Approximating discontinuous functions with the Christoffel-Darboux polynomial

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In data science, the quantity to be approximated numerically can be a discontinuous function, e.g. the solution of a nonlinear PDE with shocks, or a bang-bang optimal control. Standard numerical algorithms may face troubles when approximating such functions. In this talk, we introduce a new family of approximants designed to deal with such discontinuities. We propose to model or approximate a function of a vector x by the minimizer with respect to additional (lifting) variables y of a sum of squares of polynomials of x and y. Examples illustrate that elementary discontinuous or non-differentiable functions can be modeled exactly this way. For any Lebesgue measurable function, we describe a systematic method to construct a family of approximants of increasing degree such that their minimizers converge to the function pointwise almost everywhere and in the Lebesgue one norm. These approximants are Christoffel-Darboux polynomial kernels generated from the moments or the samples of the function. Joint work with Jean-Bernard Lasserre, Swann Marx, Edouard Pauwels and Tillmann Weisser, see hal-02085835.