Mixed Poisson processes for generative modelling and photon-starved inverse problems

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Abstract

While originally designed for generative modelling purposes, diffusion models have recently been used to solve inverse problems of the form

$$\text{Samples from } \ \pi_y(\mathrm{d}x) \propto p(y|x)\pi(\mathrm{d}x) \tag{1}$$

where $p(y|x)$ is a likelihood function and π is a prior distribution. Specifying a prior is usually difficult, yet extremely important, notably when the SNR is low and/or the inverse problem is ill-posed.

Diffusion models allow one to learn such a prior from a dataset by effectively learning a *denoiser*. Then, two main approaches exist: either estimate the prior score from this denoiser and use it in a Langevin MCMC (the *Plug-and-play approach*), or tilt the diffusion according to the observation y to sample from π_y instead of π (the *Diffusion Posterior Sampling* approach). Independently of the inverse problem at hand, both approaches rely on some Gaussian dynamics: by definition of diffusion models, the denoiser is trained to remove Gaussian noise; then, the sampling process is either a Langevin SDE or the original diffusion (with a tilted denoiser).

A difficult class of inverse problems is Poisson inverse problems: this includes medical imaging (e.g., Positron Emission Tomography) and astronomy. While the aforementioned methods can theoretically be applied to this specific case, their performances are often poor. One possible culprit is the fundamental mismatch between the Gaussian denoiser of the diffusion model and the "physical" Poisson noise driving the inverse problem.

Inspired by a recent attempt at "Poisson generative modelling", we propose to use a *mixed Poisson process* to define a new class of "diffusion" models, that is, an intrinsically Poisson interpolant rather than a Gaussian one. While mixed Poisson processes are fundamentally different from diffusion processes (they are jump processes), we show that most of the results available for diffusion models can be adapted. In particular, we may use the learned distribution as a prior to tackle inverse problems. We provide mathematical and empirical results that suggest that this new class of models is more adapted to Poisson inverse problems (especially photon-starved ones) than classical diffusion models.