

Data assimilation based on Lagrangian physics-informed Gaussian processes for the reconstruction of incompressible flows

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ABSTRACT

We present a data assimilation framework based on physics-informed Gaussian process (GP) regression that is applied to the reconstruction of incompressible fluid flows about aerodynamic profiles. Current methods for flow data assimilation exhibit lack of flexibility to integrate physical conditions and observations concurrently, while providing uncertainty quantification (UQ) of the flow estimates. They are typically mesh-based and often exhibit high computational costs without clear theoretical guarantees. On the contrary, GPs and kernel methods have been recently adapted to solve physics problems and approximate partial differential equations [1, 2, 3]. They have a well-established mathematical background and are mesh-independent. Observations from multiple data sources can be incorporated straightforwardly while providing versatile UQ counterparts.

In our methodology we approximate the Navier-Stokes equations for the vorticity while integrating velocity and vorticity measurements of Lagrangian particles to enhance the flow estimates. The forecasting step is based on GP regression through a version of the numerical method presented in [2]. Moreover, the GP prior is informed about the flow physics, such as the divergence-free velocity, profile boundary conditions, and the energy-decay of the fluid dynamics [3]. These constraints are enforced continuously at all positions within the computational domain and not at a discrete set of observations solely.

We provide numerical experiments on time-resolved incompressible flows about aerodynamic profiles. Our results indicate that this scheme can accurately propagate the initial information through subsequent assimilation steps, incorporating measurements when available. Moreover, it is competitive in comparison to its pure data-driven version. We also explore optimal sensor placement to improve the reconstructions using UQ of the estimates.

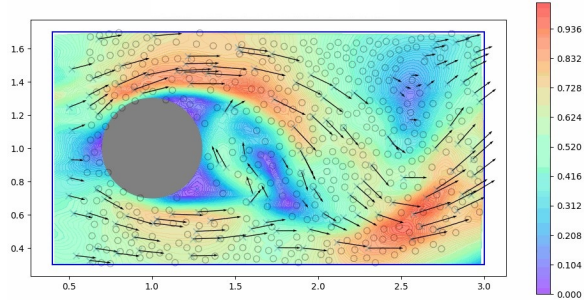


Figure: Snapshot of reconstructed velocity field (norm) of an unsteady incompressible flow past a cylinder profile using our GP-assimilation framework. Arrows represent Lagrangian velocity measurements and circles are collocation positions of vorticity forecasting from the initial condition.

REFERENCES

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