Boosted optimal weighted least-squares for the approximation of high-dimensional functions in tree tensor networks

Guillaume Perrin

One of the most challenging tasks in computational science is the approximation of high-dimensional functions $u \in L^2$, using only few point evaluations. In this work, we are interested in constructing the approximation of u in tree-based tensor format, in order to obtain an error close to the best approximation error while using as few evaluations as possible, as indeed u may be expensive to evaluate. The construction we propose relies on two main ingredients: an extension of principal component analysis to multivariate functions, and an adaptation of the works done on weighted least-squares projection, which we call boosted weighted least-squares method. The interest of the proposed approach comes from the fact that it allows to ensure almost surely the stability of the approximation with a sample size close to the complexity of the model. Quasi-optimality properties in expectation can also be obtained under additional conditions. We will illustrate on numerical examples to what extent it is possible to enrich sequentially the approximation models in order to obtain in practice controlled errors.