Sequential sensitivity analysis for computer experiments with functional inputs

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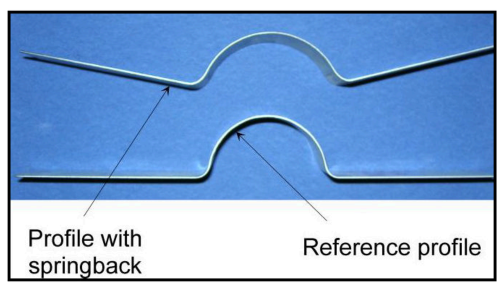
1. Application

Sheet metal forming: Standard industrial process in the forming of car parts

Occuring problems

- Springback: elastic recovery after forming, measured as mean shape deviation
- Tearing
- Wrinkling



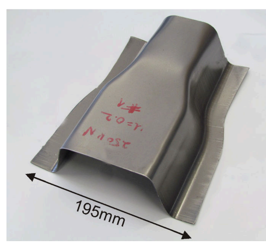
Computer experiment analysis by Finite Element simulations, e.g. by LS-DYNA

2. Extension to functional input

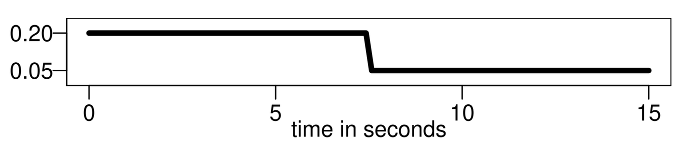
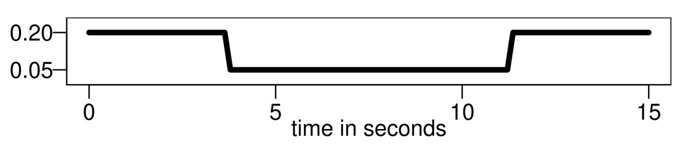
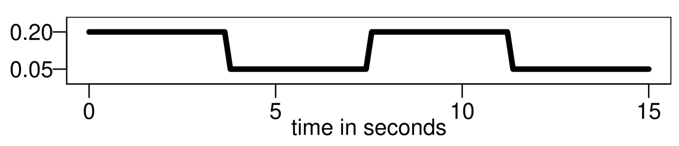
New possibility: **Variation of two inputs** – blankholder force and friction – during the forming process

→ New insights on functional influence on springback, sensitivity analysis

→ Improvements in forming, springback optimization



Test runs with different functional behaviour but equal overall mean friction show potential of functional approach

	functional input: friction setting	**scalar output: springback**
run 1		**14.7**
run 2		**0.8**
run 3		**6.0**

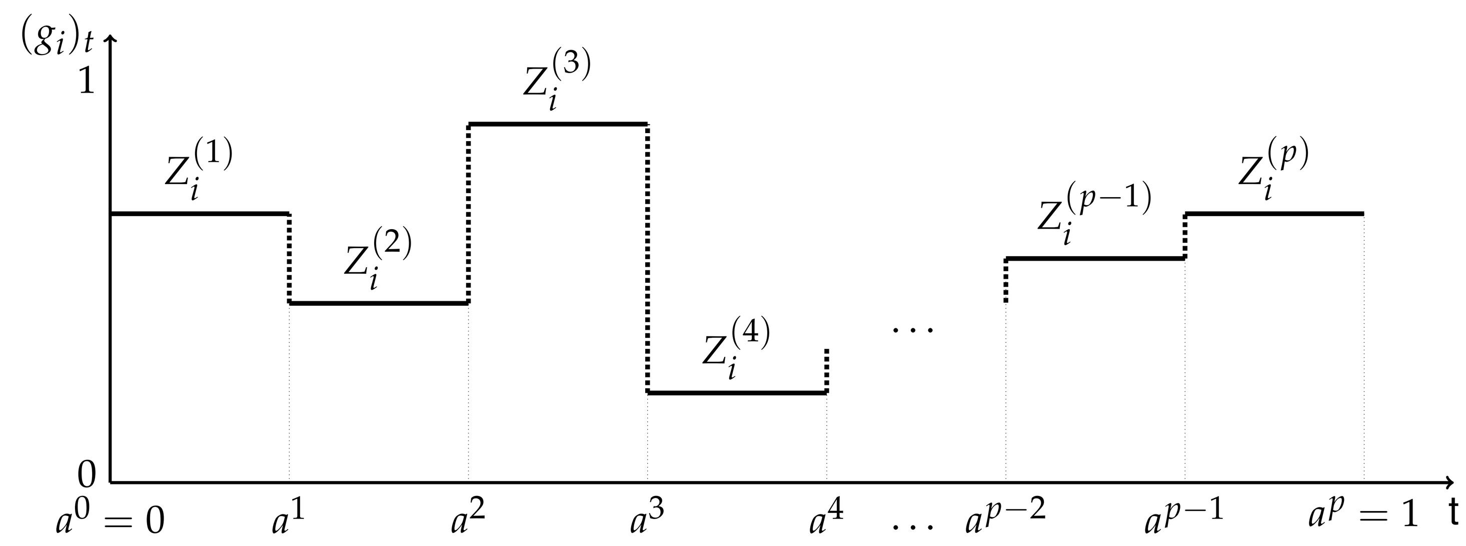
3. Assumptions

- $Y \in \mathbb{R}$ scalar response,
- $g_j : D_j \mapsto [0,1],\ j = 1,\dots,d$ functional input variables
- $D_j = [0,1]$ for each $j = 1,\dots,d$
- Connected by a black box function $\mathcal{F}^d_{[0,1]} \mapsto \mathbb{R},\ Y = f(g_1,\dots,g_d)$
- All input variables can be controlled
- Scalar inputs can be considered as constant functional inputs
- Function evaluations very time consuming

4. Functional representation

Functional input via space of **piecewise constant functions**

$$V_0^{\mathbf{a}_j} = \left\{ Z_j^{(1)} \mathbb{1}_{[0,a_j^1[}(t) + \dots + Z_j^{(p_j)} \mathbb{1}_{[a_j^{p_j-1},1]}(t),\ \text{with } Z_j^{(k)} \in [0,1],\ 1 \le k \le p_j \right\}$$



5. Sensitivity analysis

Input space transformed from functional to scalar

$$Y = f(g_1,\dots,g_d) = \tilde{f}_{a_1,\dots,a_{p_d}}\left(Z_1^{(1)},\dots,Z_1^{(p_1)},\dots,Z_d^{(1)},\dots,Z_d^{(p_d)}\right)$$

Perform sensitivity analysis on input space $Z_1^{(1)}\dots,Z_1^{(p_1)},\dots,Z_d^{(1)},\dots,Z_d^{(p_d)}$

Sensitivity analysis method: Regression coefficients, normalized to be independent of the particular partition

Definition 1. *Consider a set of splitting points* $\mathbf{a}_1,\dots,\mathbf{a}_{p_d}$ *and assume that* $g_j \in V_{\mathbf{a}_j}, j = 1,\dots,d$: $g_j(t) = \sum Z_j^{(k)} \mathbb{1}_{[a_j^{k-1},a_j^k[}(t)$. *Denote by* $\widehat{\beta}_j^k$ *and* $\widehat{\beta}_j^{(kk')}$ *the estimated first-order and second-order regression coefficients, then we define by*

$$\widehat{H}_j^k = \frac{\widehat{\beta}_j^{(k)}}{a_j^k - a_j^{k-1}} \quad \text{and} \quad \widehat{H}_j^{kk'} = \frac{\widehat{\beta}_j^{(kk')}}{(a_j^k - a_j^{k-1})(a_j^{k'} - a_j^{k'-1})},$$

the so-called ***normalized regression index*** *of* $Z_j^{(k)}$ *and the* ***normalized interaction regression index*** *of* $Z_j^{(k)}$ *and* $Z_j^{(k')}$ *resp. for* $j \in \{1,\dots,d\}, 1 \le k < k' \le p_j$.

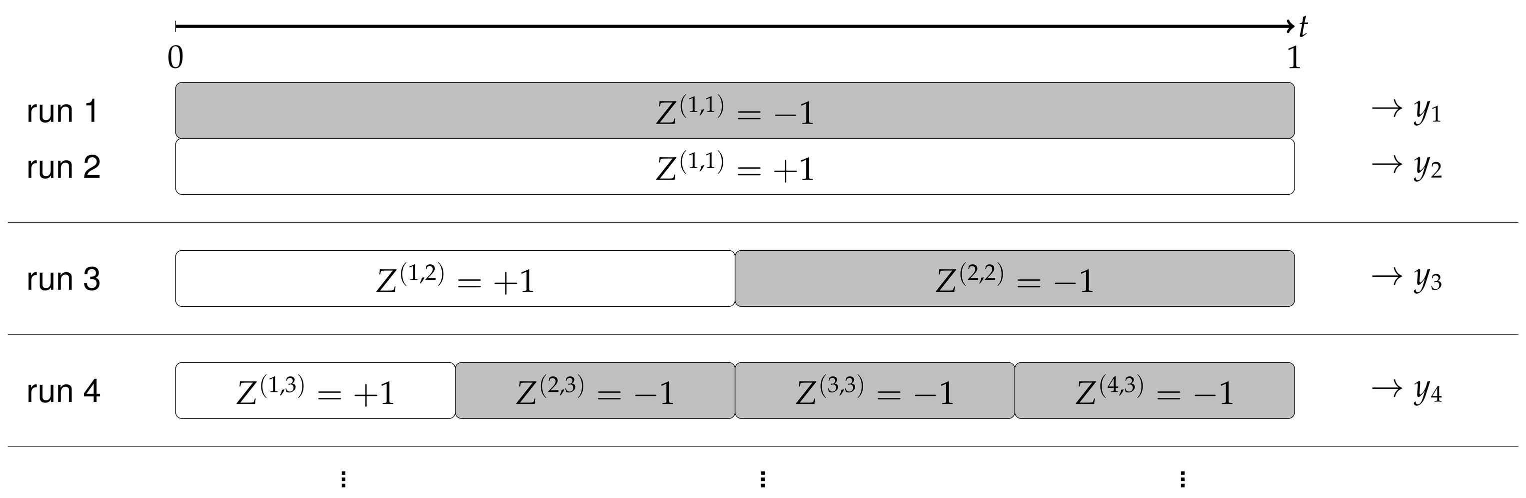
Theoretical result: When f is an integral of an input g weighted by an integrable function $w : [0,1] \mapsto \mathbb{R}$ the indices return the weights

$$f(g) = \alpha + \int_0^1 w(t) g(t)\, dt \quad \Rightarrow \quad \widehat{H}^k = \frac{\int_{a^{k-1}}^{a^k} w(t)\, dt}{a^k - a^{k-1}}, \quad \lim_{a^k - a^{k-1} \to 0} \widehat{H}^k = w(t)$$

6. Design based on sequential bifurcation

Sequential bifurcation: Very economical screening method that saves runs by grouping factors

Implementation in functional design: Start with one interval and then split interesting intervals in sequential steps



$$\widehat{H}^{1,1} = \frac{0.5(y_2 - y_1)}{1},\ \widehat{H}^{1,2} = \frac{0.5(y_3 - y_1)}{0.5},\ \widehat{H}^{2,2} = \frac{0.5(y_2 - y_3)}{0.5},\ \dots$$

Literature

[1] Bettonvil, B. (1995): Factor screening by sequential bifurcation. Communications in Statistics-Simulation and Computation 24 (1), S. 165–185.

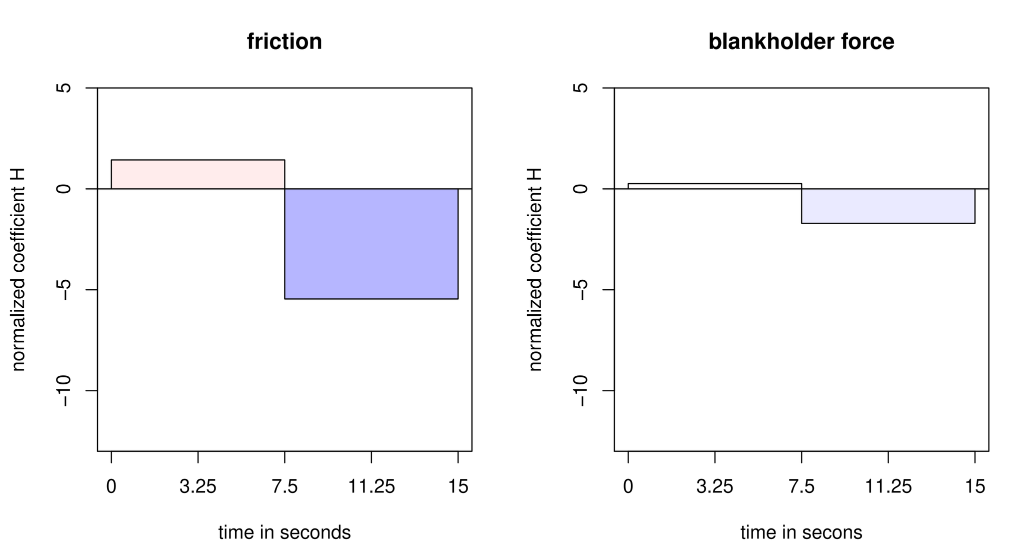
[2] Fruth, J.; Roustant, O.; Kuhnt, S. (2014): Sequential designs for sensitivity analysis of functional inputs in computer experiments. Online on HAL: http://hal.archives-ouvertes.fr/hal-00943509.

[3] Ramsay, J. O.; Silverman, B. W. (2005): Functional data analysis. 2nd ed. New York: Springer.

[4] Saltelli, A.; Chan, K.; Scott, E. M. (Eds.) (2000): Sensitivity analysis. Chichester: Wiley.

Application results: Sensitivity analysis of the functional influence of friction and blankholder force on springback in three sequential steps using 8 evaluation runs each

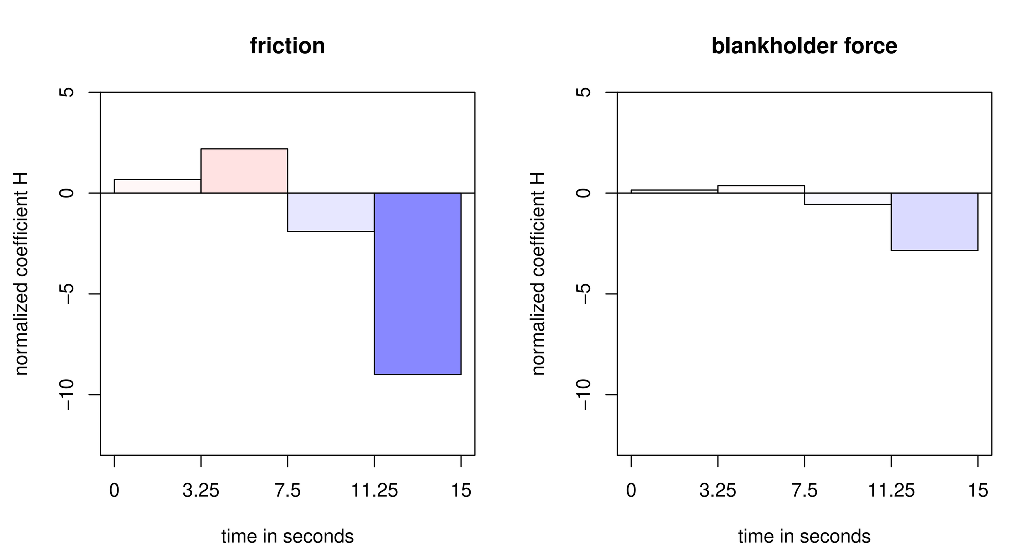
Step 1: Two intervals for both inputs



→ Stronger influence of friction than of blankholder force

→ General: positive influence in first, negative in second half

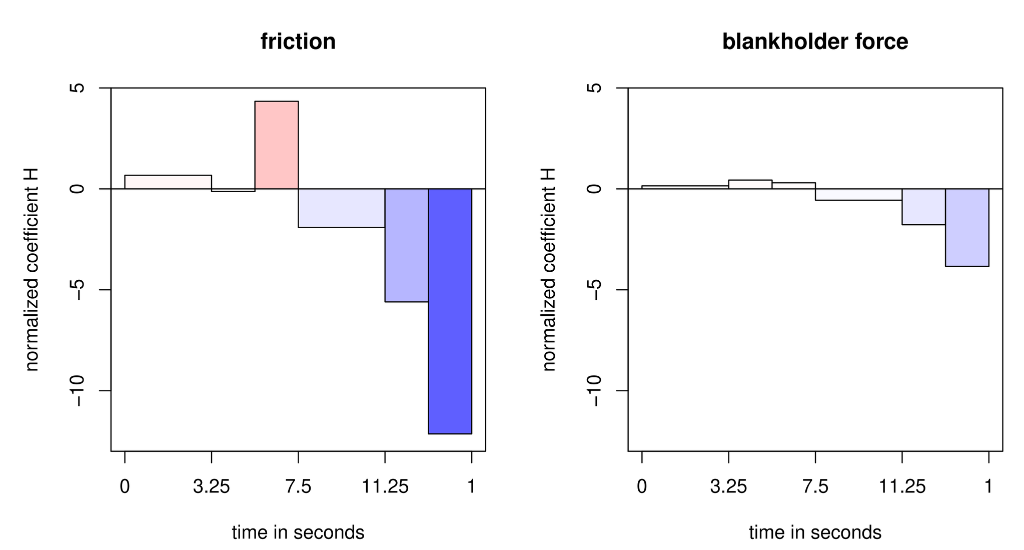
Step 2: Split all intervals for both inputs



→ Last interval has greatest impact

→ First and third intervals have small influence

Step 3: Split each second and fourth interval



→ Overall for both factors: clear change after half of the time

→ Magnitude increases towards the end of both halves