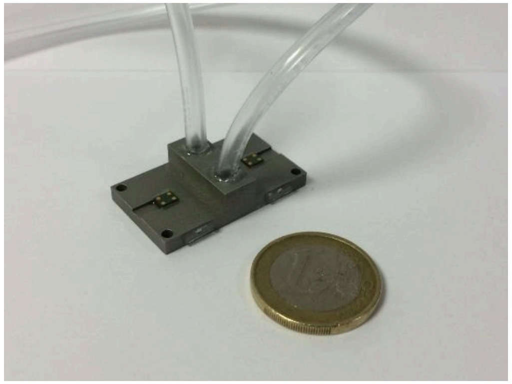
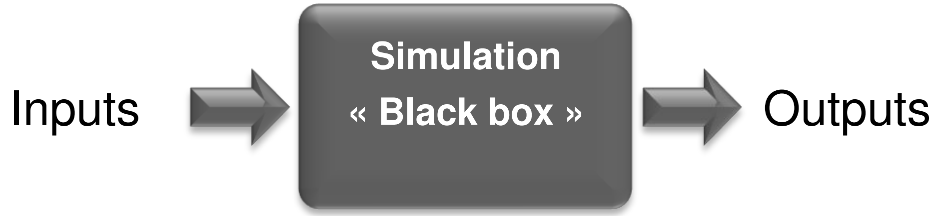


OPTIMIZATION OF A PHOTOACOUSTIC GAS SENSOR USING MULTIFIDELITY RBF METAMODELING

MASCOTNUM 2017 | Durantin Cédric | 22/03/2017

CONTEXT

- **Component behaviour are described by numerical models.**



$$Y: \left| \begin{array}{l} \boldsymbol{\mathcal{X}} \subset \mathbb{R}^d \to \mathbb{R}^m \\ \mathbf{x} \mapsto \mathbf{z} = y(\mathbf{x}) \end{array} \right.$$

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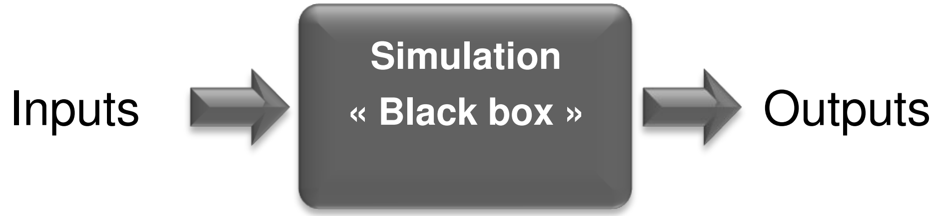

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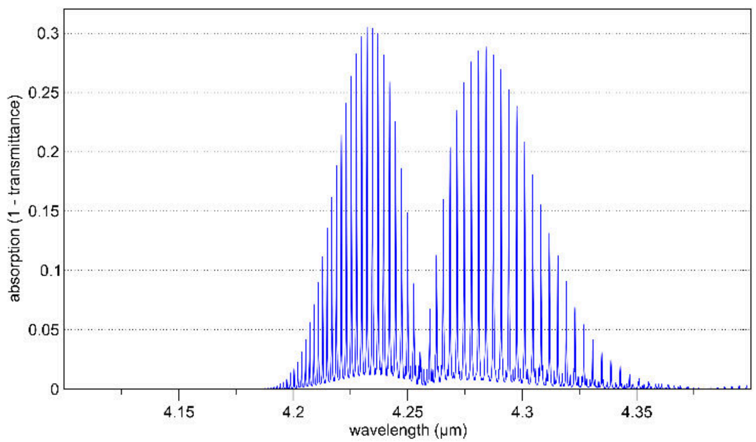
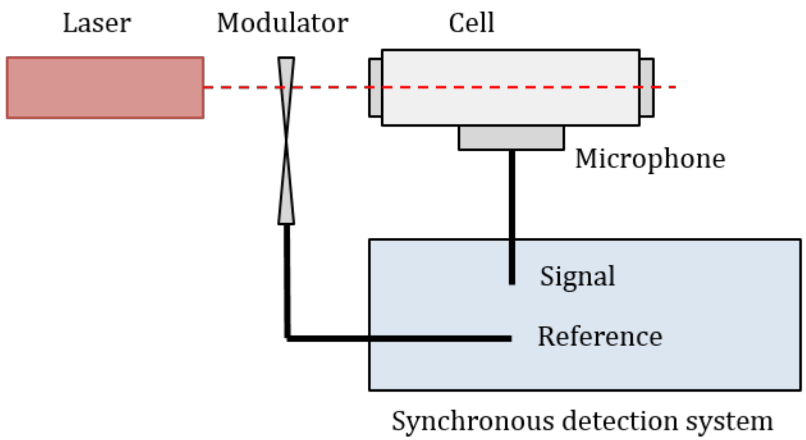
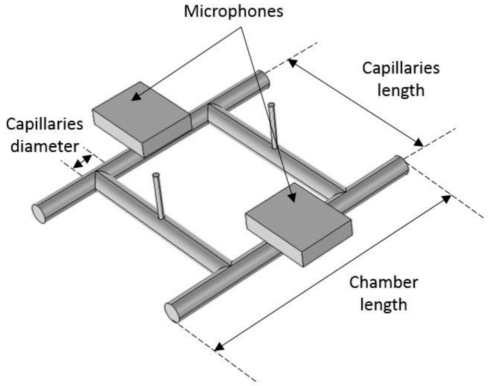
- **Absorption spectroscopy principle is the basis of optical sensors.**



MAIN OPTIMIZATION PROBLEM

- **Gas sensor design: the miniaturized photoacoustic cell**





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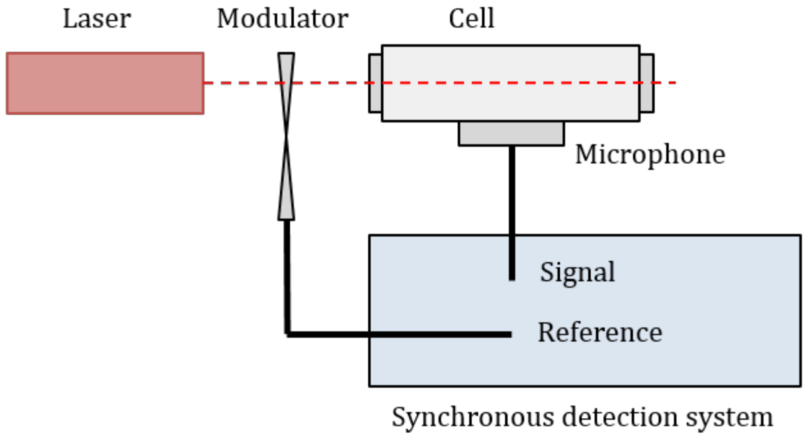
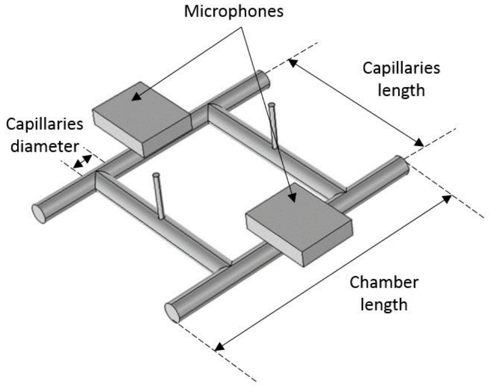
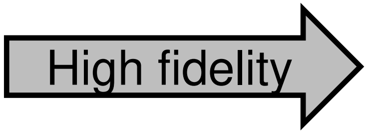
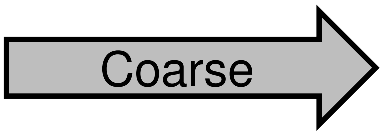




- **Two different simulation model are available:**



Fluids mechanics:
Navier-Stokes equations
75 min



Acoustic :
Helmholtz equation
3 min

PRELIMINARY RESULTS

		High fidelity	Low fidelity
Resonance frequency	RBF	1,07±0,52	0,07±0,01
	Kriging	2,94±0,32	0,12±0,03
Signal	RBF	3,26±1,10	0,04±0,01
	Kriging	3,23±1,08	0,08±0,03

OUTLINE

- Radial Basis Function
- RBF-based multifidelity metamodel
- Benchmark with co-kriging
- RBF optimization algorithm extended to multifidelity

RADIAL BASIS FUNCTION (RBF)

- **Objective function is decomposed on a basis of function :**

$$\hat{Y}(\mathbf{x}) = \sum_{i=1}^{n} \beta_i \varphi\left(\left\|\mathbf{x} - \mathbf{x}_i\right\|\right) + Q(\mathbf{x})$$

RBF	$\varphi\left(\|\mathbf{x} - \mathbf{x}'\|\right)$	$Q(\mathbf{x})$
Cubic	$\left(\|\mathbf{x} - \mathbf{x}'\|\right)^3$	$\alpha_1 p_1(\mathbf{x}) + \alpha_2 p_2(\mathbf{x})$
Gaussian	$\exp\left(-\sum_{i=1}^{d} \gamma_i \|\mathbf{x}_i - \mathbf{x}'_i\|^2\right), \quad \gamma_i > 0$	$\{\emptyset\}$

M. J. D. Powell. Radial basis functions for multivariable interpolation: a review. In J. C. Mason and M. G. Cox, editors, *Algorithms for Approximation*, pages 143-167 Clarendon Press, New York, NY, USA, 1987.

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- Interpolating condition

- $(-1)^{l_0+1} \boldsymbol{\beta}^T \boldsymbol{\Phi} \boldsymbol{\beta} > 0 \quad \forall \boldsymbol{\beta} \in \mathcal{V}_{l_0} \setminus \{0\}$

$$\sum_{i=1}^{n} \beta_i Q(\mathbf{x}_i) = 0 \quad \forall Q \in \Pi_{l_0}$$

$$\boldsymbol{\Phi}_{ij} := \varphi(\| \mathbf{x}_i - \mathbf{x}_j \|) \quad i, j = 1, \ldots, n$$

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$$LOO = \sum_{i=1}^{n} \left(\frac{\beta_i}{\mathbf{\Phi}_{ii}^{-1}} \right)^2$$

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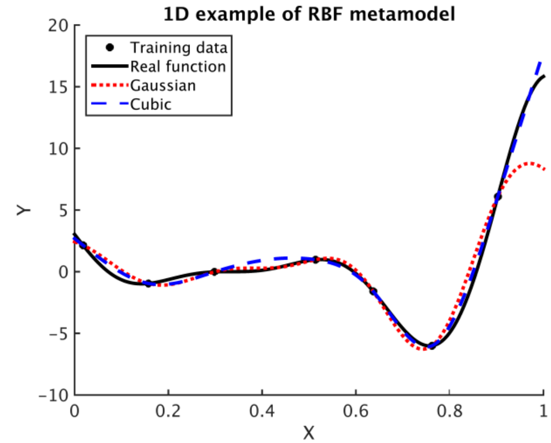

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RBF-BASED MULTIFIDELITY METAMODEL

- **Different fidelity level between models involves different computational time:**
  - 2D and 3D modeling.
  - Multiple physical approaches.
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$$\hat{y}_e(\mathbf{x}) = \rho \hat{y}_c(\mathbf{x}) + \hat{y}_d(\mathbf{x})$$

$\hat{y}_c \rightarrow \{\mathcal{X}_c, \mathbf{z}_c\}$ $\quad$ $\hat{y}_d \rightarrow \{\mathcal{X}_e, \mathbf{z}_e - \rho \mathbf{z}_c\}$

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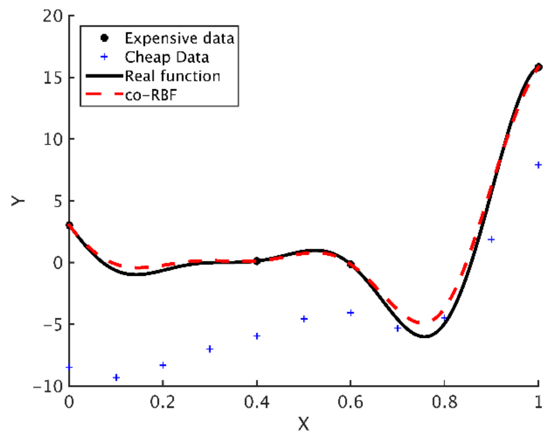

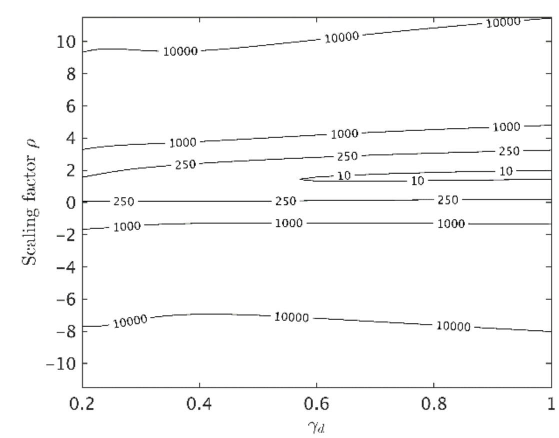

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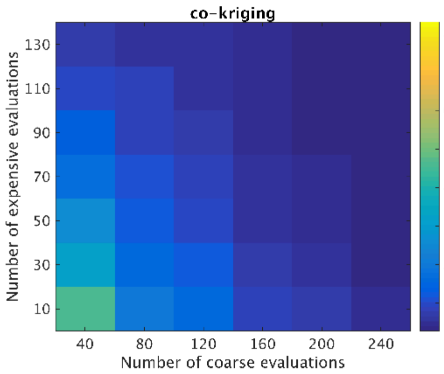
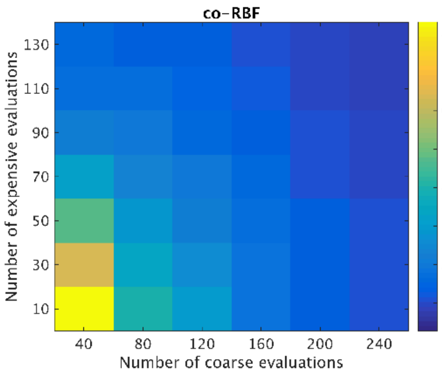
BENCHMARK : ANALYTICAL EXAMPLE

$$y_e = \frac{2\pi T_u \left(H_u - H_l\right)}{\log\left(r / r_w\right)\left[1 + \frac{2LT_u}{\log\left(r / r_w\right) r_w^2 K_w} + T_u / T_l\right]} \qquad y_c = \frac{5\pi T_u \left(H_u - H_l\right)}{\log\left(r / r_w\right)\left[1.5 + \frac{2LT_u}{\log\left(r / r_w\right) r_w^2 K_w} + T_u / T_l\right]}$$

Mean value of the prediction error over 20 random initial designs





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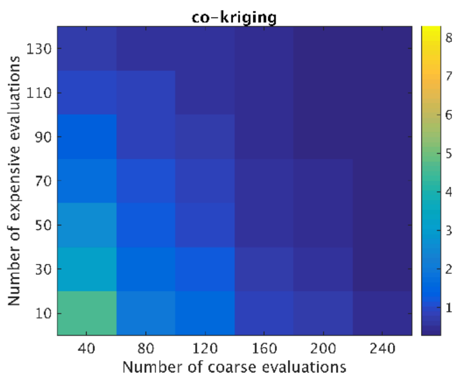

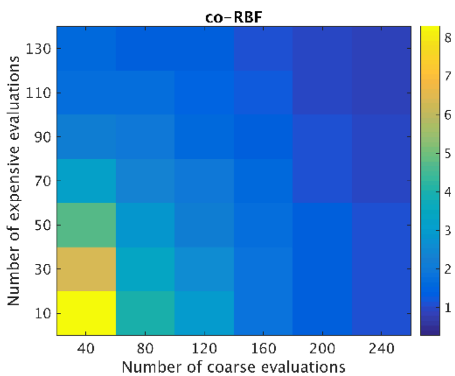

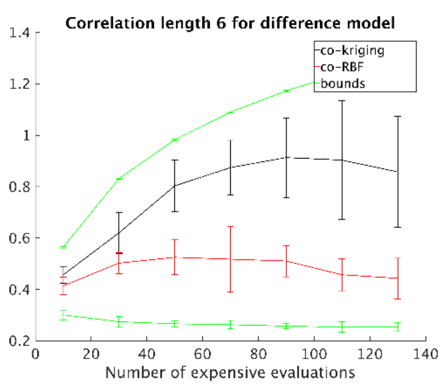

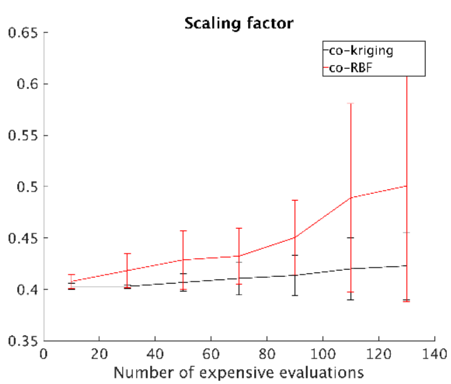

BENCHMARK : PHOTOACOUSTIC SIGNAL

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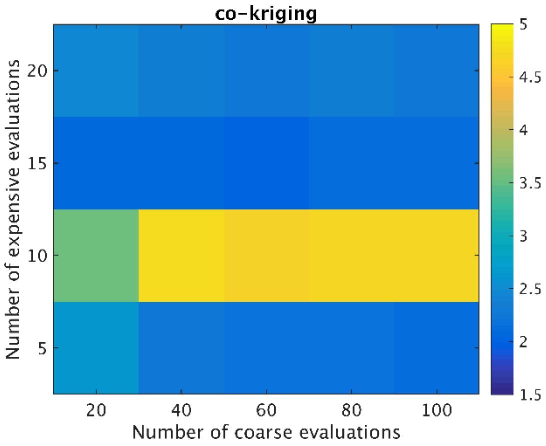

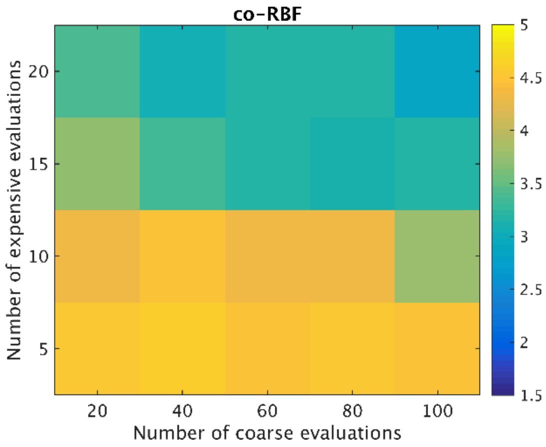

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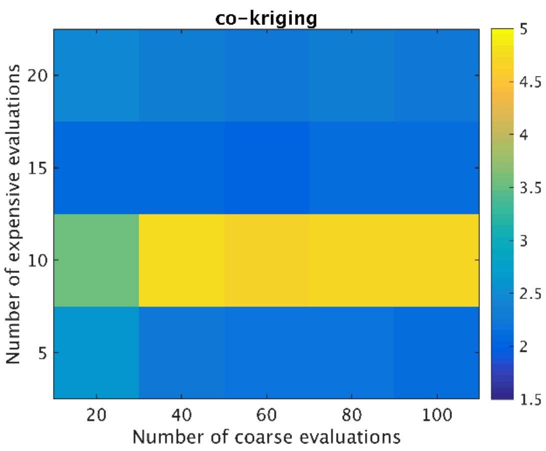

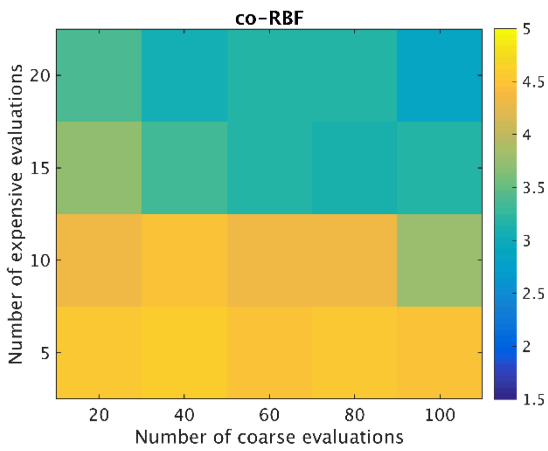

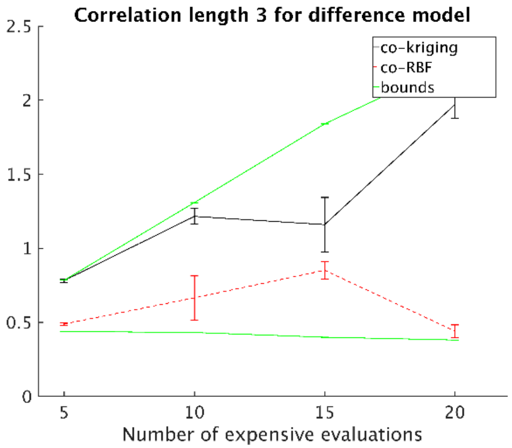

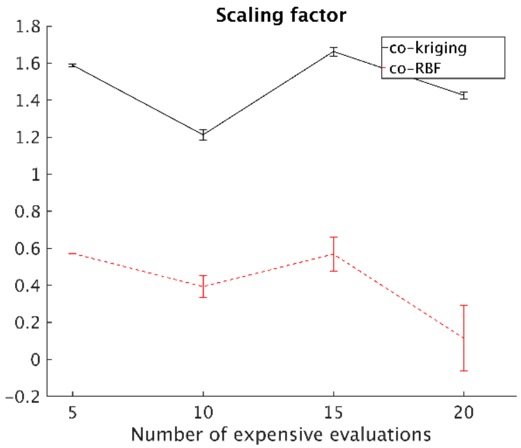

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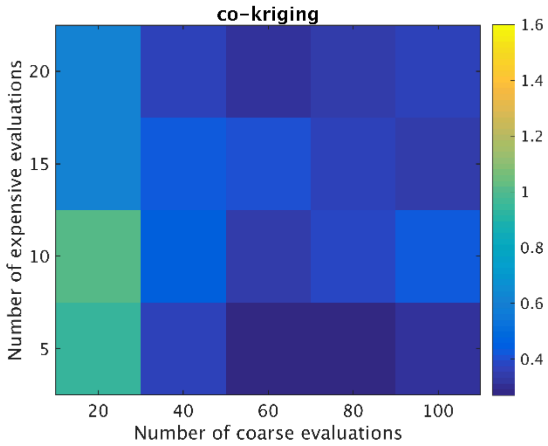

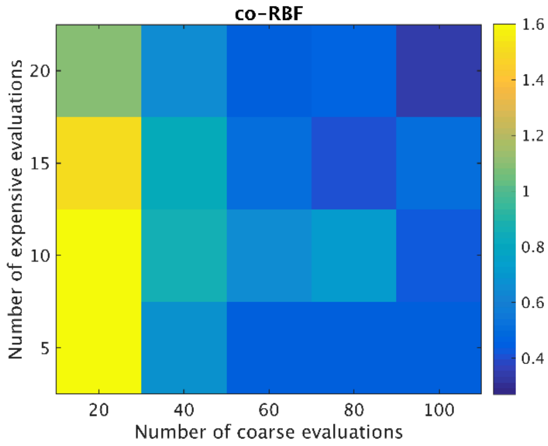

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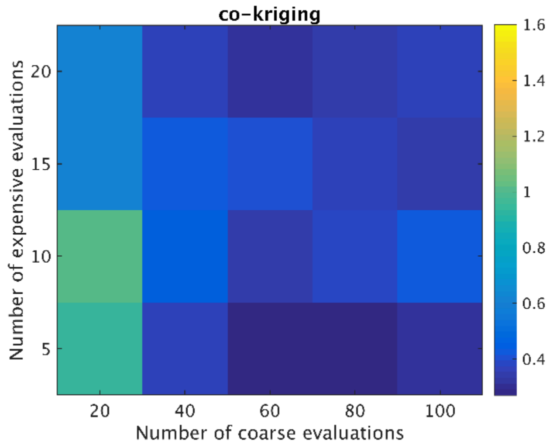

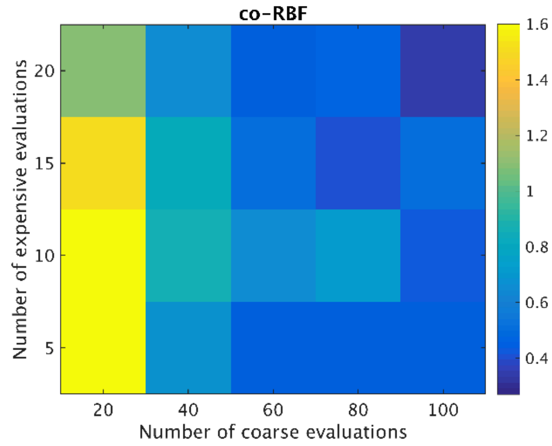

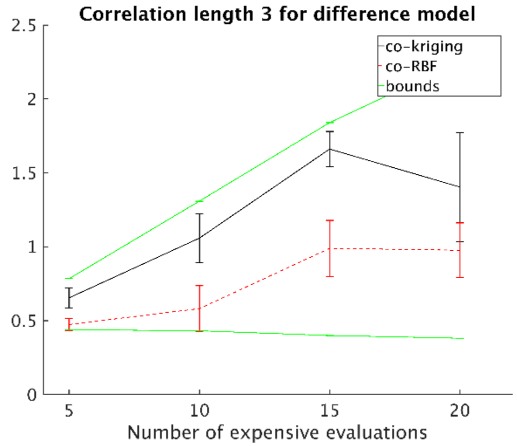

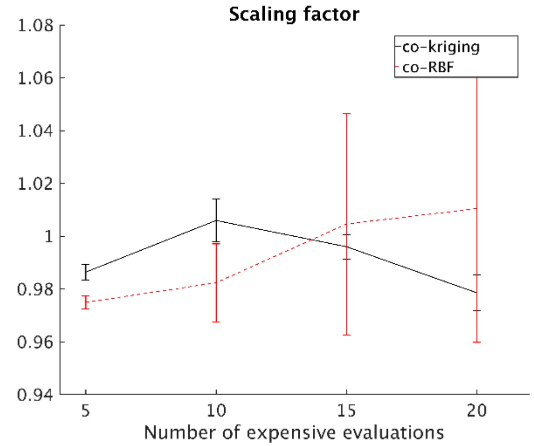

BENCHMARK CONCLUSION

- **Difference in computational time and accuracy:**

Output	Model	5 points	10 points	15 points	20 points
Resonance frequency	RBF	8.0 ± 1.1	3.2 ± 1.6	1.4 ± 0.1	1.1 ± 0.5
	co-RBF	0.44 ± 0.11	0.43 ± 0.11	0.51 ± 0.12	0.33 ± 0.08
	co-kriging	0.33 ± 0.05	0.42 ± 0.08	0.34 ± 0.1	0.37 ± 0.1
Signal	RBF	6.0 ± 2.4	5.0 ± 3	3.9 ± 1.9	3.2 ± 1.2
	co-RBF	4.48 ± 1.6	3.75 ± 1.72	3.20 ± 1.65	2.83 ± 1.37
	co-kriging	2.16 ± 1.07	4.70 ± 5.14	2.13 ± 0.66	2.32 ± 1.03

OPTIMIZATION USING CO-RBF

- **A criterion to refine RBF toward the minimum of the function is defined by Gutmann.**

$$\langle s, s \rangle = (-1)^{l_0+1} \boldsymbol{\beta}^T \boldsymbol{\Phi} \boldsymbol{\beta} \qquad h(t) = \langle s_t, s_t \rangle - \langle s, s \rangle$$

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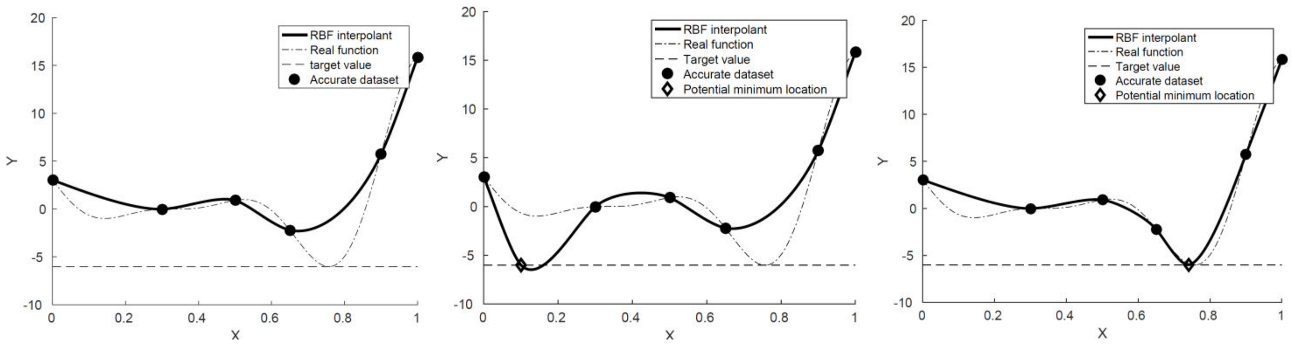

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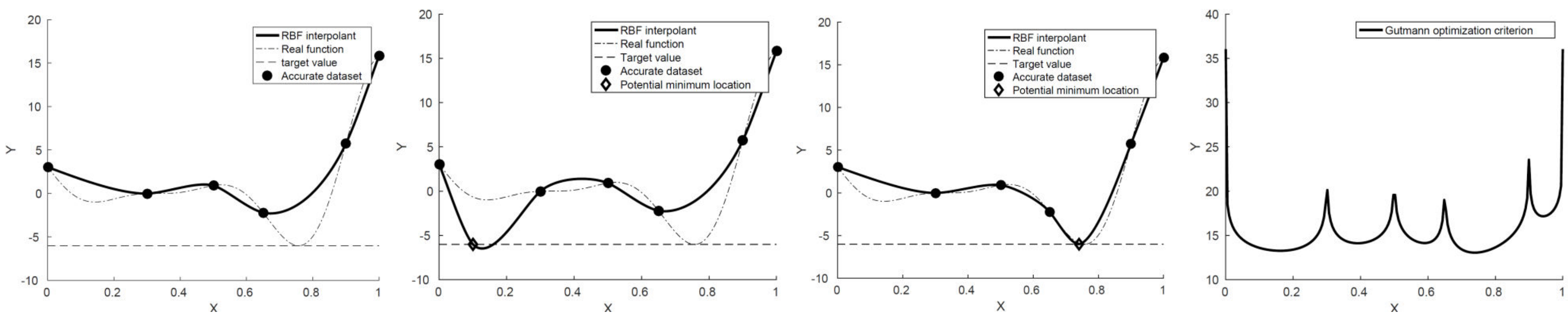

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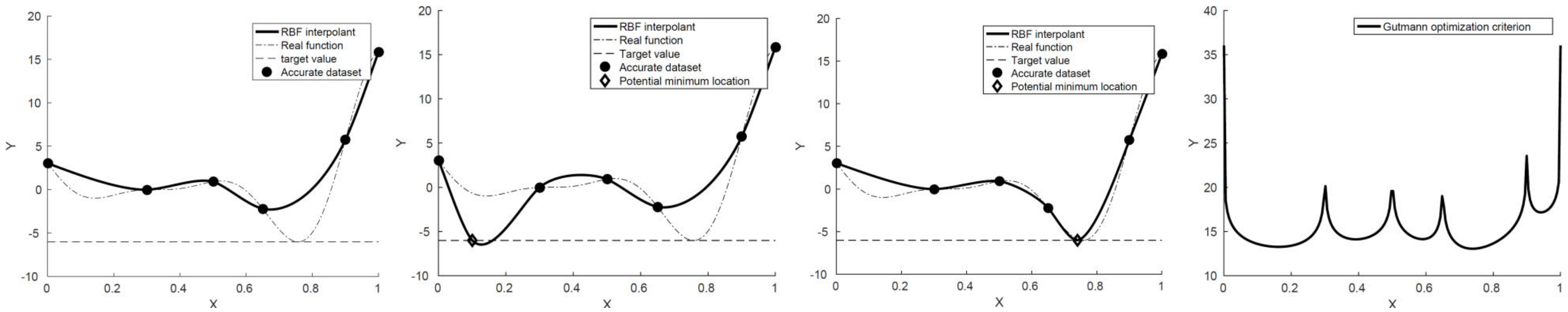

$$h(t) = \frac{(-1)^{l_0+1}}{\left[\hat{y}(t) - f^*\right]} \times \left[\varphi(0) - \begin{pmatrix} \mathbf{u}_t^T & \mathbf{F}_t^T \end{pmatrix} \begin{pmatrix} \boldsymbol{\Phi} & \mathbf{F} \\ \mathbf{F}^T & \mathbf{0} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{u}_t \\ \mathbf{F}_t \end{pmatrix} \right]$$

$$\mathbf{u}_t = \left(\varphi\left(\|t - x_1\|\right), \ldots, \varphi\left(\|t - x_n\|\right) \right)^T, \mathbf{F}_t = \left(p_1(t), \ldots, p_{l_0+1}(t) \right)$$

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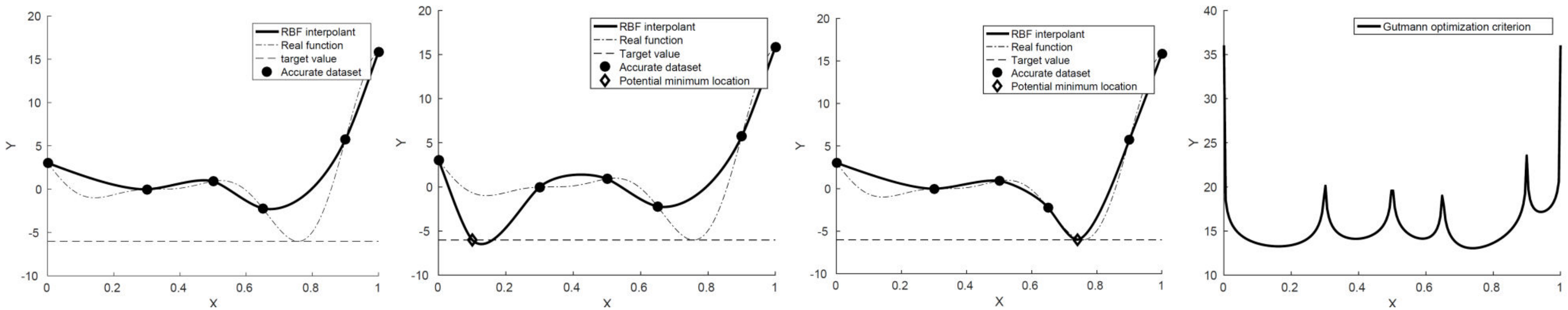


$$h(t) = \frac{(-1)^{l_0+1}}{\left[\hat{y}(t) - f^*\right]} \times \left[\varphi(0) - \begin{pmatrix} \mathbf{u}_t^T & \mathbf{F}_t^T \end{pmatrix} \begin{pmatrix} \boldsymbol{\Phi} & \mathbf{F} \\ \mathbf{F}^T & \mathbf{0} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{u}_t \\ \mathbf{F}_t \end{pmatrix} \right]$$

$$\mathbf{u}_t = \left(\varphi(\|t - x_1\|), \ldots, \varphi(\|t - x_n\|)\right)^T, \mathbf{F}_t = \left(p_1(t), \ldots, p_{l_0+1}(t)\right)$$

- **In the co-RBF framework, only the difference model is refined using the detailed simulation.**

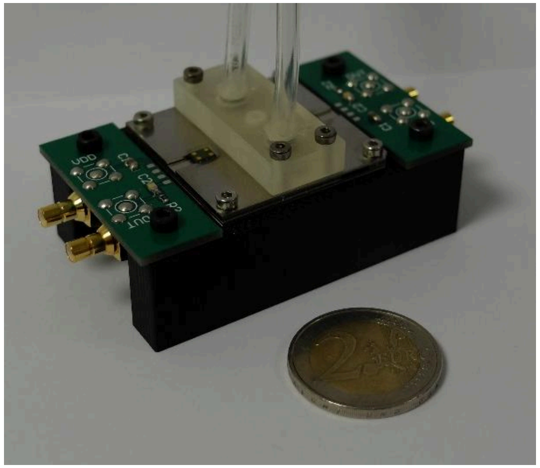
H.-M. Gutmann. "A radial basis function method for global optimization". Journal of Global Optimization, 2001.

OPTIMIZATION OF THE PHOTOACOUSTIC GASSENSOR

- **Signal optimization:**

Initial DOE	Minimum value		Minimum location	
	Co-kriging	Co-RBF	Co-kriging	Co-RBF
1	-1.88	-1.91	$[0.02, 4.18e^{-4}, 0.0179]$	$[0.02, 5.00e^{-4}, 0.0177]$
2	-1.87	-1.90	$[0.02, 5.00e^{-4}, 0.0159]$	$[0.02, 4.97e^{-4}, 0.0173]$
3	-1.91	-1.90	$[0.02, 4.86e^{-4}, 0.0188]$	$[0.02, 4.73e^{-4}, 0.0200]$
4	-1.90	-1.88	$[0.02, 4.54e^{-4}, 0.0200]$	$[0.02, 5.00e^{-4}, 0.0200]$
5	-1.87	-1.91	$[0.02, 4.58e^{-4}, 0.0200]$	$[0.02, 4.46e^{-4}, 0.0200]$

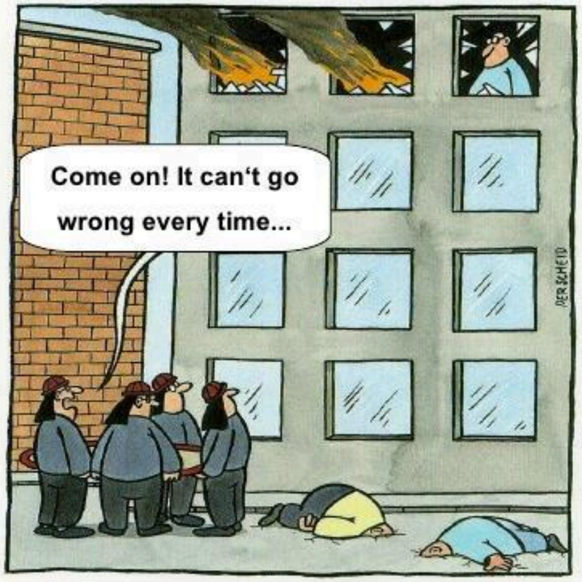
CONCLUSION



- **A new multifidelity metamodel:**
  - RBF-based
  - Optimization method extended to multifidelity framework
  - Validation on a real engineering design problem

- **Perspectives:**
  - Test on a higher dimensional problem
  - Adapt other RBF-based optimization method to multifidelity framework.

Forecasting and optimization are complex



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