^{\delta}_{d}(\delta \underline{x},\delta \underline{\theta})\cdot \underline{u} - \underline{u}\|_2 \leq t/2\}$$

of a feature $f$ with tolerance value $t$, the quality $\lambda$ of a deviation $(\delta \underline{x},\delta \underline{\theta})$ is defined as

$$\lambda=\{\lambda \geq 0 \mid \max_u \|T^{\delta}_{d}(\delta \underline{x},\delta \underline{\theta})\cdot \underline{u} - \underline{u}\|_2 = \lambda \cdot t/2\}.$$

For the fast calculation of the quality measure, the domain D should be convex. This is ensured, if the function $\|T^{\delta}(\delta \underline{x},\delta \underline{\theta})\cdot \underline{u} - \underline{u}\|_2$ is quasi-convex, what for the here considered tolerances is the case.

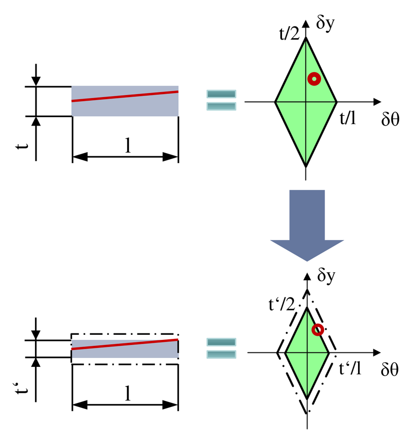

Figure 9: Quality evaluation of a deviating line with quality $\lambda = t'/t$.

### 7. Sensitivity Analysis based on deviation quality measure

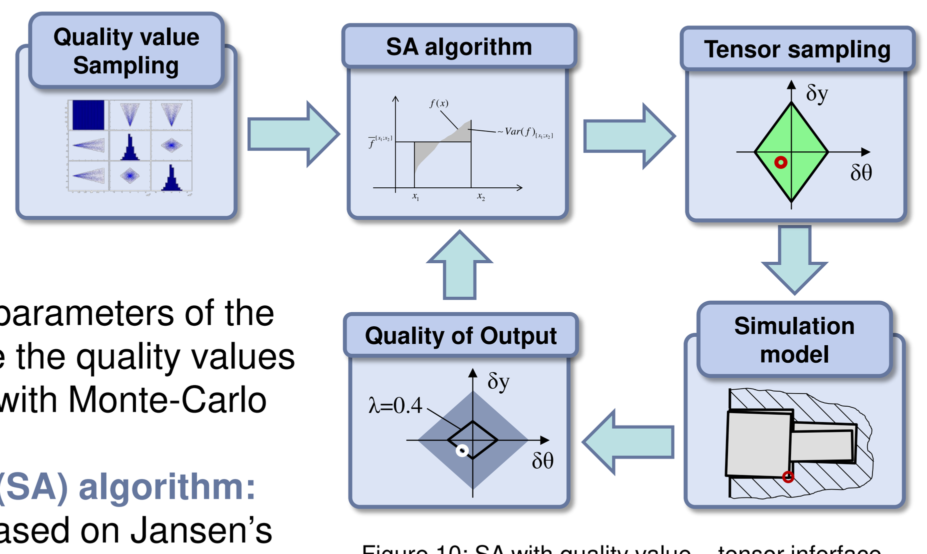

Figure 10: SA with quality value – tensor interface

**Sampling:** The input parameters of the sensitivity analysis are the quality values for deviating features with Monte-Carlo Methods.

**Sensitivity Analysis (SA) algorithm:** variance-based SA, based on Jansen's formula.

**Interface SA – Simulation model:** The simulation model has to translate the quality values from the SA algorithm into matching deviation parameters $(\delta \underline{x},\delta \underline{\theta})$.

### 8. Application Example: Tolerance Scaling of Pin-Hole-Connection

- Two-dimensional simulation-model
- Deviating pin and hole with clearance fit (figure 13)
- Geometric tolerances: Coaxiality (figure 12, figure 4)
- Size tolerances: diameters of pin & hole (figure 12)
- Model output: size of the deviation domain of the pin, approximated by a monte-carlo sampling (200 samples)
- Sensitivity analysis: 5000 samples

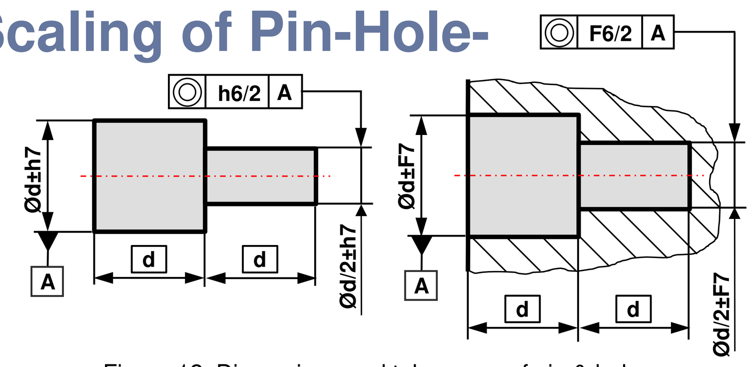

Figure 12: Dimensions and tolerances of pin & hole

**Question:** How scale the sensitivity values with the international fit system?

**International fit system**

The tolerance zone t is defined by a letter and a number (figure 12), dependent on the nominal dimension N (figure 11 and table below).

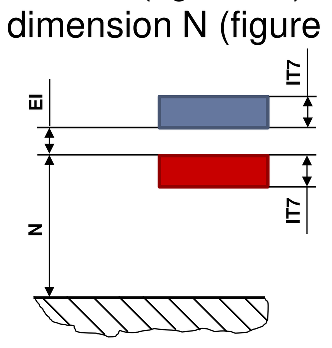

Figure 11: Tolerance zone IT 7 for pin (red) and hole (blue) for the same nominal dimension N

**Scaling of d in mm:**
10, 20, 30, 40, 50
**Tolerancing System:**
Basic shaft system (DIN 7157)
**Clearance fit:**
Pin tolerance: h7
Hole tolerance: F7

| d | d/2 | IT7(d) | IT7(d/2) | IT6/2(d/2) | F: EI(d) | F: EI(d/2) |
|---|---|---|---|---|---|---|
| mm | mm | µm | µm | µm | µm | µm |
| 10 | 5 | 15 | 12 | 4 | 13 | 10 |
| 20 | 10 | 21 | 15 | 4,5 | 20 | 13 |
| 30 | 15 | 21 | 18 | 5,5 | 20 | 16 |
| 40 | 20 | 25 | 21 | 6,5 | 25 | 20 |
| 50 | 25 | 25 | 21 | 6,5 | 25 | 20 |

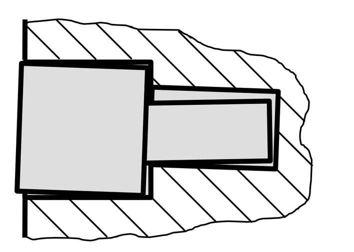
Figure 13: Exemplary clearance fit of pin & hole

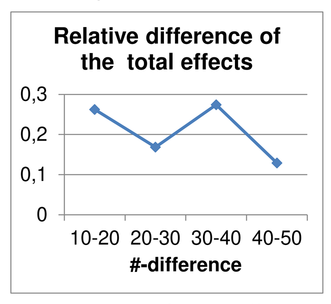

Figure 14: Relative difference of the total effects sum

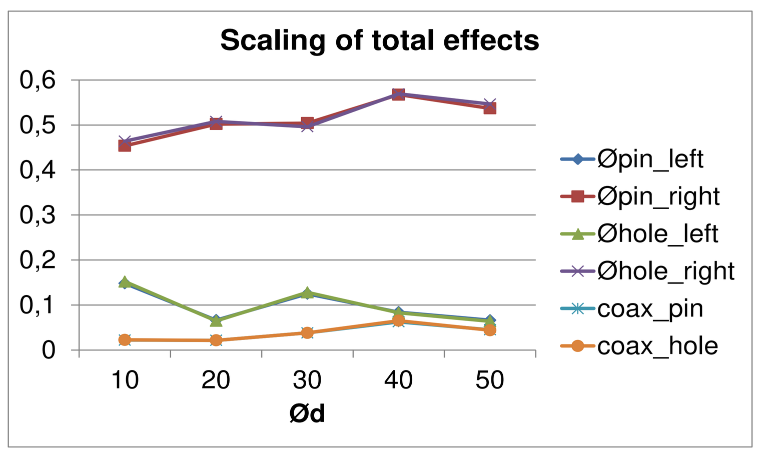

Figure 15: Scaling of the total effects values dependent on Ød

**Interpretation:** The relative difference is for 10-20 and 30-40 very high (figure 14). There, the main contribution of Øpin/hole_right increases significantly (figure 15).

[1] V. Srinivasan, "Role of Statistics in Achieving Global Consistency of Tolerances", *Proceedings of the 6th CIRP International Seminar on Computer-Aided Tolerancing*, University of Twente, Netherlands, 1999.
[2] W. Jorden, W. Schütte, *Form- und Lagetoleranzen – Handbuch für Studium und Praxis*, 7th edition, Carl Hanser Verlag, München, 2012.
[3] J. Shah, "Conceptual development of form features and feature models", *Research in Engineering Design*, 2: 93-108, 1991.
[4] G. Ameta, S. Serge and M. Giordano, "Comparison of Spatial Math Models for Tolerance Analysis: Tolerance-Maps, Deviation Domain, and TTRS", *Jour. of Comp. and Inf. Sc. in Eng.*, **11**(2): 021004, pp. 8, 2011.
[5] Z. Wu, "Sensitive Factor for Position Tolerance", *Research in Engineering Design*, 9: 228-234, 1997.



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