Piecewise deterministic processes and the interacting particle system method

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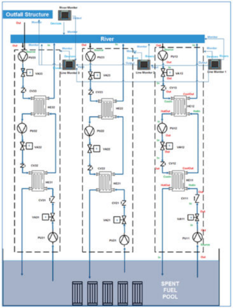
Reliability assessment for complex systems

- Reliability assessment for power generation systems or subsystems
- Simulation tool:
  PyCaTSHOO developped by EDF
- Rare event issue:
  Monte-Carlo is inefficient
- Goal:
  Accelerate reliability assessment

Example :
Spent fuel pool



$\Rightarrow$ variance reduction method needed, possibles solutions:

- Importance sampling (IS)
- Particle filters methods:
  - Interacting particle system (IPS)
  - Sequential Monte-Carlo sampler (SMC)

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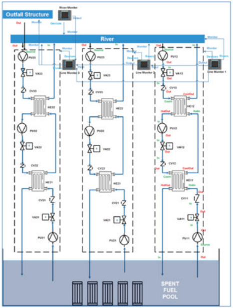

⇒ variance reduction method needed, possibles solutions:

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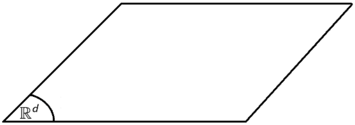
Description of the system

- Dynamical system characterized by physical variables (temperature, pressure, water level)
- System failure = physical variables hit a critical region
- Components are in different statuses (On, Off, broken), which determine the dynamic of physical variables
- Partial failures \ repairs \ control mechanisms

$\rightarrow$ Piecewise deterministic Markovian processes (PDMP) can model the state of the system.

See the books of *Davis 1984*, and *Zhang et al. 2015*



- State of the system at time t:

$$Z_t = (X_t, M_t) \in \mathbb{R}^d \times \mathbb{M}$$

X_t : physical variables ($\simeq$continuous)
M_t: statuses of all components (discrete)

- X_t is piecewise deterministic :

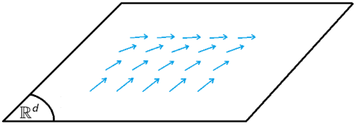
$$\frac{\partial X_t}{\partial t} = F_{M_t}(X_t)$$

- M_t follow a jump process
  Jumps' times :

$$\mathbb{P}_{Z_{S_k}}(T_k \leq t) = 1 - \exp\left[-\lambda_{M_S} t\right]$$

- Jumps' destination :

$$\mathbb{P}\left(Z_S \in B | Z_S^- = z^-\right) = \int_B K_{z^-}(z)\, d\nu_{z^-}(z)$$



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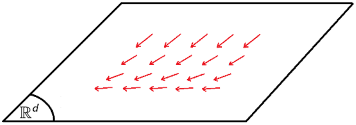
Piecewise Deterministic Markovian processes (PDMP)



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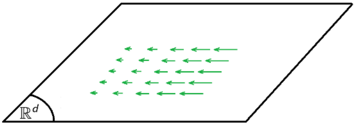

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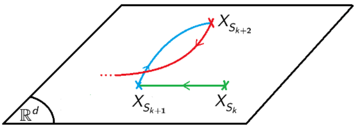

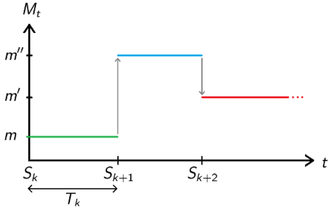

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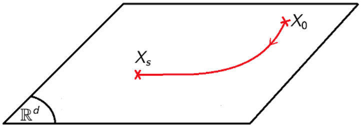
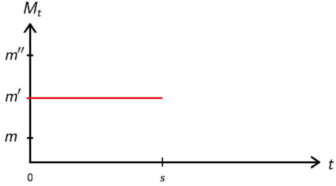
PDMP is degenerate process





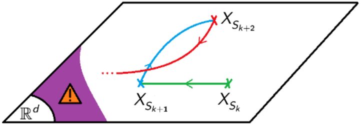
- $\mathbf{Z}_s = (Z_t)_{t\in[0;s)}$: trajectory of the state up to a time s
- For a trajectory $\mathbf{a}_s$ with no spontaneous jumps we have

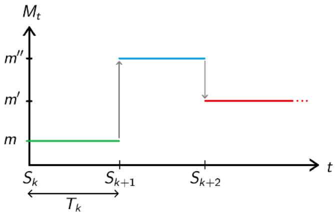
$$\mathbb{P}(\mathbf{Z}_s = \mathbf{a}_s) = \exp\left[-\lambda_{M_0} s\right] > 0$$

- If λ_{M_0} and s are small:

$$\exists\, \mathbf{a}_s,\ \mathbb{P}(\mathbf{Z}_s = \mathbf{a}_s) \simeq 1$$

$\rightarrow$ Slows down the exploration of the space of trajectories





- $\mathcal{A}$: set of the trajectories with system failure before t_f
- system failure = hit the danger zone
- Goal : accelerate the estimation of

$$p = \mathbb{P}_{z_o}\left(\mathbf{Z}_{t_f} \in \mathcal{A}\right)$$

Problematic

- We want to adapt IPS to PDMP
- IPS relies on our capacity to simulate *many different trajectories* on short periods of time

→ Very unlikely for PDMP with low jump intensity.
In particular for reliable systems:

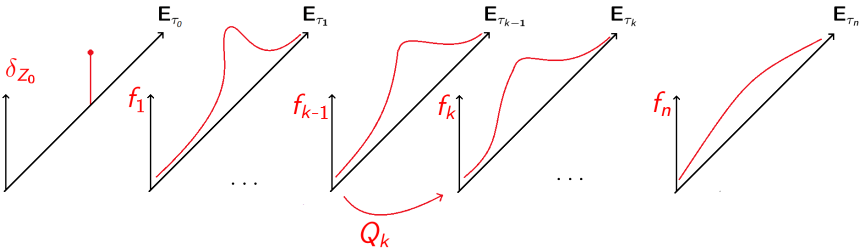
$$\exists \mathbf{a}_s, \ \mathbb{P}(\mathbf{Z}_s = \mathbf{a}_s) \simeq 1$$

How to force the differentiation of the simulated trajectories?

IPS : background

- Discretize time : $0 = \tau_0 < \ \dots \ < \ \tau_k < \ \cdots < \ \tau_n = t_f$
- $\mathbf{E}_{\tau_k}$: the set of trajectories up to time τ_k
- Markov:

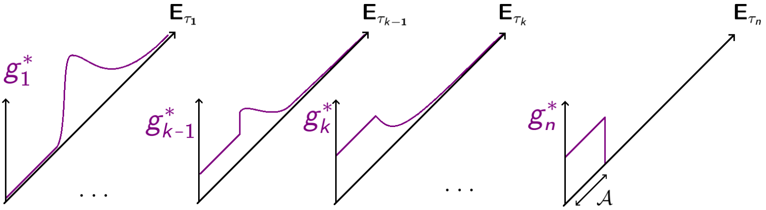
$$f_k(\mathbf{Z}_{\tau_k}) = \prod_{s=1}^{k} Q_s(\mathbf{Z}_{\tau_s}|\mathbf{Z}_{\tau_{s\text{-}1}})$$



- For a distribution g and a function h : $g(h) = \int h \, dg$

- Discretize time : $0 = \tau_0 < \ \dots \ < \ \tau_k < \ \cdots < \ \tau_n = t_f$
- $\mathbf{E}_{\tau_k}$: the set of trajectories up to time τ_k
- The target distributions:

$$g_k^*(\mathbf{Z}_{\tau_k}) \propto \prod_{s=1}^{k} G_s^*(\mathbf{Z}_{\tau_s}) \prod_{s=1}^{k} Q_s(\mathbf{Z}_{\tau_s}|\mathbf{Z}_{\tau_{s\text{-}1}})$$

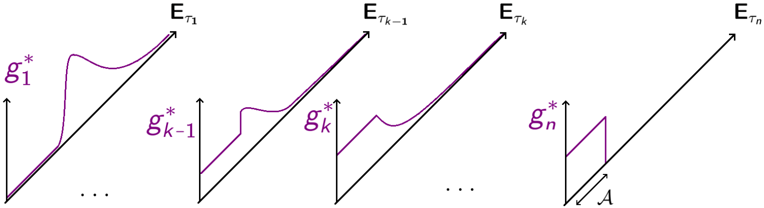


- If $\mathcal{A} \subset \text{supp}\left(\prod_{s=1}^{n\text{-}1} G_s^*\right)$ then:

$$p = \mathbb{E}\left[\mathbb{1}_{\mathcal{A}}(\mathbf{Z}_{\tau_n})\right] = g_{n\text{-}1}^* Q_n\left(\frac{\mathbb{1}_{\mathcal{A}}(\mathbf{Z}_{\tau_n})}{\prod_{s=1}^{n\text{-}1} G_s^*(\mathbf{Z}_{\tau_s})}\right)\prod_{k=1}^{n-1} g_{k\text{-}1}^* Q_k\left(G_k^*\right)$$

- IPS yields some empirical approximations of $g^*_k(h)$ and $g^*_{k\text{-}1}Q_k(h)$:

$$\widehat{g^*_k}(h) = \frac{1}{N}\sum_{i=1}^{N} h(\tilde{\mathbf{Z}}^i_{\tau_k}) \quad \text{and} \quad \widehat{g^*_{k\text{-}1}Q_k}(h) = \frac{1}{N}\sum_{i=1}^{N} h(\mathbf{Z}^i_{\tau_k})$$



- p is estimated by:

$$\hat{p} = \widehat{g^*_{n\text{-}1}Q_n}\left(\frac{\mathbb{1}_{\mathcal{A}}(\mathbf{Z}_{\tau_n})}{\prod_{s=1}^{n-1} G_s(\mathbf{Z}_{\tau_s})}\right)\prod_{k=1}^{n-1}\widehat{g^*_{k\text{-}1}Q_k}(G_k) \quad \underset{n\to\infty}{\longrightarrow} \quad \mathcal{N}\left(p, \frac{\sigma^2_{IPS^*}}{N}\right)$$

Instrumental target distributions

- We want to estimate $p = \mathbb{E}\left[\mathbb{1}_{\mathcal{A}}(\mathbf{Z}_{\tau_n})\right]$. A good choice of target distribution would be :

$$g_k^*(\mathbf{Z}_{\tau_k}) = \frac{\mathbb{E}\left[\mathbb{1}_{\mathcal{A}}(\mathbf{Z}_{\tau_n})|\mathbf{Z}_{\tau_k}\right]}{p} \prod_{s=1}^{k} Q_s(\mathbf{Z}_{\tau_s}|\mathbf{Z}_{\tau_{s\text{-}1}})$$

- But we do not know $\mathbb{E}\left[\mathbb{1}_{\mathcal{A}}(\mathbf{Z}_{\tau_n})|\mathbf{Z}_{\tau_k}\right]$, so we use instead

$$g_k(\mathbf{Z}_{\tau_k}) \propto \frac{U_k(\mathbf{Z}_{\tau_k})}{p} \prod_{s=1}^{k} Q_s(\mathbf{Z}_{\tau_s}|\mathbf{Z}_{\tau_{s\text{-}1}})$$

where $U_s(\mathbf{Z}_{\tau_s})$ is an approximation of $\mathbb{E}\left[\mathbb{1}_{\mathcal{A}}(\mathbf{Z}_{\tau_n})|\mathbf{Z}_{\tau_s}\right]$

- Potential function: $G_s(\mathbf{Z}_{\tau_s}) = \frac{U_s(\mathbf{Z}_{\tau_s})}{U_{s\text{-}1}(\mathbf{Z}_{\tau_{s\text{-}1}})}$, approximates $G_s^*(\mathbf{Z}_{\tau_s})$

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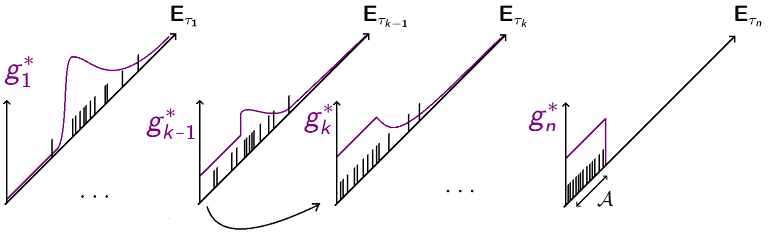

Recycle the trajectories of step k-1, to get the trajectories of step k

- p is estimated by:

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IPS : Sequentially approximate the target distributions

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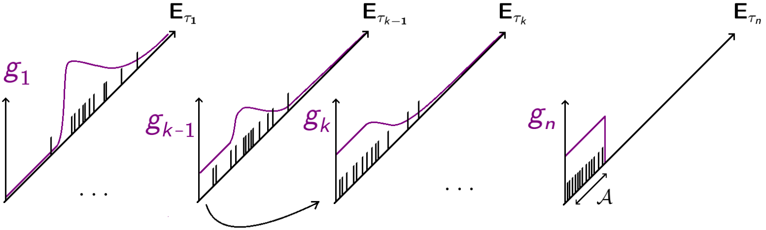

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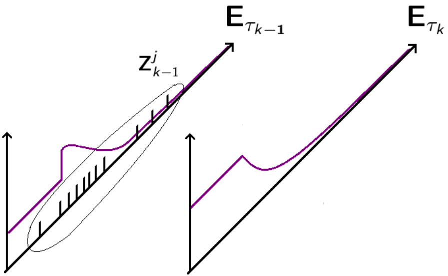
The IPS algorithm

- Start with $k = 1$, and $\mathbf{Z}^j_{\tau_0} = \tilde{\mathbf{Z}}^j_{\tau_0} = z_0$ $(\forall j)$.
- While $k \leq n$ repeat these 2 steps incrementing k each time:
  1. Simulate the trajectories $\mathbf{Z}^1_{\tau_k}, \dots, \mathbf{Z}^N_{\tau_k}$ with
  $$\mathbf{Z}^j_{\tau_k} \sim Q_k(\mathbf{Z}^j_{\tau_k} | \tilde{\mathbf{Z}}^j_{\tau_{k\text{-}\mathbf{1}}})$$
  2. Re-sample the trajectories by drawing with replacement:
  $$\tilde{\mathbf{Z}}^j_{\tau_k} \sim \sum_{j=1}^{N} \frac{G_k(\mathbf{Z}^j_{\tau_k})}{\sum_{i=1}^{N} G_k(\mathbf{Z}^i_{\tau_k})} \delta_{Z^j_{\tau_k}}(.)$$

See *Del Moral et al. 2004*, or *Garnier and Del Moral 2005*

Benefits of the re-sampling at each step



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- Forget low-potential trajectories
- Multiply high-potential trajectories

$\Rightarrow$ We focus computational power on trajectories that are likely to have a greater impact on g_k's estimations

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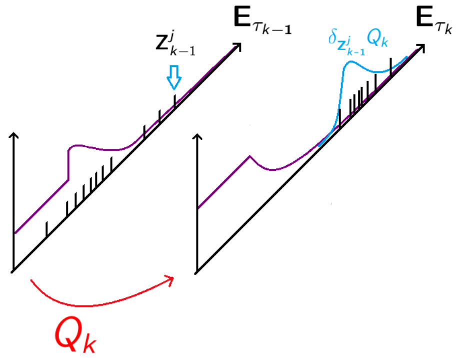

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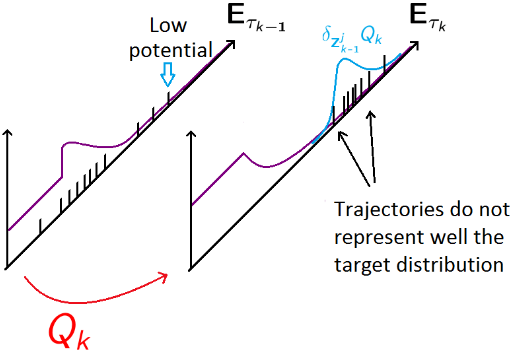

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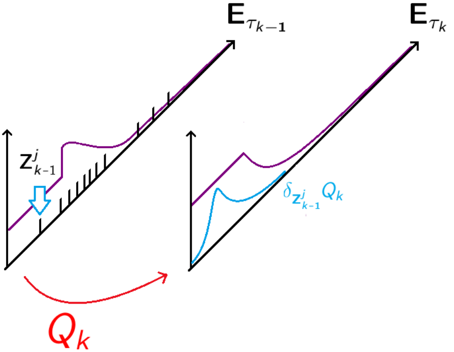

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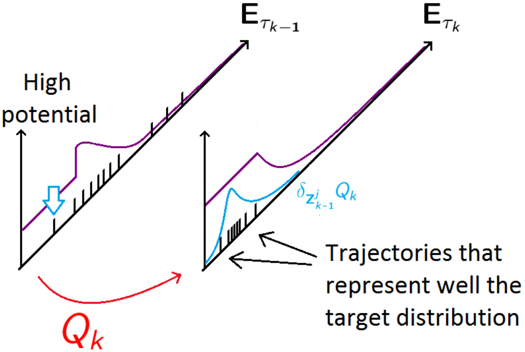

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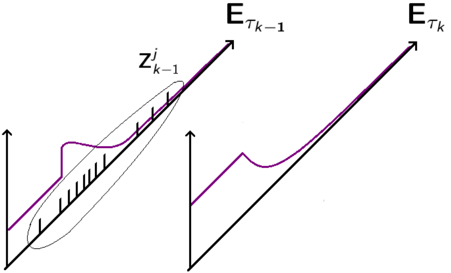

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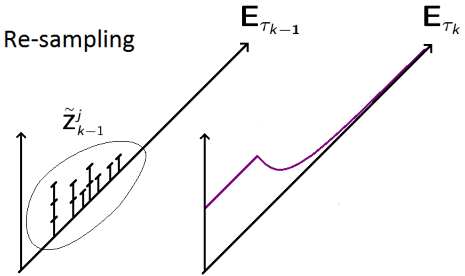

Re-sampling : $\tilde{\mathbf{Z}}^j_{\tau_k} \sim \sum_{j=1}^{N} \frac{G_k(\mathbf{Z}^j_{\tau_k})}{\sum_{i=1}^{N} G_k(\mathbf{Z}^i_{\tau_k})} \delta_{Z^j_{\tau_k}}(.)$

- Forget low-potential trajectories
- Multiply high-potential trajectories

$\Rightarrow$ We focus computational power on trajectories that are likely to have a greater impact on g_k's estimations

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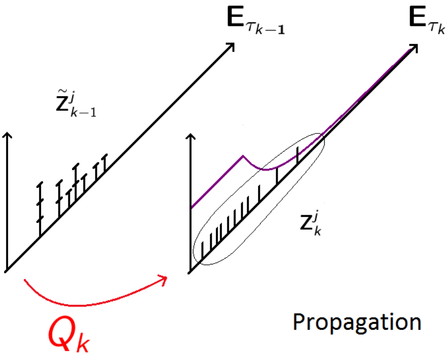

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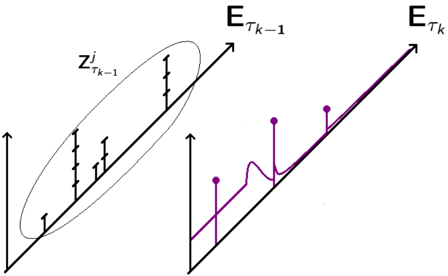
Issue with PDMP



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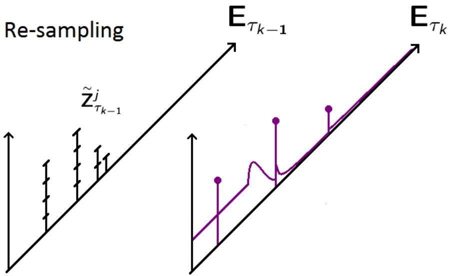

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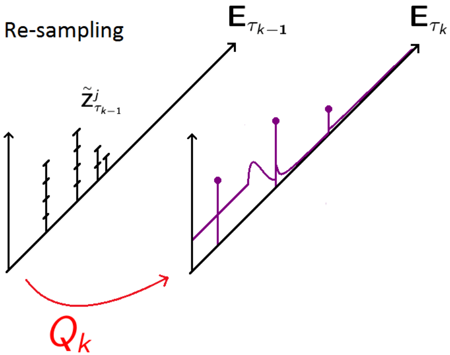

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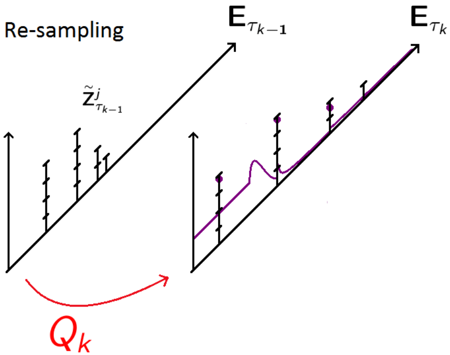

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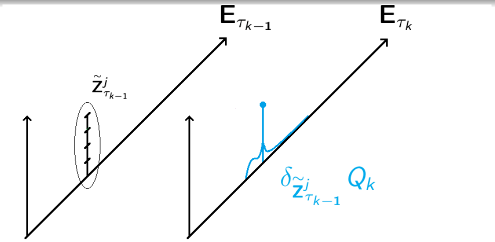
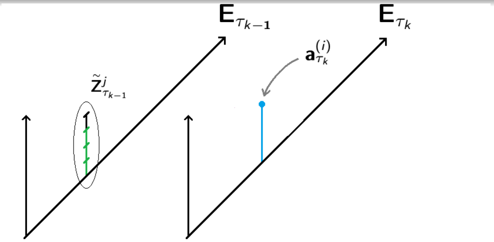
Force trajectories' differentiation



- Focus on the propagation of a i^th cluster with N^i_{k-1} trajectories clumped together: $\tilde{\mathbf{Z}}^{i_j}_{\tau_{k-1}} = \mathbf{a}^{(i)}_{\tau_{k-1}}$
  i_j : index of the j^{th} trajectory in the i^{th} cluster
  $\mathbf{a}^{(i)}_{\tau_k}$: the trajectory continuing $\mathbf{a}^{(i)}_{\tau_{k-1}}$ with the largest probability p_i
- Simulate $\mathbf{Z}^{i_1}_{\tau_k} = \mathbf{a}^{(i)}_{\tau_k}$, and the remaining trajectories avoiding $\mathbf{a}^{(i)}_{\tau_k}$

$$\forall j \geq 2, \qquad \mathbf{Z}^{i_j}_{\tau_k} \sim \mathbf{Z}_{\tau_k} \,\big|\, \mathbf{Z}_{\tau_{k-1}} = \mathbf{a}^{(i)}_{\tau_{k-1}},\ \mathbf{Z}_{\tau_k} \neq \mathbf{a}^{(i)}_{\tau_k}$$

- Reweight: $W^{i_1}_k = p_i \frac{N^{(i)}_{k-1}}{N}$ and $\forall j \geq 2,\ W^{i_j}_k = (1-p_i)\frac{N^{(i)}_{k-1}}{N(N^{(i)}_{k-1}-1)}$



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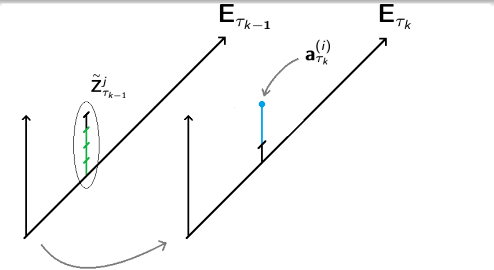

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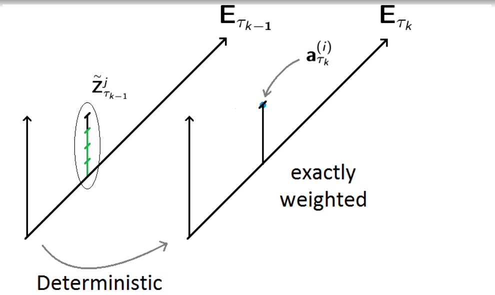

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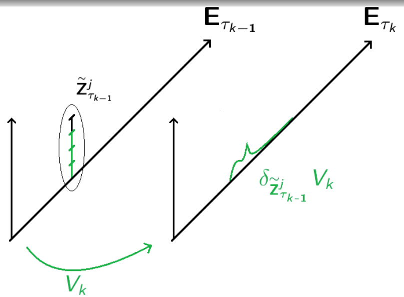

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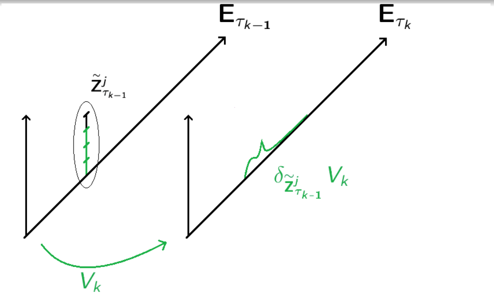

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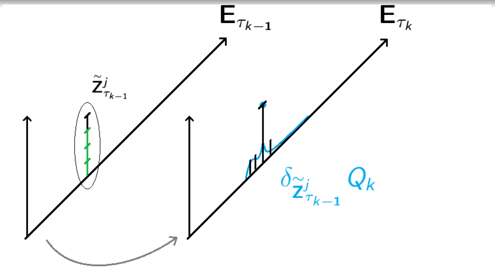

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How to simulate a trajectory $\mathbf{Z}_{\tau_k}$ avoiding $\mathbf{a}_{\tau_k}$

- Denote τ the time at which $\mathbf{Z}_{\tau_k}$ differ from $\mathbf{a}_{\tau_k}$:

$$\forall t < \tau, \quad Z_t = a_t \quad \text{and} \quad Z_\tau \neq \mathbf{a}_\tau$$

- $\tau_{k\text{-}1} < \tau \leq \tau_k \;\Leftrightarrow\; \mathbf{Z}_{\tau_{k\text{-}1}} = \mathbf{a}_{\tau_{k\text{-}1}}$ and $\mathbf{Z}_{\tau_k} \neq \mathbf{a}_{\tau_k}$

- We can compute the cdf of $\tau|\tau_{k\text{-}1} < \tau \leq \tau_k$. (*Labeau 1996*)

$$\mathbb{P}\Big(\tau < t \Big| \tau_{k\text{-}1} < \tau \leq \tau_k\Big) = \frac{1 - \mathbb{P}(\mathbf{Z}_t = \mathbf{a}_t | \mathbf{Z}_{\tau_{k\text{-}1}} = \mathbf{a}_{k\text{-}1})}{1 - \mathbb{P}(\mathbf{Z}_{\tau_k} = \mathbf{a}_{\tau_k} | \mathbf{Z}_{\tau_{k\text{-}1}} = \mathbf{a}_{k\text{-}1})}$$

- To simulate $\mathbf{Z}_{\tau_k}$ knowing $\tau_{k\text{-}1} < \tau \leq \tau_k$:
  1. Simulate $\tau|\tau_{k\text{-}1} < \tau \leq \tau_k$ by inverse method and set $\mathbf{Z}_\tau^- = \mathbf{a}_\tau^-$
  2. Simulate $Z_\tau|\tau$
  3. Simulate $Z_{(\tau,\tau_k]}|Z_\tau$

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  2. Simulate $Z_\tau | \tau$
  3. Simulate $Z_{(\tau, \tau_k]} | Z_\tau$

The IPS method

- Start with $k=1$, and $\mathbf{Z}^j_{\tau_0} = \tilde{\mathbf{Z}}^j_{\tau_0} = z_0$ $(\forall j)$.
- While $k \leq n$ repeat these 2 steps incrementing k each time:
  1. Simulate the trajectories $\mathbf{Z}^1_{\tau_k}, \dots, \mathbf{Z}^N_{\tau_k}$ with
  $$\mathbf{Z}^j_{\tau_k} \sim Q_k\left(\mathbf{Z}^j_{\tau_k} \middle| \tilde{\mathbf{Z}}^j_{\tau_{k\text{-}1}}\right)$$
  2. Re-sample the trajectories :
  $$\tilde{\mathbf{Z}}^j_{\tau_k} \sim \sum_{j=1}^{N} \frac{G_k(\mathbf{Z}^j_{\tau_k})}{\sum_{i=1}^N G_k(\mathbf{Z}^i_{\tau_k})} \delta_{Z^j_{\tau_k}}(.)$$
- Finally p is estimated by :
$$\hat{p} = \widehat{g^*_n Q_n}\left(\frac{\mathbb{1}_{\mathcal{A}}(\mathbf{Z}_{\tau_n})}{\prod_{s=1}^{n-1} G_s(\mathbf{Z}_{\tau_s})}\right) \prod_{k=1}^{n-1} \widehat{g^*_k Q_k}\left(G_k(\mathbf{Z}_{\tau_k})\right)$$

where $\widehat{g^*_{k\text{-}1} Q_k}(B) = \frac{1}{N} \sum_{j=1}^N \delta_{Z^j_{\tau_k}}(B)$

The IPS method with weights

- Start with $k=1$, and $\mathbf{Z}^j_{\tau_0}=\tilde{\mathbf{Z}}^j_{\tau_0}=z_0$, $\tilde{W}^j_0=\frac{1}{N}$ $(\forall j)$.
- While $k\leq n$ repeat these 2 steps incrementing k each time:
  1. Simulate the trajectories $\mathbf{Z}^1_{\tau_k},\dots,\mathbf{Z}^N_{\tau_k}$ with
  $$\mathbf{Z}^j_{\tau_k}\sim Q_k\left(\mathbf{Z}^j_{\tau_k}\middle|\tilde{\mathbf{Z}}^j_{\tau_{k\text{-}1}}\right)\quad\text{and set}\quad W^j_k=\tilde{W}^j_{k\text{-}1}$$
  2. Re-sample the trajectories :
  $$\tilde{\mathbf{Z}}^j_{\tau_k}\sim\sum_{j=1}^N\frac{G_k(\mathbf{Z}^j_{\tau_k})W^j_{k\text{-}1}}{\sum_{i=1}^N G_k(\mathbf{Z}^i_{\tau_k})W^i_{k\text{-}1}}\delta_{Z^j_{\tau_k}}(.)\quad\text{and}\quad\tilde{W}^j_k=\frac{1}{N}$$
- Finally p is estimated by :

$$\hat{p}=\widehat{g^*_nQ_n}\left(\frac{\mathbb{1}_{\mathcal{A}}(\mathbf{Z}_{\tau_n})}{\prod_{s=1}^{n-1}G_s(\mathbf{Z}_{\tau_s})}\right)\prod_{k=1}^{n-1}\widehat{g^*_kQ_k}\left(G_k(\mathbf{Z}_{\tau_k})\right)$$

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- Start with $k=1$, and $\mathbf{Z}^j_{\tau_0} = \tilde{\mathbf{Z}}^j_{\tau_0} = z_0$, $\tilde{W}^j_0 = \frac{1}{N}$ $(\forall j)$.
- While $k \leq n$ repeat these 2 steps incrementing k each time:
  1. Simulate the trajectories $\mathbf{Z}^1_{\tau_k}, \dots, \mathbf{Z}^N_{\tau_k}$ with

$$\mathbf{Z}^j_{\tau_k} \sim Q_k\left(\mathbf{Z}^j_{\tau_k} \middle| \tilde{\mathbf{Z}}^j_{\tau_{k\text{-}1}}\right) \text{ and set } W^j_k = \tilde{W}^j_{k\text{-}1}$$

  2. Re-sample the trajectories :

$$\tilde{\mathbf{Z}}^j_{\tau_k} \sim \sum_{j=1}^{N} \frac{G_k(\mathbf{Z}^j_{\tau_k}) W^j_{k\text{-}1}}{\sum_{i=1}^N G_k(\mathbf{Z}^i_{\tau_k}) W^i_{k\text{-}1}} \delta_{Z^j_{\tau_k}}(.) \text{ and } \tilde{W}^j_k = \frac{1}{N}$$

- Finally p is estimated by :

$$\hat{p} = \widehat{g^*_n Q_n}\left(\frac{\mathbb{1}_{\mathcal{A}}(\mathbf{Z}_{\tau_n})}{\prod_{s=1}^{n-1} G_s(\mathbf{Z}_{\tau_s})}\right) \prod_{k=1}^{n-1} \widehat{g^*_k Q_k}\left(G_k(\mathbf{Z}_{\tau_k})\right)$$

where $\widehat{g^*_{k\text{-}1} Q_k}(B) = \sum_{j=1}^N W^j_k \, \delta_{Z^j_{\tau_k}}(B)$

Include Memorization method in the IPS method

- Start with $k=1$, and $\mathbf{Z}^j_{\tau_0} = \tilde{\mathbf{Z}}^j_{\tau_0} = z_0$, $\tilde{W}^j_0 = \frac{1}{N}$ $(\forall j)$.
- While $k \leq n$ repeat these 2 steps incrementing k each time:

  1. For each cluster, if $N^{(i)}_{k\text{-}1} = 1$ then $\mathbf{Z}^{i_1}_{\tau_k} \sim Q_k$, else:

     Set $\mathbf{Z}^{i_1}_{\tau_k} = \mathbf{a}^i_{\tau_k}$ and $W^{i_1}_k = p_i \frac{N^{(i)}_{k\text{-}1}}{N}$,
     and $\forall j \leq 2$ simulate the trajectories $\mathbf{Z}^{i_j}_{\tau_k}$ with

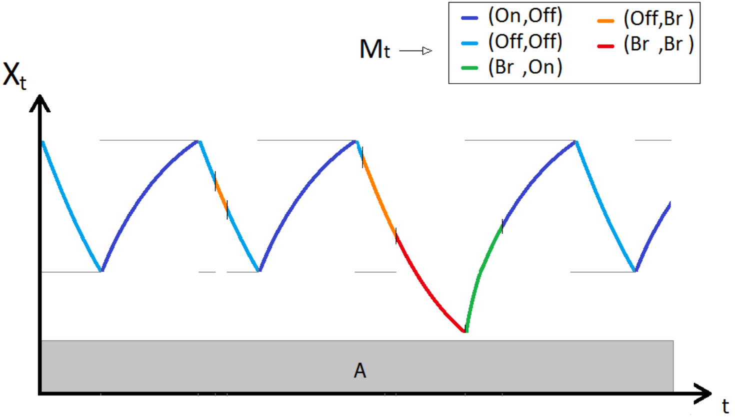
     $$\mathbf{Z}^{i_j}_{\tau_k} \sim V_k\left(\mathbf{Z}^{i_j}_{\tau_k} | \tilde{\mathbf{z}}^{(i)}_{\tau_{k\text{-}1}}\right) \quad \text{and set} \quad W^{i_j}_k = (1-p_i)\frac{N^{(i)}_{k\text{-}1}}{N(N^{(i)}_{k\text{-}1}-1)}$$

  2. Re-sample the trajectories :

     $$\tilde{\mathbf{Z}}^j_{\tau_k} \sim \sum_{j=1}^{N} \frac{G_k(\mathbf{Z}^j_{\tau_k})W^j_{k\text{-}1}}{\sum_{i=1}^N G_k(\mathbf{Z}^i_{\tau_k})W^i_{k\text{-}1}} \delta_{Z^j_{\tau_k}}(.), \quad \text{and} \quad \tilde{W}^j_k = \frac{1}{N}$$

- Finally p is still estimated using $\widehat{g^*_{k\text{-}1}Q_k}(B) = \sum_{j=1}^N W^j_k \delta_{Z^j_{\tau_k}}(B)$

- System : Room heated by 2 components, $p \simeq 2.73 \times 10^{-5}$



- System : Room heated by 2 components, $p \simeq 2.73 \times 10^{-5}$

		IPS	IPS + Memorization
$n = 5$	$\hat{p}$	2.86×10^{-5}	2.70×10^{-5}
	$\hat{\sigma}^2$	1.78×10^{-9}	1.37×10^{-10}
$n = 10$	$\hat{p}$	2.85×10^{-5}	2.64×10^{-5}
	$\hat{\sigma}^2$	1.08×10^{-9}	$1,07 \times 10^{-10}$
$n = 20$	$\hat{p}$	2.41×10^{-5}	2.81×10^{-5}
	$\hat{\sigma}^2$	5.86×10^{-10}	1.20×10^{-10}

Table: Mean results, obtain from 100 runs of the methods with $N = 10^4$ residual re-sampling was used

Conclusion and perspective

- IPS+ Memorization is unbiased and satisfies a CLT
- Include the memorization method in the propagation step of the IPS yields smaller variances
- IPS + Memorization can be generalized to PDMP with boundaries

- Is the generalization to other particle filter methods possible? SMC?
  - The memorization method unbalances the weights
    $\rightarrow$ Re-sampling would be triggered at each steps

Thank you for your attention

The SMC algorithm

Start with $k = 1$, and $\mathbf{Z}^j_{\tau_0} = \tilde{\mathbf{Z}}^j_{\tau_0} = z_0$, $\tilde{W}^j_k = \frac{1}{N}$ $(\forall j)$.

While $k \leq n$ repeat this steps incrementing k each time:

1. Simulate the trajectories $\mathbf{Z}^1_{\tau_k}, \dots, \mathbf{Z}^N_{\tau_k}$ with

$$\mathbf{Z}^j_{\tau_k} \sim Q_k(\mathbf{Z}^j_{\tau_k} | \tilde{\mathbf{Z}}^j_{\tau_{k-1}})$$

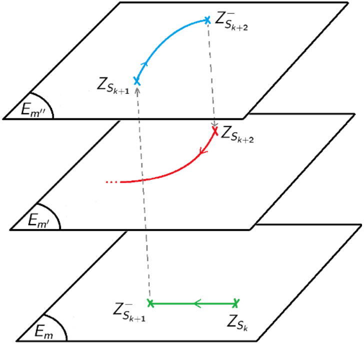
2. If $ESS < 0.2N$: Re-sampling step

$$\tilde{\mathbf{Z}}^j_{\tau_k} \sim \sum_{j=1}^{N} \frac{G_k(\mathbf{Z}^j_{\tau_k}) \tilde{W}^j_{k-1}}{\sum_{i=1}^N G_k(\mathbf{Z}^i_{\tau_k}, \tilde{\mathbf{Z}}^i_{\tau_{k-1}}) \tilde{W}^i_{k-1}} \delta_{Z^j_{\tau_k}}(.), \quad \text{and} \quad \tilde{W}^j_k = \frac{1}{N}$$

else : Importance sampling step

$$\tilde{\mathbf{Z}}^j_{\tau_k} = \mathbf{Z}^j_{\tau_k}, \quad \text{and} \quad \tilde{W}^j_k = \frac{G_k(\mathbf{Z}^j_{\tau_k}) \tilde{W}^j_{k-1}}{\sum_{i=1}^N G_k(\mathbf{Z}^i_{\tau_k}, \tilde{\mathbf{Z}}^i_{\tau_{k-1}}) \tilde{W}^i_{k-1}}$$

PDMP (the case without boundaries)



- State space :

$$E = \bigcup_{m \in \mathbb{M}} E_m = \bigcup_{m \in \mathbb{M}} \left\{ (x, m), x \in \mathbb{R}^d \right\}$$

- Between jumps :

$$Z_{S+t} = \Phi_{Z_S}(t)$$

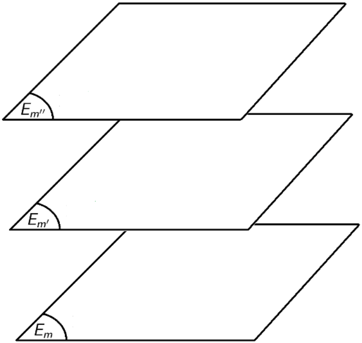
- Jumps' times :

$$\mathbb{P}_{Z_S}(T \leq t) = 1 - \exp\left[-\lambda_{M_S} t \right]$$

- Jumps' destination :

$$\mathbb{P}\left(Z_S \in B | Z_S^- = z^- \right) = \int_B K_{z^-}(z)\, d\nu_{z^-}(z)$$

PDMP with boundaries



- In a mode M the position X is restricted to an open $\Omega_M \subset \mathbb{R}^d$
- The state space becomes:

$$E = \bigcup_{m \in \mathbb{M}} E_m = \bigcup_{m \in \mathbb{M}} \Big\{(x,m), x \in \Omega_m\Big\}$$

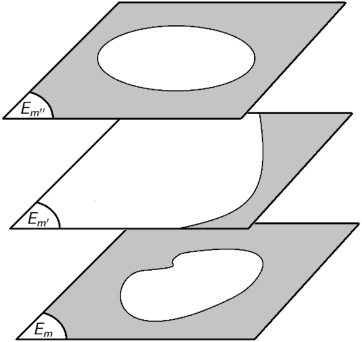
- Jumps' times :

$$\mathbb{P}_z(T \leq t) = \begin{cases} 1 - \exp\left[-\lambda_m t\right] & \text{if } t < t_z^* \\ 1 & \text{if } t \geq t_z^* \end{cases}$$

t_z^* : Boundary hitting time starting from z

- Boundaries model automatic control mechanisms

PDMP with boundaries



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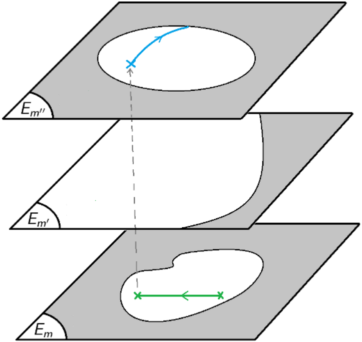

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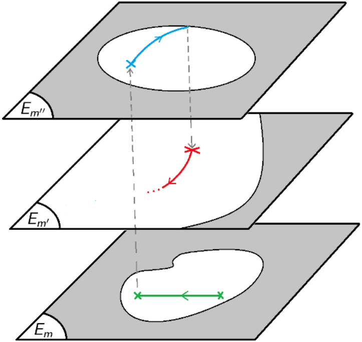

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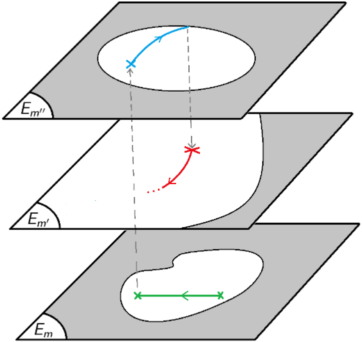
Model power generation systems



- Component statuses :

$$\mathbb{M} = \{On, Off, Failed\}^{N_c}$$

- Intensities :
  $j(m, m^+)$: transition from m to m^+ (failure or repair)

$$\lambda_m = \sum_{m^+ \in \mathbb{M}} \lambda_m^{j(m,m^+)}$$

- Kernel :
  if $z = (x, m)$ is not on a boundary

$$K_z\big((x^+, m^+)\big) = \frac{\lambda_m^{j(m,m^+)}}{\lambda_m} \mathbb{1}_{x=x^+}$$