Stable high-order randomized cubatures in arbitrary dimension

Giovanni Migliorati

We propose and analyse randomized cubature formulae for the numerical integration of functions with respect to a given probability measure on any domain in any dimension. Such cubatures are constructed using weighted least-squares approximants of the integrand function, they are exact (i.e. they integrate exactly any function in a given finite-dimensional subspace of L^2) and they can have strictly positive cubature weights under some conditions. For such cubatures we derive some convergence rates, and discuss similarities and advantages over other types of cubatures from the literature.