

Plan d'expérience avec entrées fonctionnelles dépendantes pour la génération de spectres gamma en prospection minière de l'uranium

Workshop RT-UQ « Expériences et simulations : comment les planifier ? » – 6-7 novembre 2025

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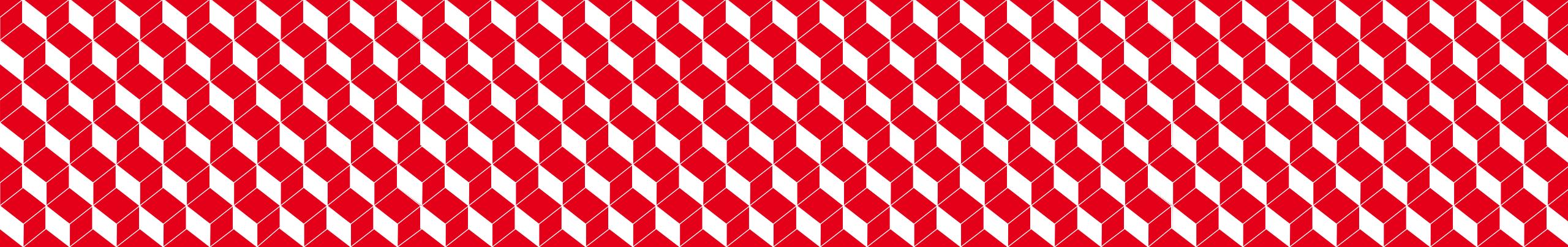
¹CEA, DES, IRESNE, DTN, SMTA, LMN, Cadarache, France

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³Orano Mining, Châtillon, France

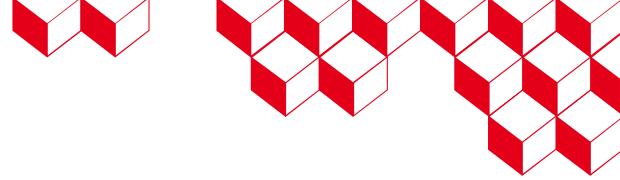
⁴I2M, Marseille, France

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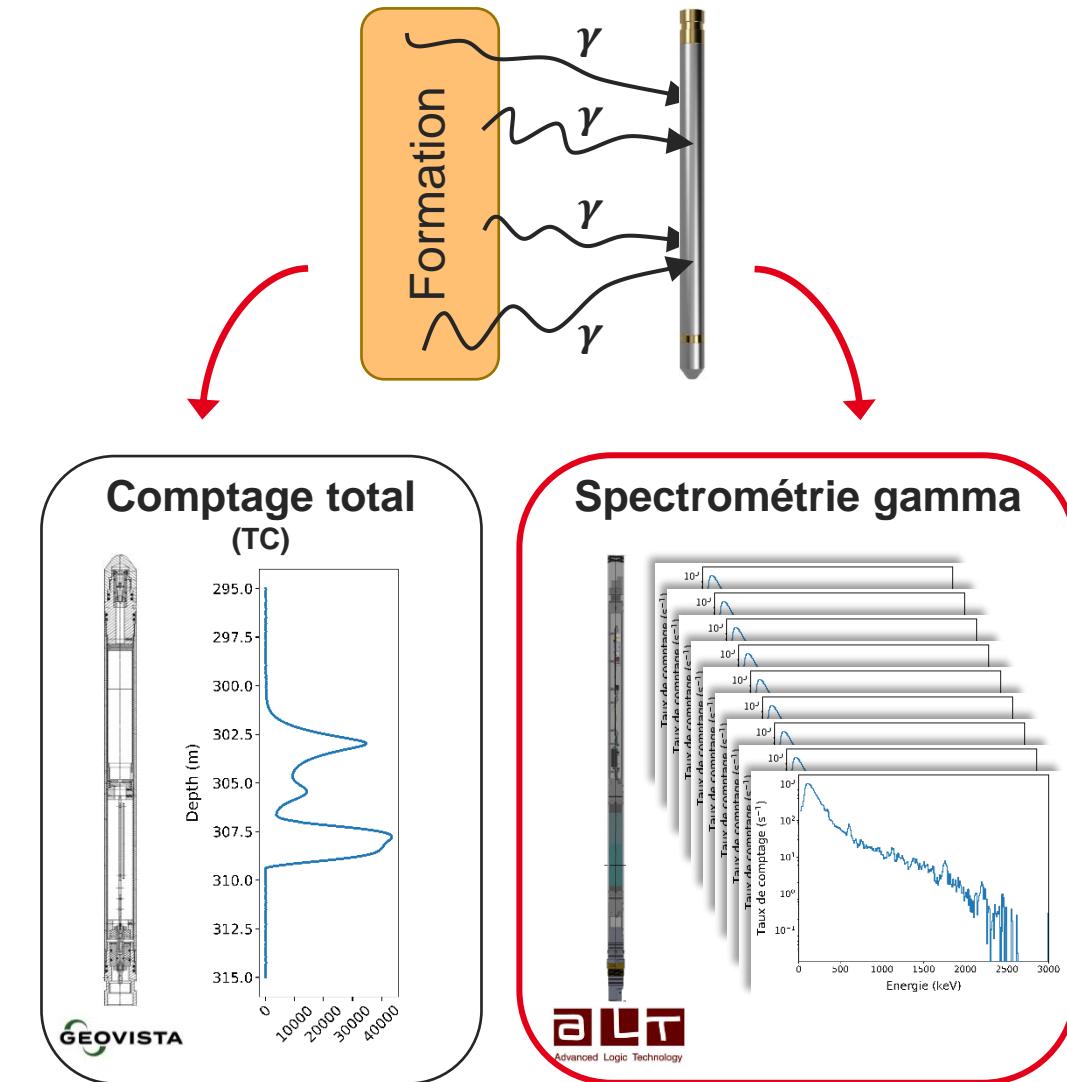
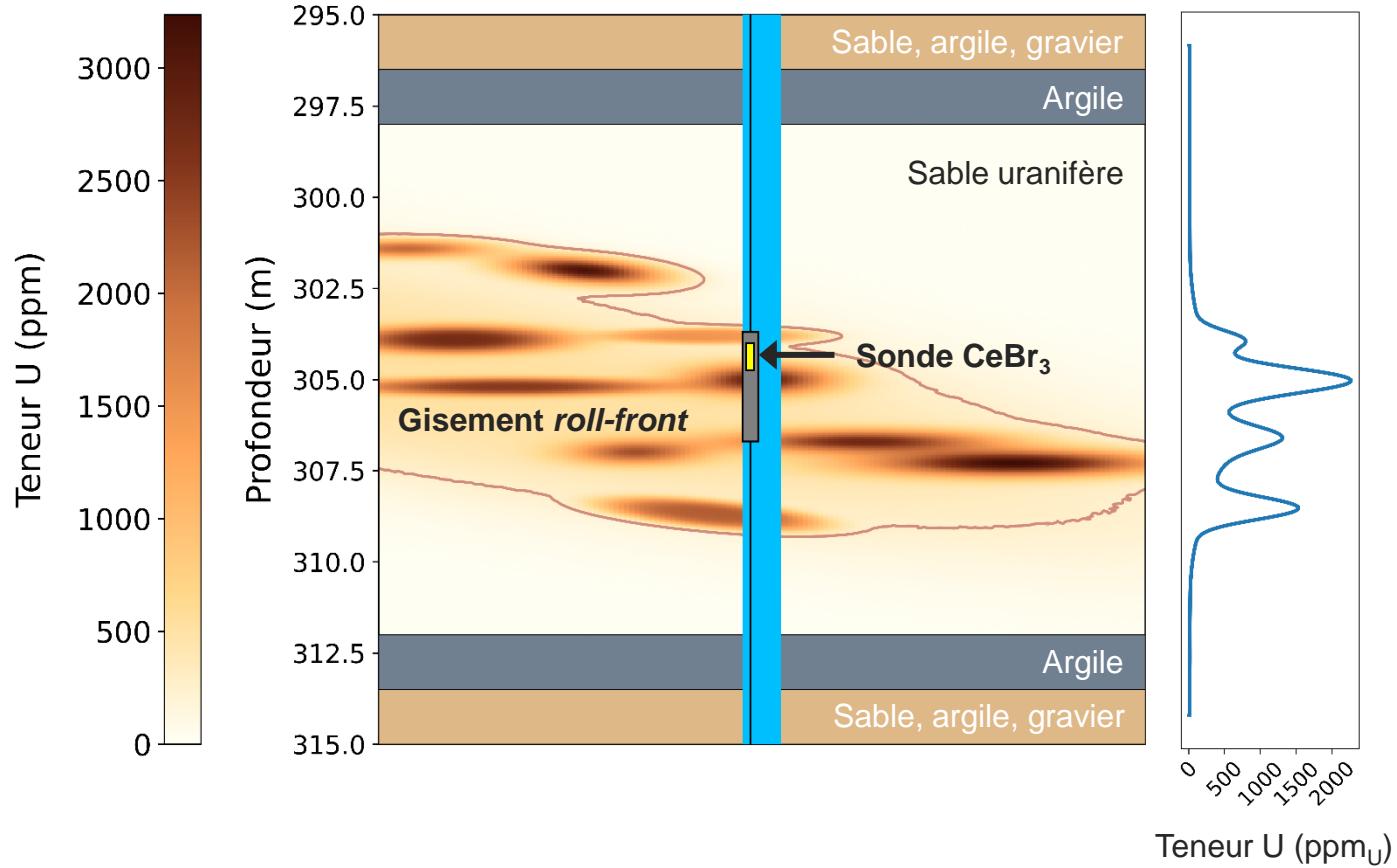


1 ■ Contexte & Enjeux

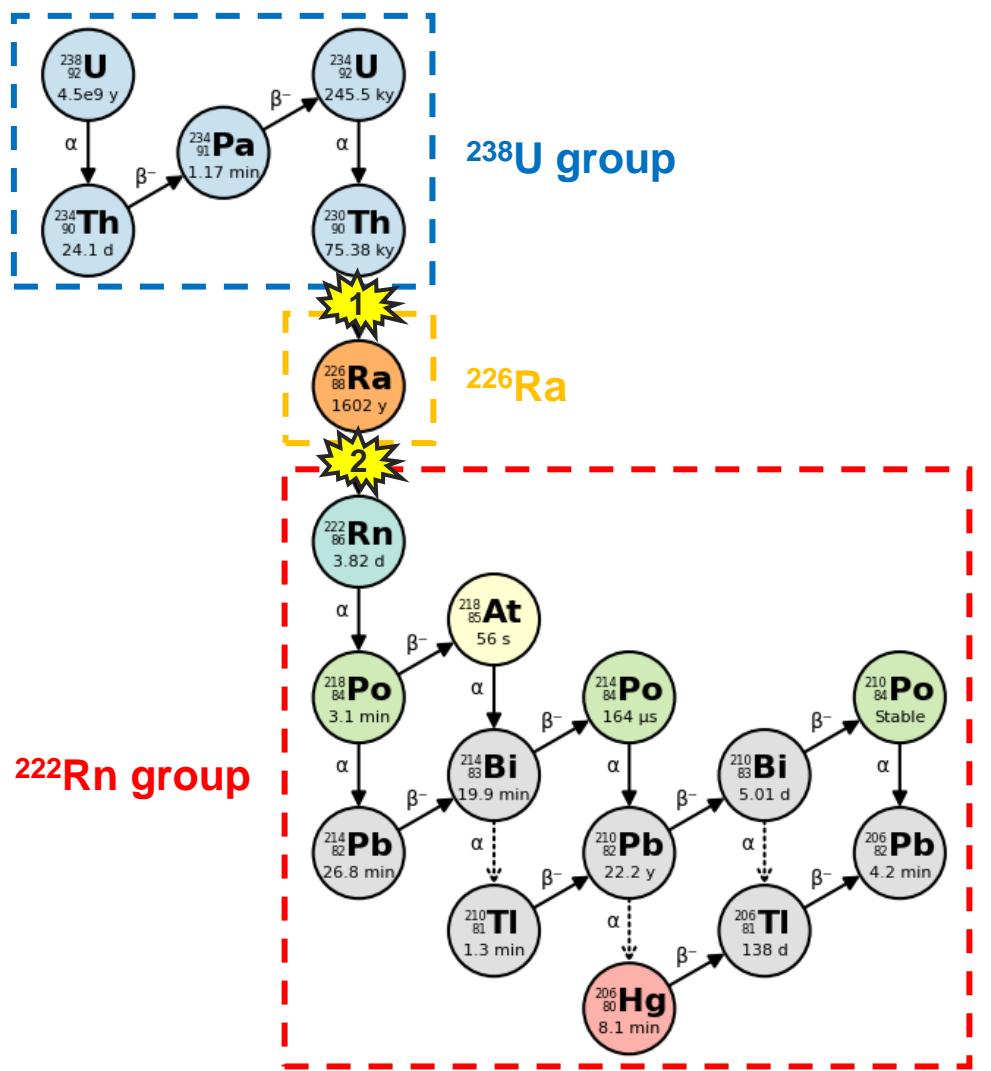
Contexte : la mesure gamma diagraphique dans les fronts d'oxydo-réduction



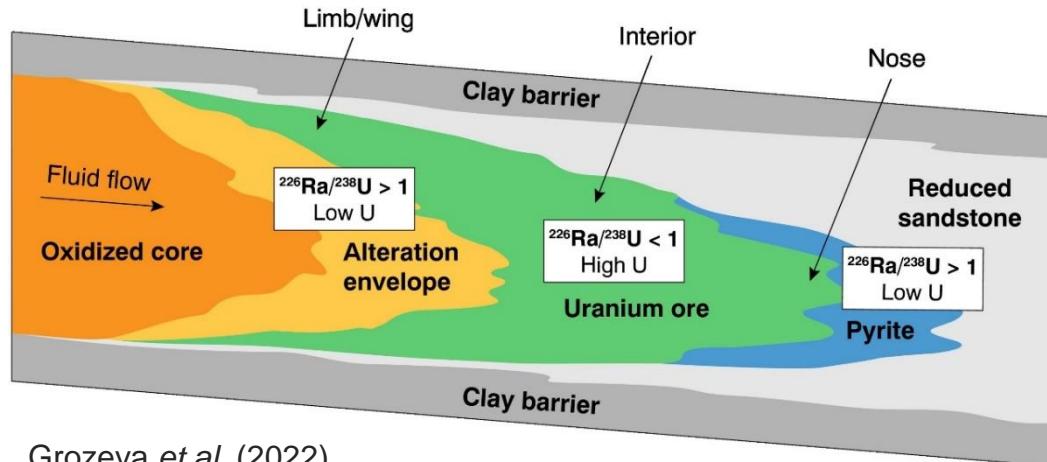
Vue en coupe d'une mesure gamma diagraphique en puits de forage



Déséquilibre radioactif dans les chaînes de l'uranium



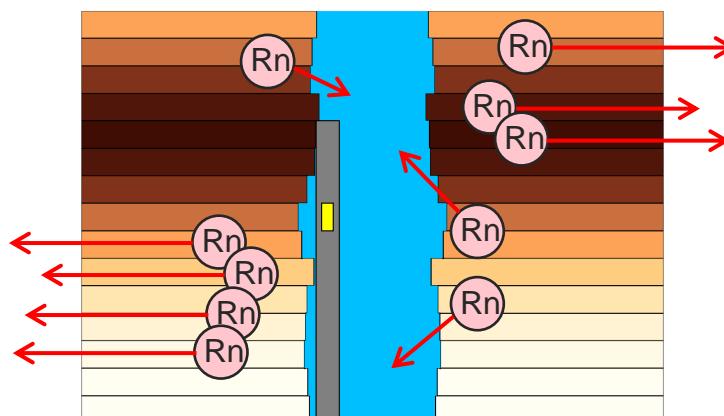
1! Vue en coupe d'un front d'oxydo-réduction



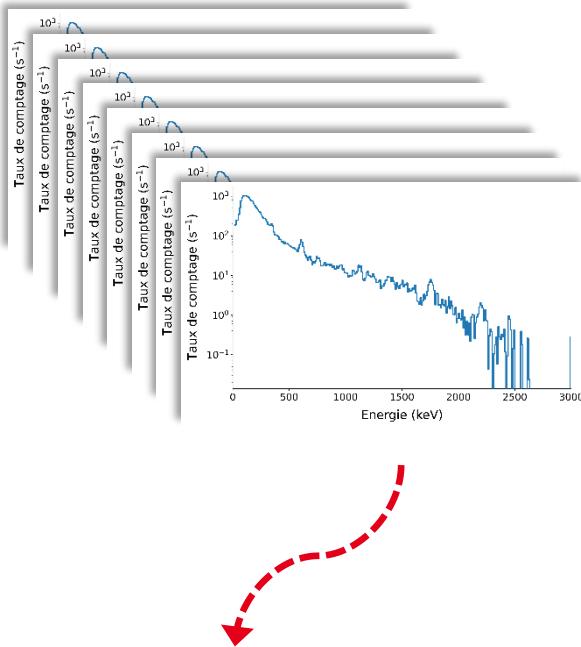
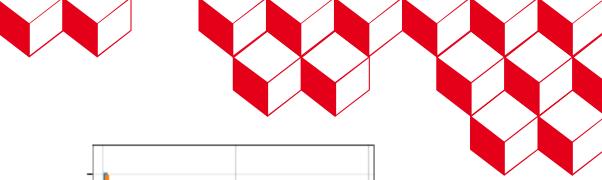
Grozeva et al. (2022)

<https://doi.org/10.1016/j.gexplo.2022.106961>

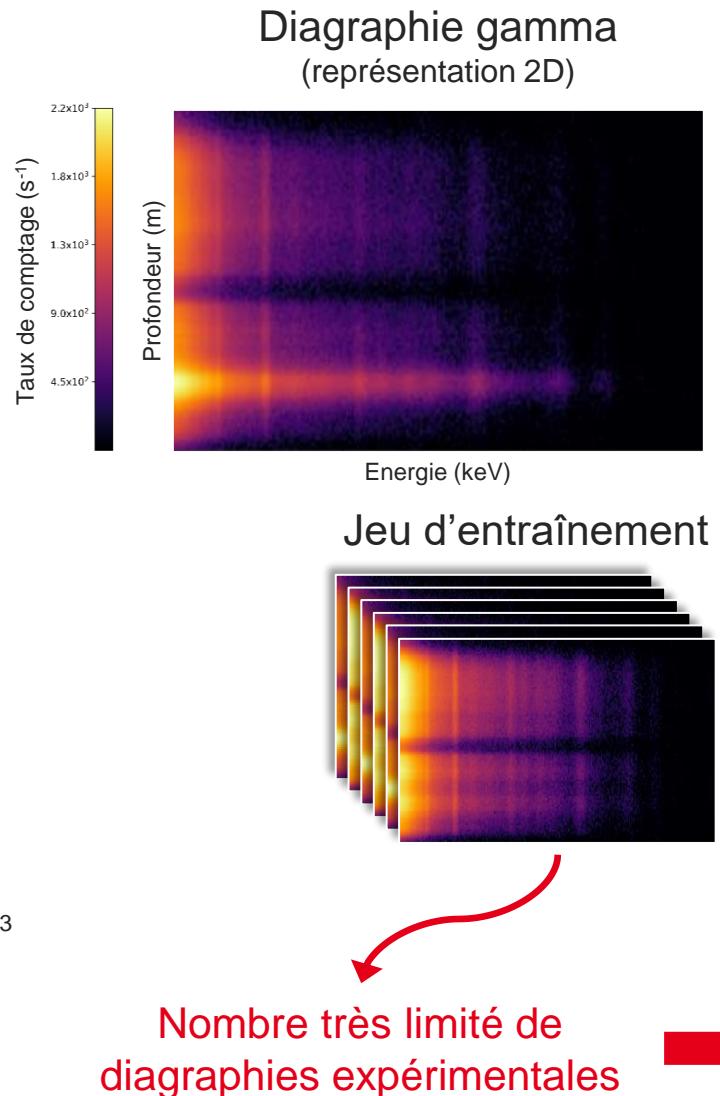
2! Migration du radon dans le puits de forage



Reconstruction automatique de profils de teneur U



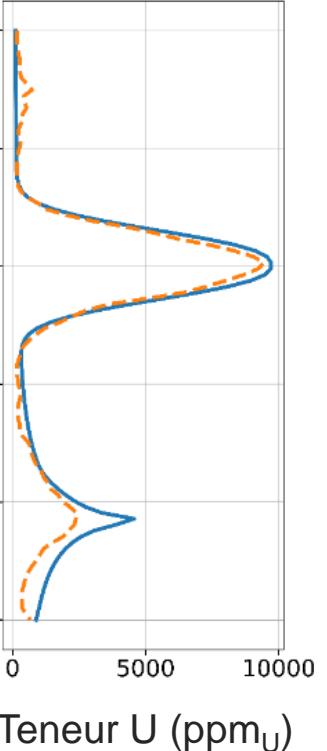
- Déséquilibre radioactif
- Bruit statistique élevé
- Faible résolution en énergie de la sonde CeBr₃



Prédiction



Entraînement

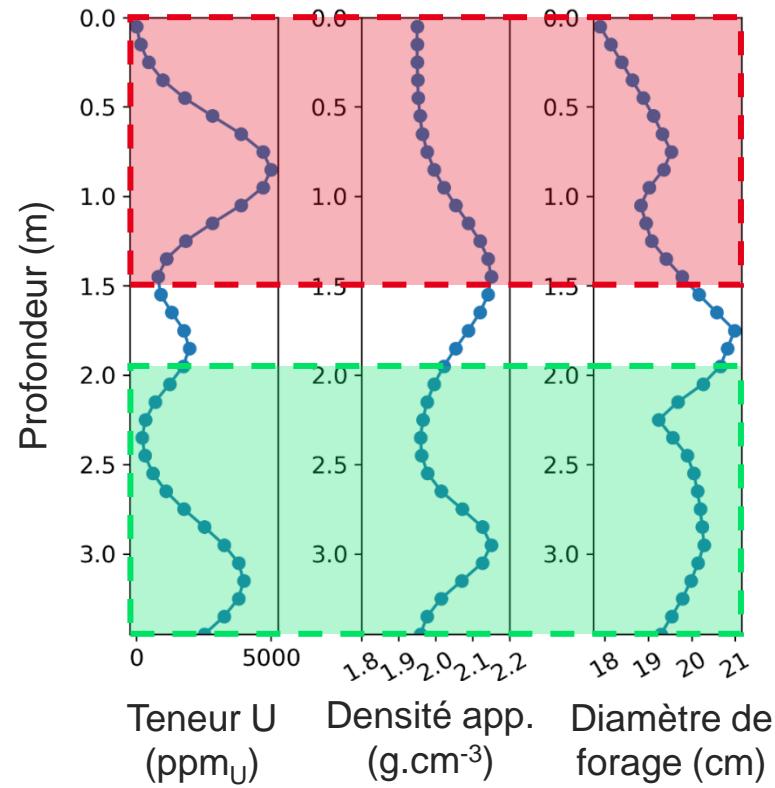


Construction d'une base de données diagraphiques simulées

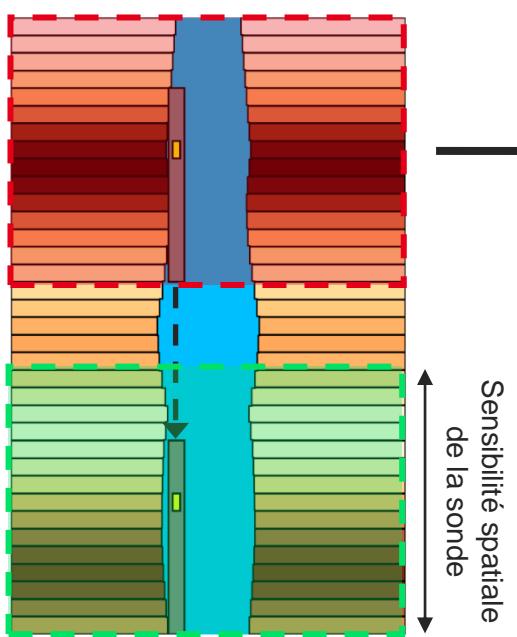
Simulation d'une diagraphie gamma spectrale



Profils géologiques (1D)



Géométrie du puits

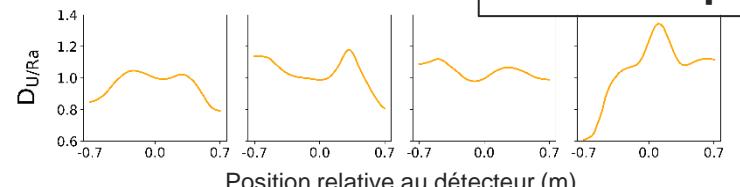


MCNP
MONTE CARLO N-PARTICLE

Sensibilité spatiale
de la sonde

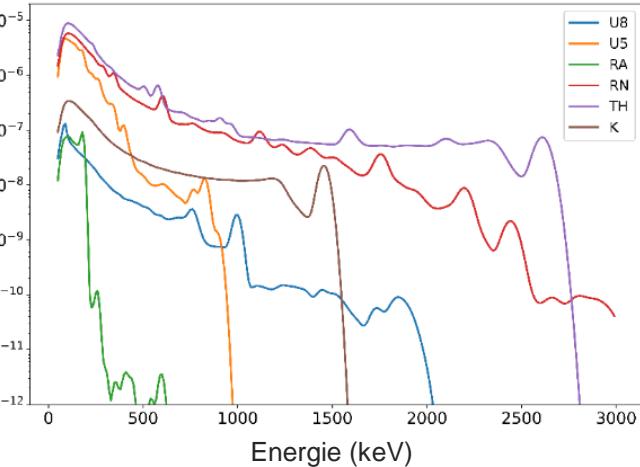
Activités a_i

Profils de teneur U et
déséquilibre



Taux de comptage par secondes
(s⁻¹.Bq⁻¹)

Taux de comptage (s⁻¹)



6 Spectres fondamentaux

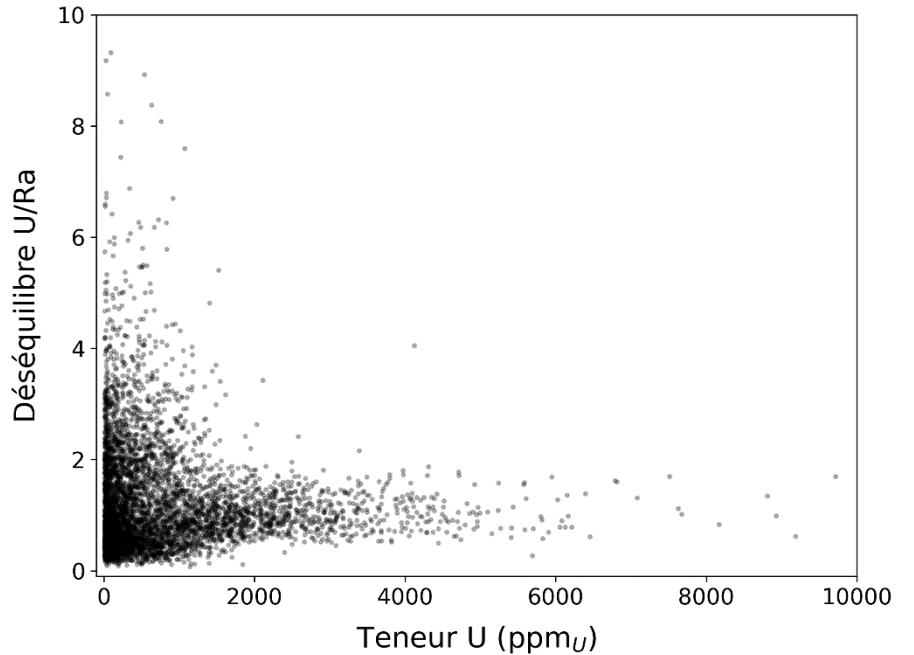
05/11/2025

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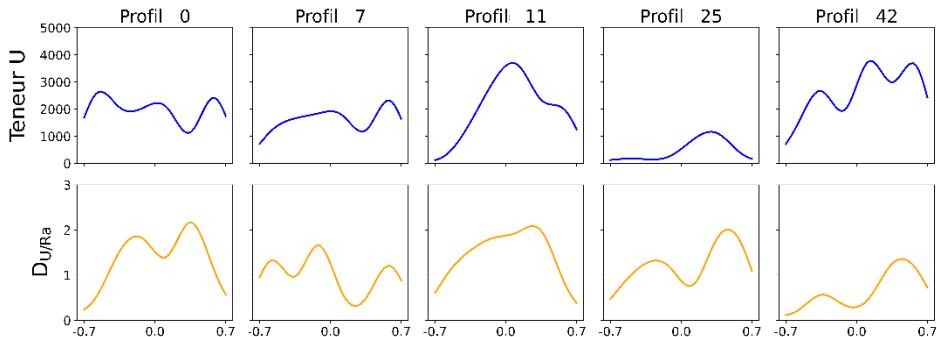
Relation teneur U – déséquilibre U/Ra



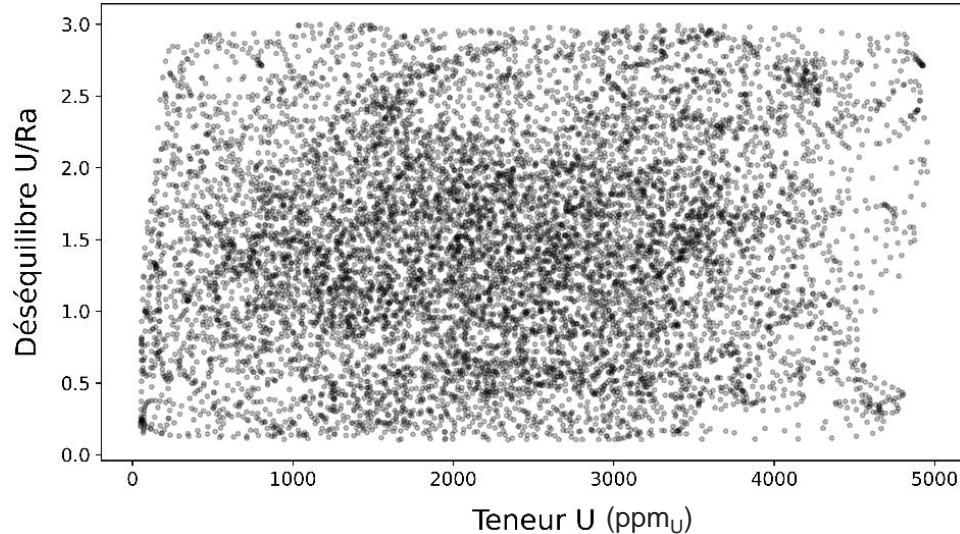
Distribution jointe de U et $D_{U/Ra}$
issue de mesures sur échantillons



Profils construits à partir de B-splines



↓
Distribution des points agrégés



2

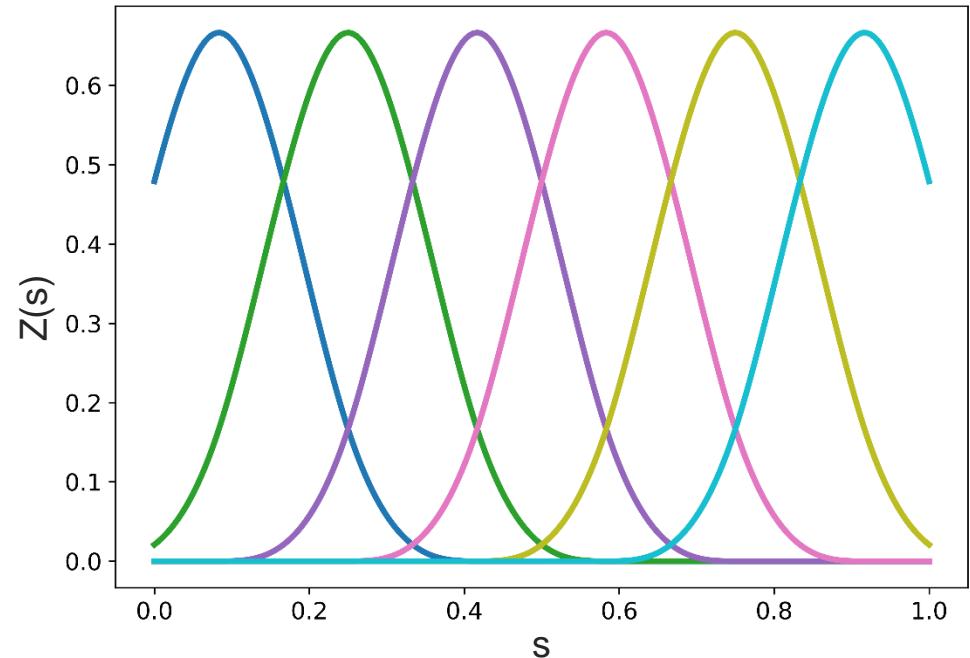
Génération d'entrées fonctionnelles liées par une copule gaussienne

Génération de profils à l'aide de B-splines



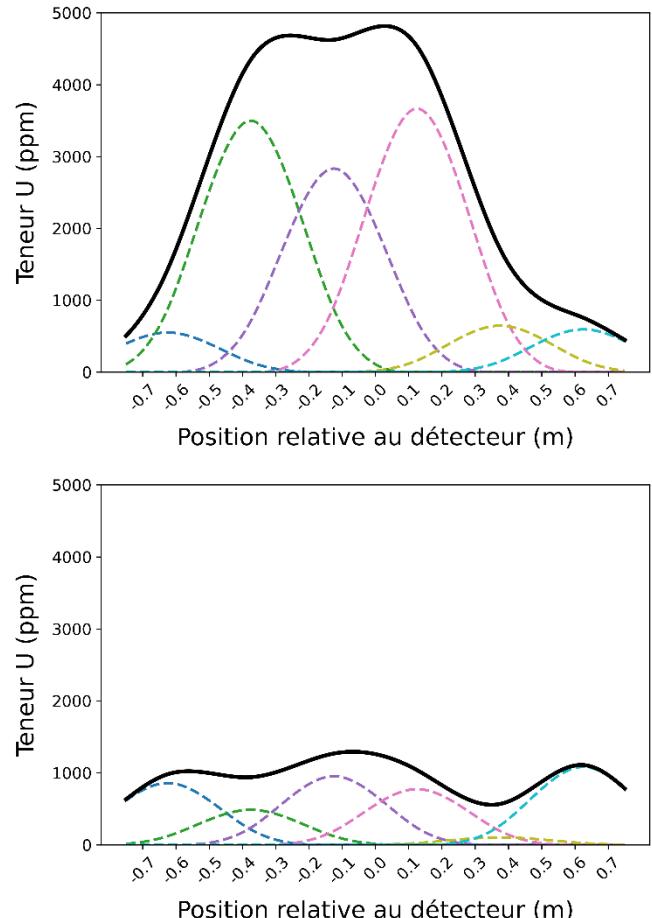
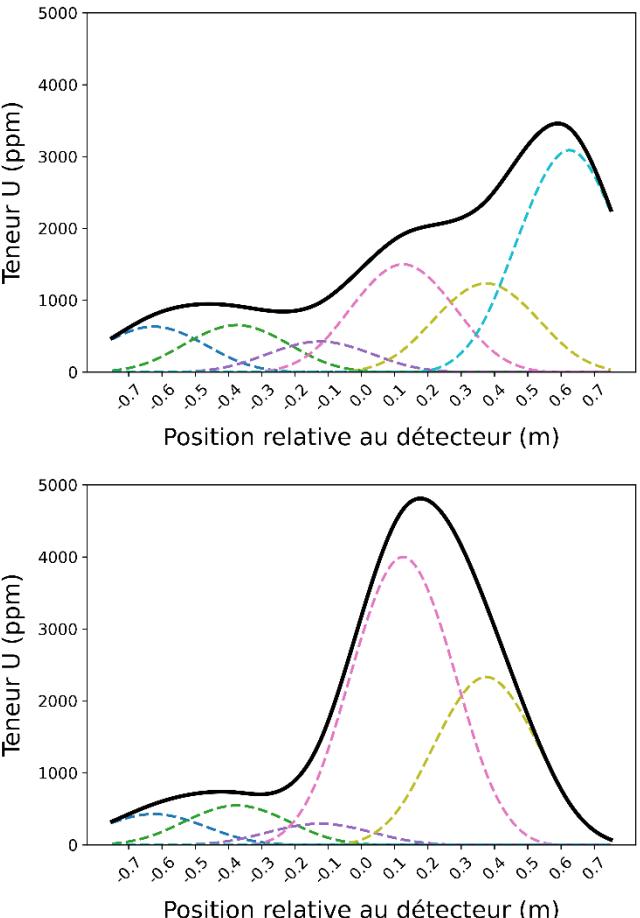
Courbes de teneur U générées par combinaison linéaire de B-splines et mise à l'échelle

Fonctions de base B-splines

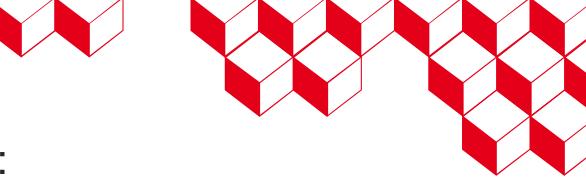


De Boor (1978)

<https://doi.org/10.2307/2006241>



$$Z(s) = \sum_{k=1}^K \beta_k B_k(s)$$



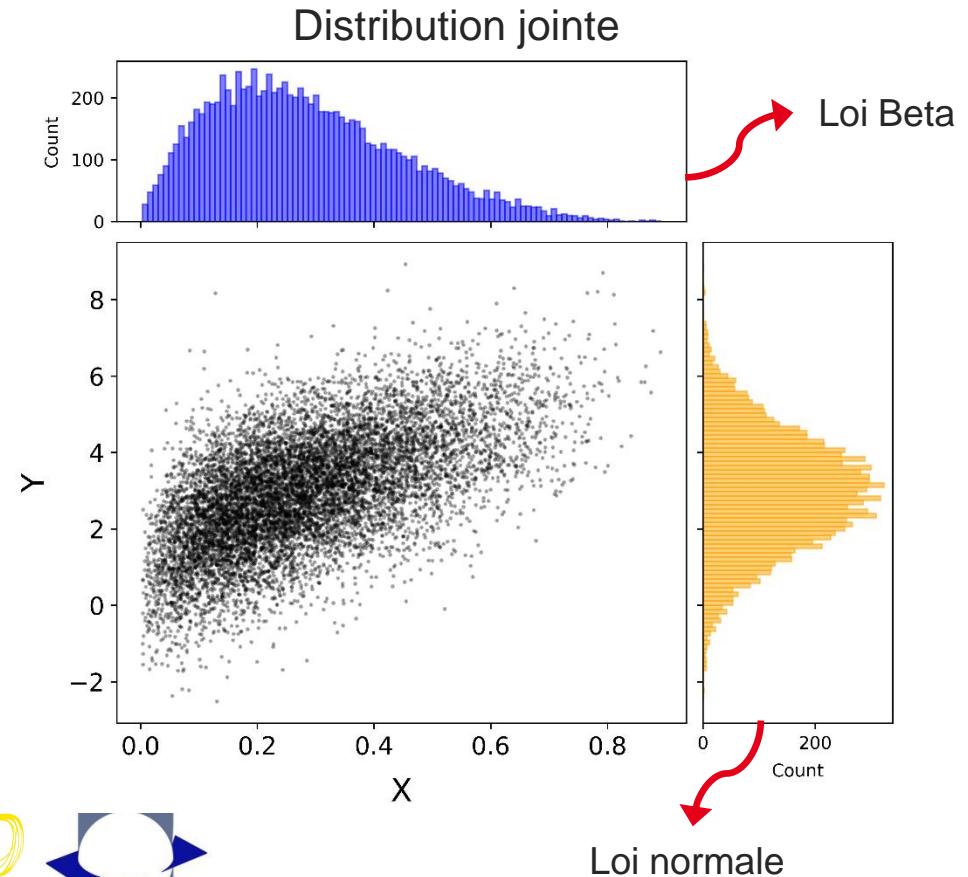
Copule gaussienne

Théorème de Sklar : $F(x, y) = C(F_X(x), F_Y(y))$

Copule gaussienne : $\forall (u, v) \in [0,1]^2, C_\rho(u, v) = \Phi_\rho(\Phi^{-1}(u), \Phi^{-1}(v))$

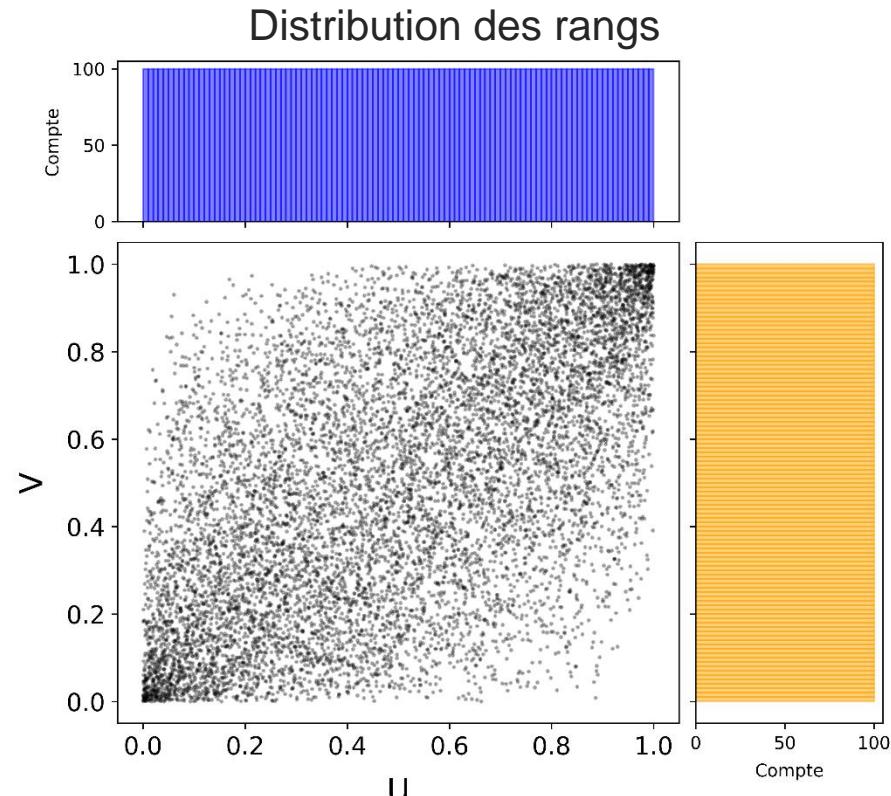
avec :

- Φ_ρ CDF de la loi normale multivariée centrée de matrice de covariance $\Sigma_\rho = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$
- Φ CDF de la loi normale centrée réduite

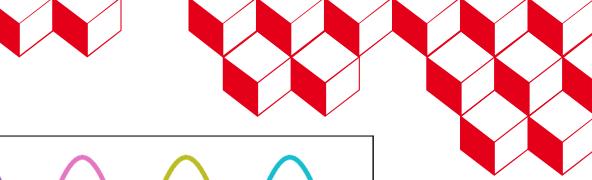


Génération de $(X, Y) \sim C_\rho$:

- $(Z_U, Z_V) \sim \mathcal{N}(0, \Sigma_\rho)$
- $U := \Phi(Z_U), \quad V := \Phi(Z_V)$
- $X := F_X^{-1}(U), \quad Y := F_Y^{-1}(V)$



Génération des entrées fonctionnelles



M points par courbe

Matrice des B-splines discrétisées suivant M : $\mathbf{B} \in \mathbb{R}^{K \times M}$

$\boldsymbol{\beta}^U, \boldsymbol{\beta}^V$: coefficients de la décomposition en B-splines

$$\begin{pmatrix} \boldsymbol{\beta}^U \\ \boldsymbol{\beta}^V \end{pmatrix} \sim \mathcal{N} \left(0, \begin{pmatrix} I_m & \rho I_m \\ \rho I_m & I_m \end{pmatrix} \right) \rightarrow \begin{pmatrix} \mathbf{Z}^U \\ \mathbf{Z}^V \end{pmatrix} = \begin{pmatrix} \mathbf{B}^T \boldsymbol{\beta}^U \\ \mathbf{B}^T \boldsymbol{\beta}^V \end{pmatrix} \sim \mathcal{N} \left(0, \begin{pmatrix} \mathbf{B}^T \mathbf{B} & \rho \mathbf{B}^T \mathbf{B} \\ \rho \mathbf{B}^T \mathbf{B} & \mathbf{B}^T \mathbf{B} \end{pmatrix} \right)$$

$$\left. \begin{pmatrix} Z_S^U \\ Z_S^V \end{pmatrix} \right|_{S=S} = \begin{pmatrix} {B_{\cdot,s}}^T \boldsymbol{\beta}_U \\ {B_{\cdot,s}}^T \boldsymbol{\beta}_V \end{pmatrix} \sim \mathcal{N} \left(0, \|B_{\cdot,s}\|_2^2 \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right)$$

$$F_{(Z_S^U, Z_S^V)}(z_U, z_V) = \frac{1}{M} \sum_{s=1}^M \Phi_\rho \left(\frac{z_U}{\|B_{\cdot,s}\|_2}, \frac{z_V}{\|B_{\cdot,s}\|_2} \right)$$

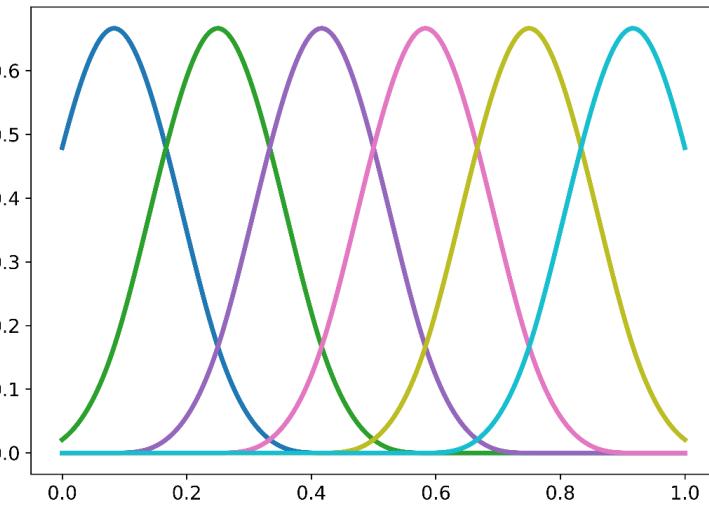
Copule non préservée si $\|B_{\cdot,s}\|_2$ varie avec s



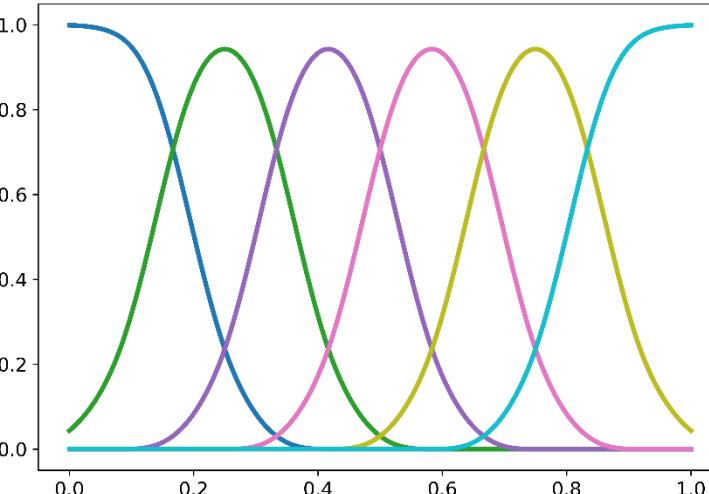
Solution = standardisation des B-splines :

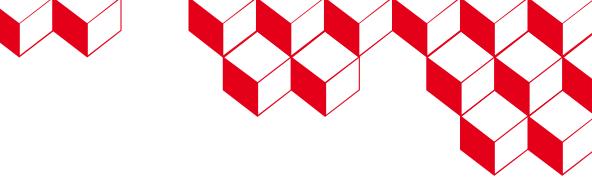
$$\forall s \in \mathbb{R}, \tilde{B}_{\cdot,s} = \frac{B_{\cdot,s}}{\|B_{\cdot,s}\|_2} \Rightarrow \|\tilde{B}_{\cdot,s}\|_2 = 1 \Rightarrow F_{(Z_S^U, Z_S^V)}(z_U, z_V) = \Phi_\rho(z_U, z_V)$$

Base classique



Base standardisée



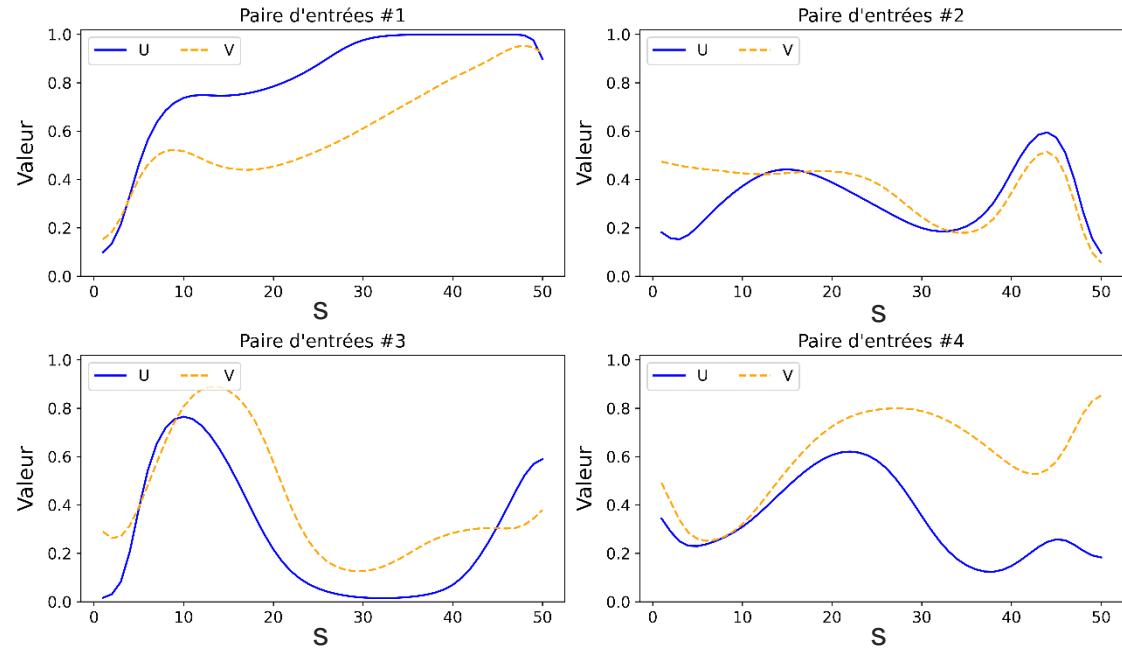


Génération des entrées fonctionnelles

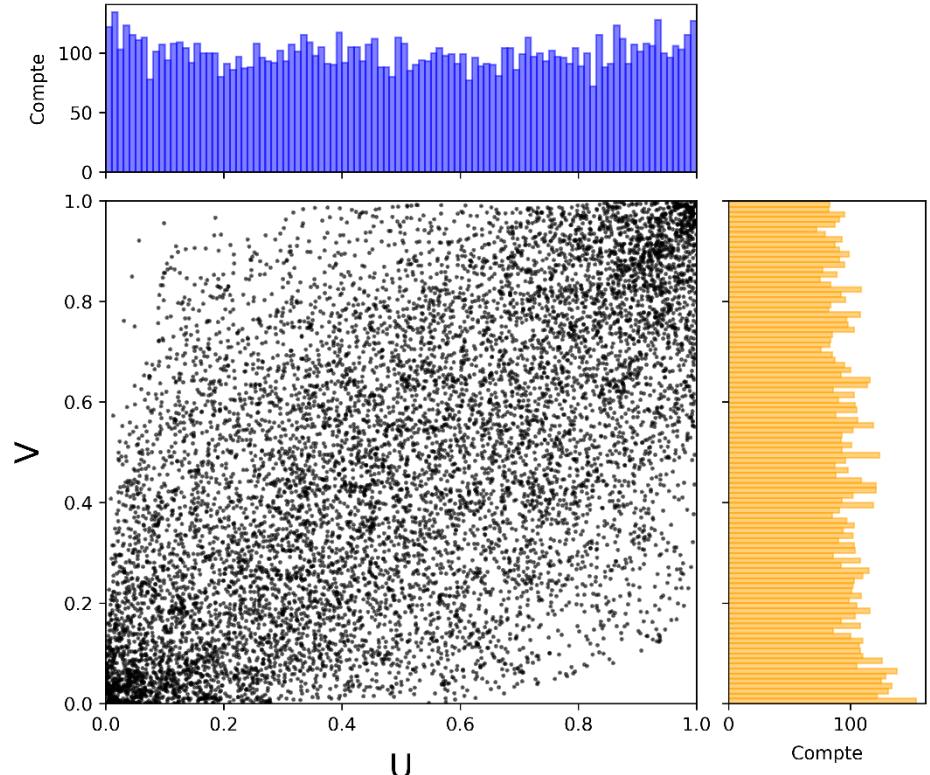
Tirage aléatoire

$$\begin{aligned} \left(\begin{matrix} \boldsymbol{\beta}^U \\ \boldsymbol{\beta}^V \end{matrix} \right) &\sim \mathcal{N} \left(0, \begin{pmatrix} I_m & \rho I_m \\ \rho I_m & I_m \end{pmatrix} \right) \rightarrow \left(\begin{matrix} \mathbf{Z}^U \\ \mathbf{Z}^V \end{matrix} \right) = \left(\begin{matrix} \tilde{\mathbf{B}}^T \boldsymbol{\beta}_U \\ \tilde{\mathbf{B}}^T \boldsymbol{\beta}_V \end{matrix} \right) \sim \mathcal{N} \left(0, \begin{pmatrix} \tilde{\mathbf{B}}^T \tilde{\mathbf{B}} & \rho \tilde{\mathbf{B}}^T \tilde{\mathbf{B}} \\ \rho \tilde{\mathbf{B}}^T \tilde{\mathbf{B}} & \tilde{\mathbf{B}}^T \tilde{\mathbf{B}} \end{pmatrix} \right) \rightarrow \mathbf{U} = (U_1, \dots, U_s) \text{ avec } U_k = \Phi^{-1}(\mathbf{Z}_{U,k}) \sim \mathcal{U}(0,1) \\ &\quad \mathbf{V} = (V_1, \dots, V_s) \text{ avec } V_k = \Phi^{-1}(\mathbf{Z}_{V,k}) \sim \mathcal{U}(0,1) \end{aligned}$$

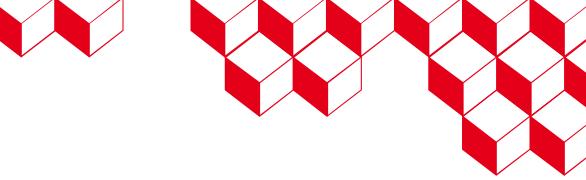
Courbes générées (espace des rangs)



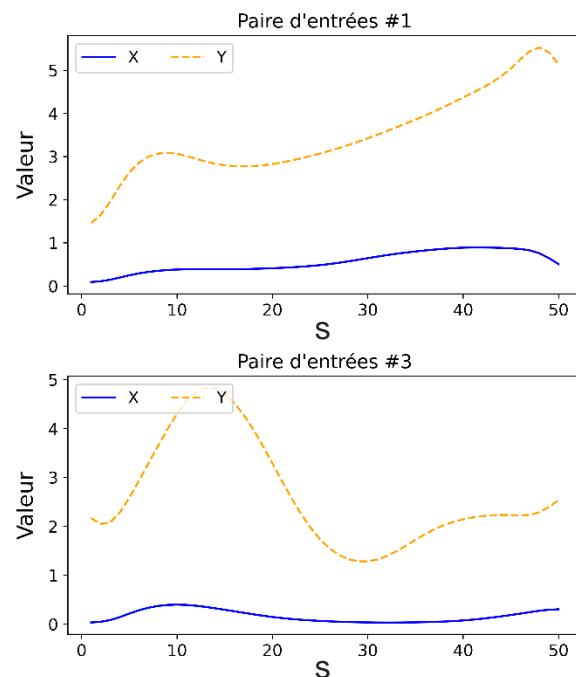
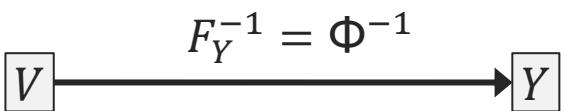
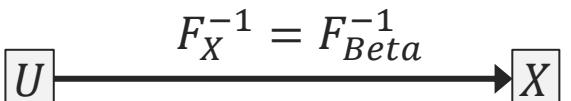
Distribution des points agrégés (espace des rangs)



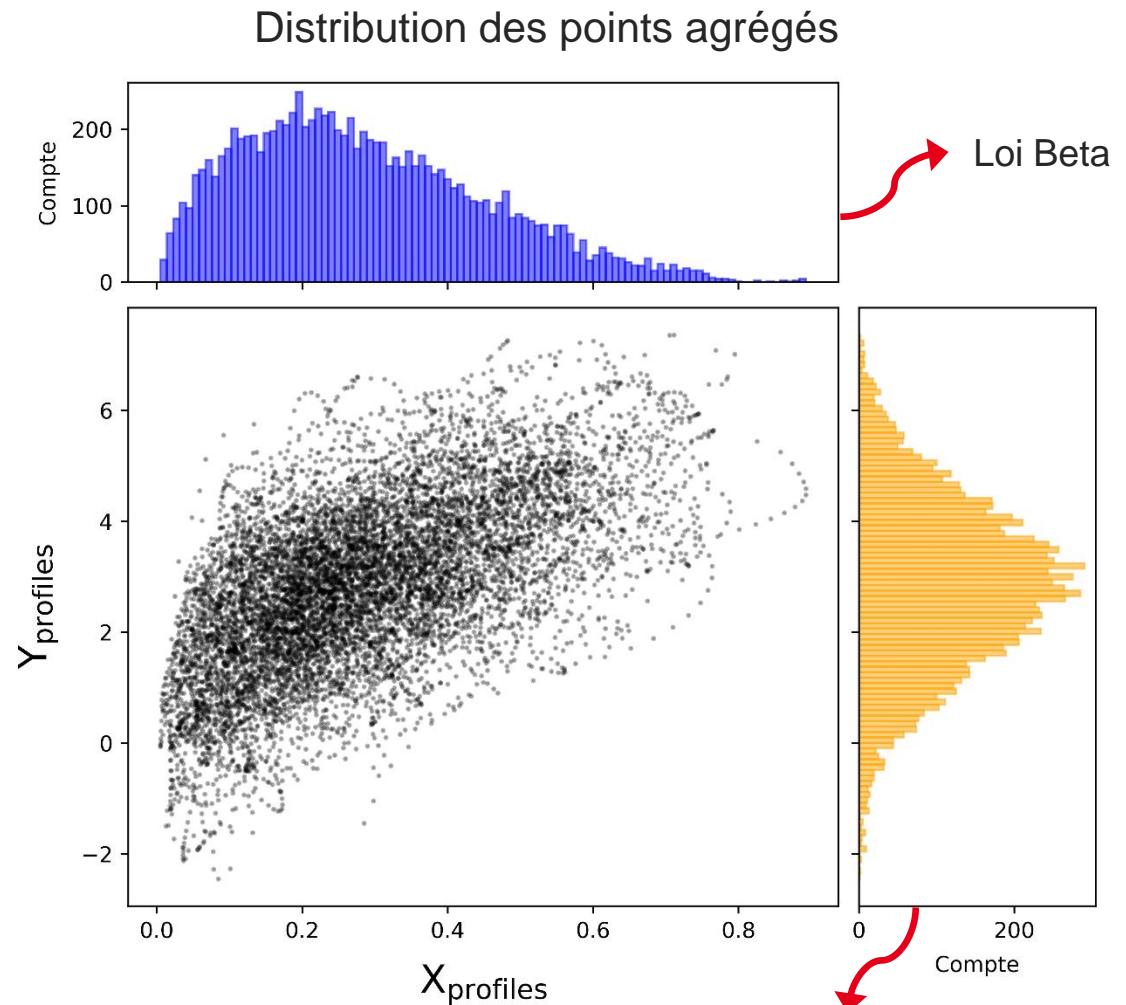
200 courbes
50 points par courbe
 $\rho = 0.6$



Génération des entrées fonctionnelles

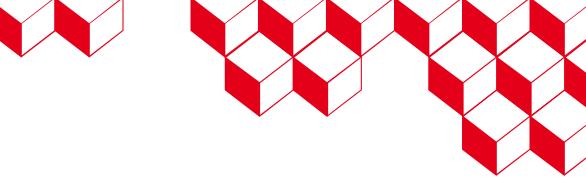


200 courbes
50 points par courbe
 $\rho = 0.6$



2

Génération d'entrée fonctionnelles liées par une copule archimédienne



Copule archimédienne

$$\forall (u, v) \in [0,1]^2, C_\theta(u, v) = \psi_\theta(\psi_\theta^{-1}(u) + \psi_\theta^{-1}(v))$$

Pour la copule de Clayton : $\psi_\theta(x) = (1 + x)^{-1/\theta}$

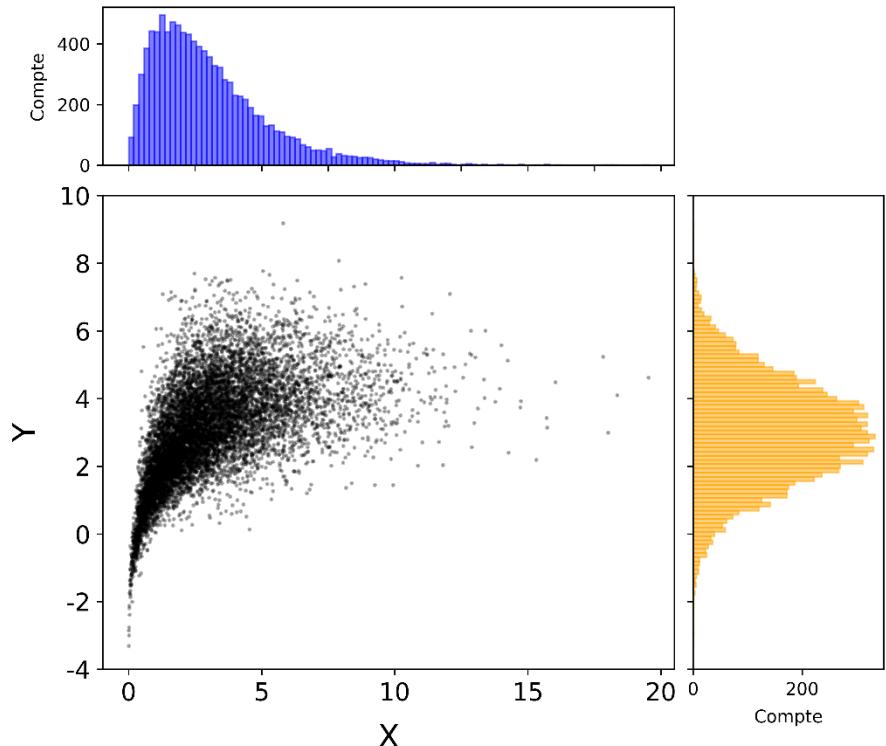
Génération de $(U, V) \sim C_\theta$:

- $E_1, E_2 \sim Exp(1)$
- $W \sim F_{w,\theta}$
- $U := \psi_\theta(E_1/W), \quad V := \psi_\theta(E_2/W)$

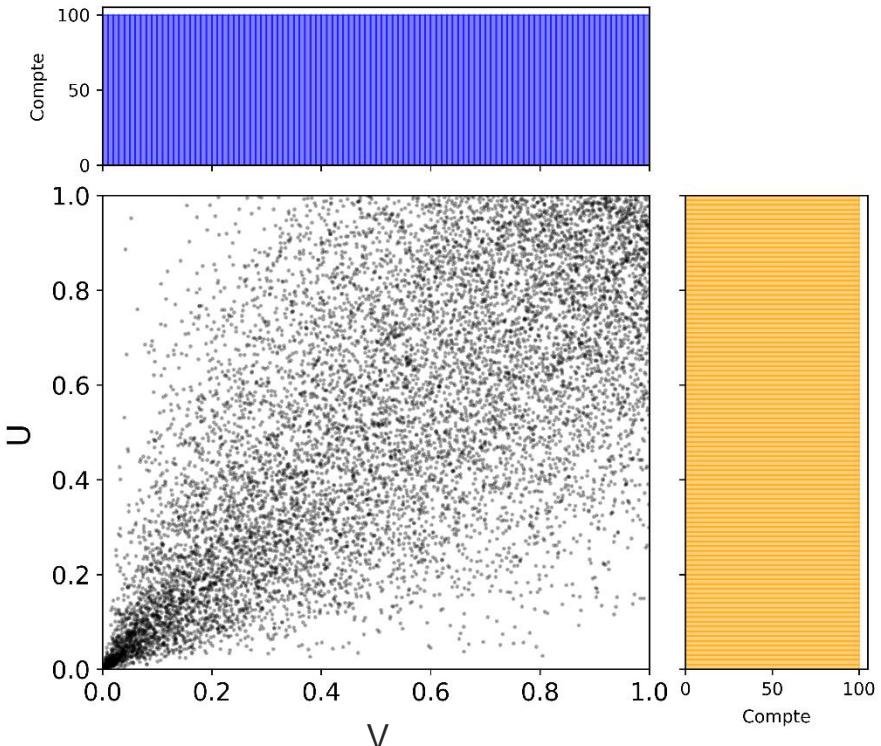
Exemple de la copule de Clayton :

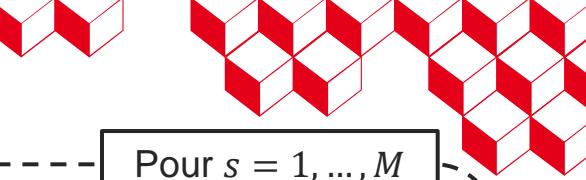
- Générateur : $\psi_\theta(x) = (1 + x)^{-1/\theta}$
- Fragilité : $W \sim \Gamma(1/\theta, 1)$

Distribution jointe



Distribution des rangs





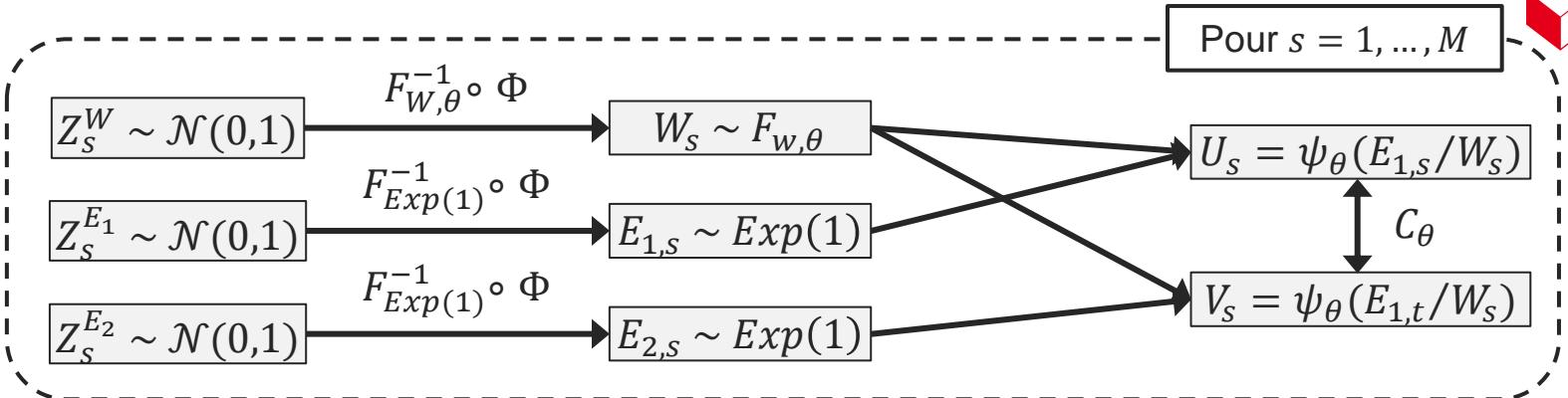
Génération des entrées fonctionnelles

Tirage aléatoire

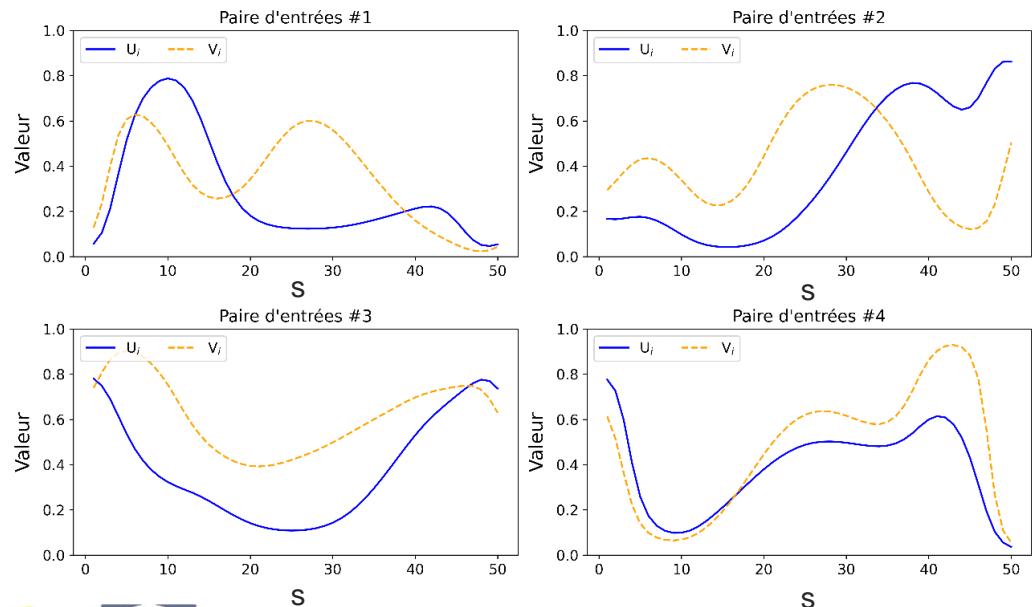
$$\boldsymbol{\beta}^W \sim \mathcal{N}(0, I_m) \rightarrow Z^W \sim \mathcal{N}(0, \tilde{\mathbf{B}}^T \tilde{\mathbf{B}})$$

$$\boldsymbol{\beta}^{E_1} \sim \mathcal{N}(0, I_m) \rightarrow Z^{E_1} \sim \mathcal{N}(0, \tilde{\mathbf{B}}^T \tilde{\mathbf{B}})$$

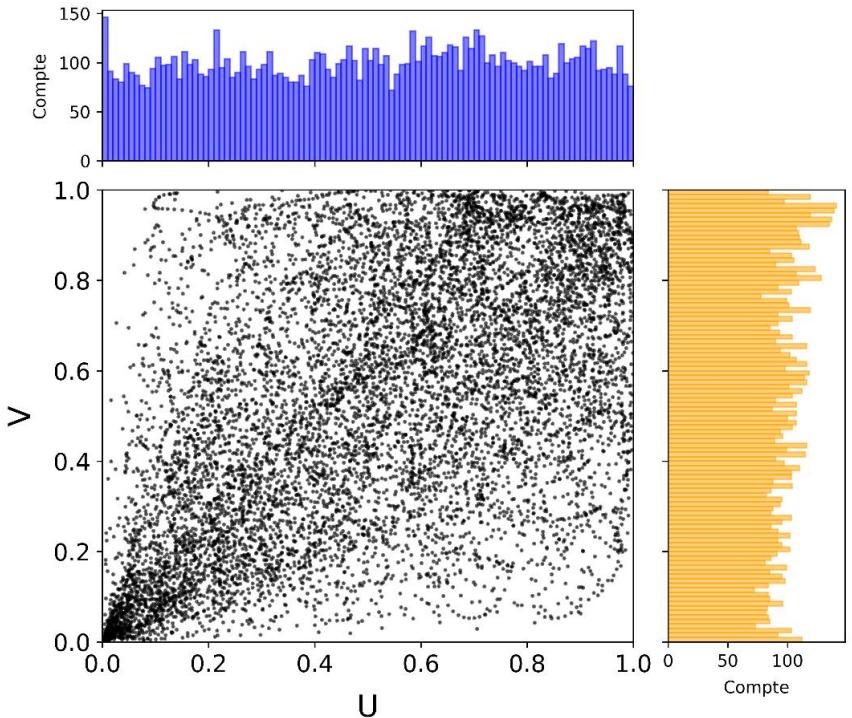
$$\boldsymbol{\beta}^{E_2} \sim \mathcal{N}(0, I_m) \rightarrow Z^{E_2} \sim \mathcal{N}(0, \tilde{\mathbf{B}}^T \tilde{\mathbf{B}})$$

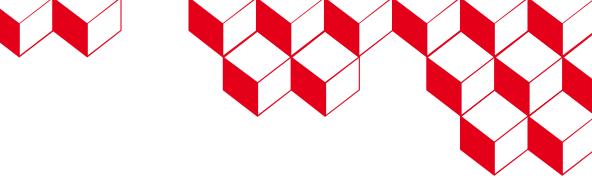


Courbes générées (espace des rangs)

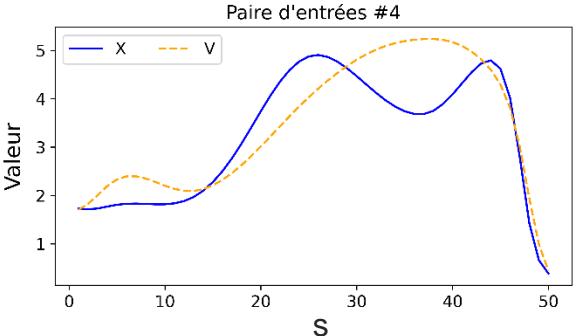
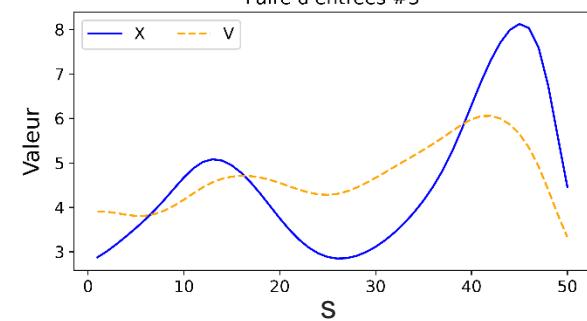
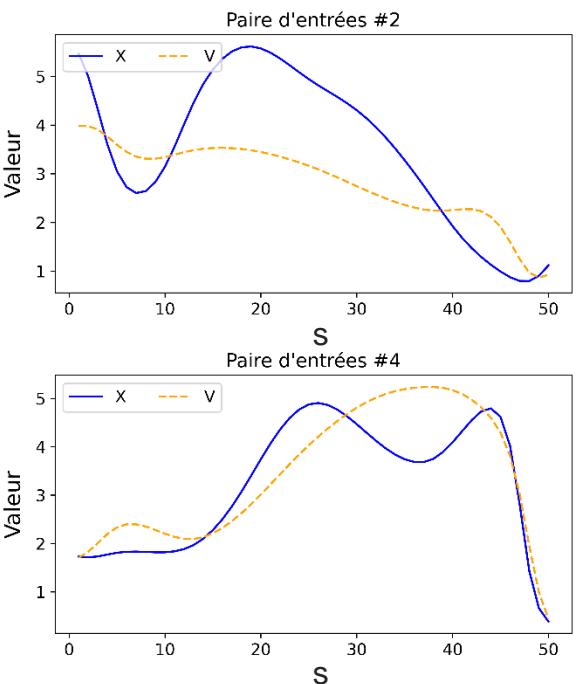
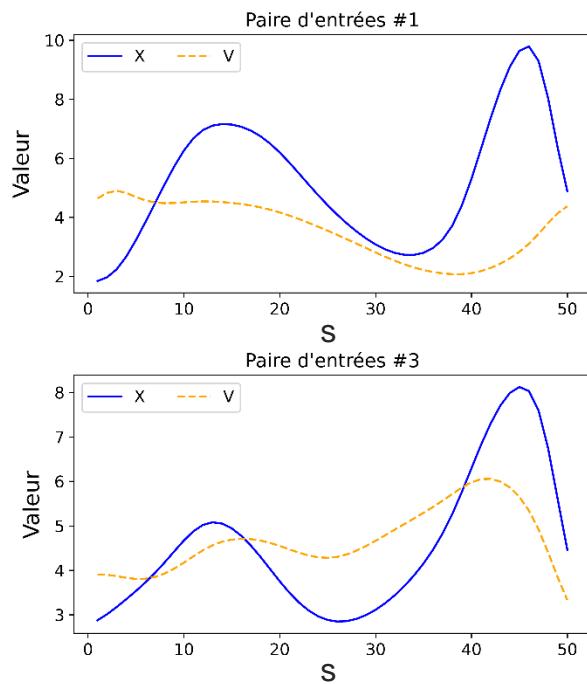
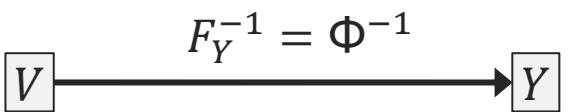
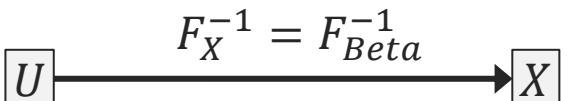


Distribution des points agrégés (espace des rangs)

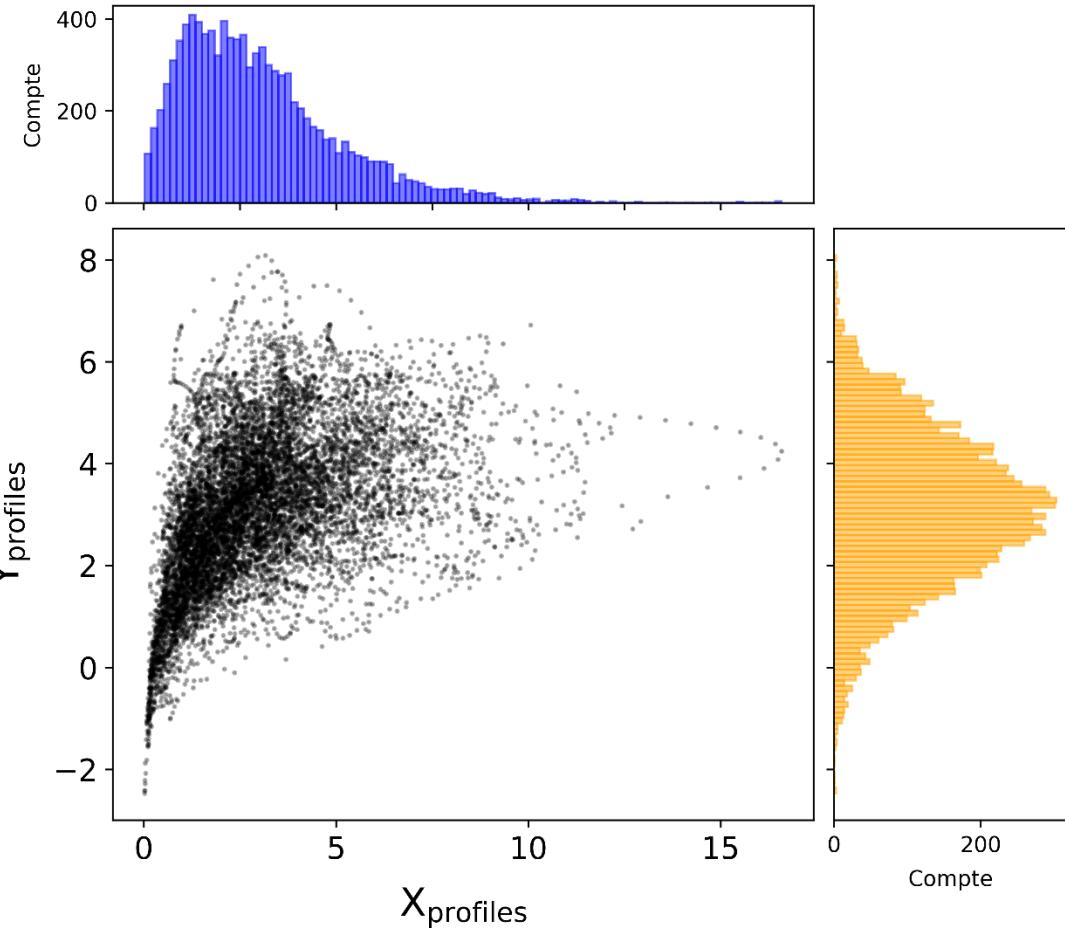


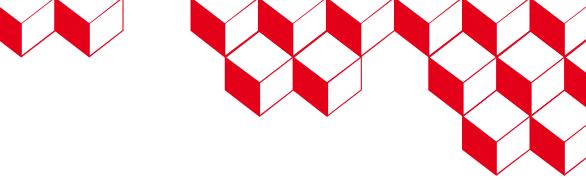


Génération des entrées fonctionnelles



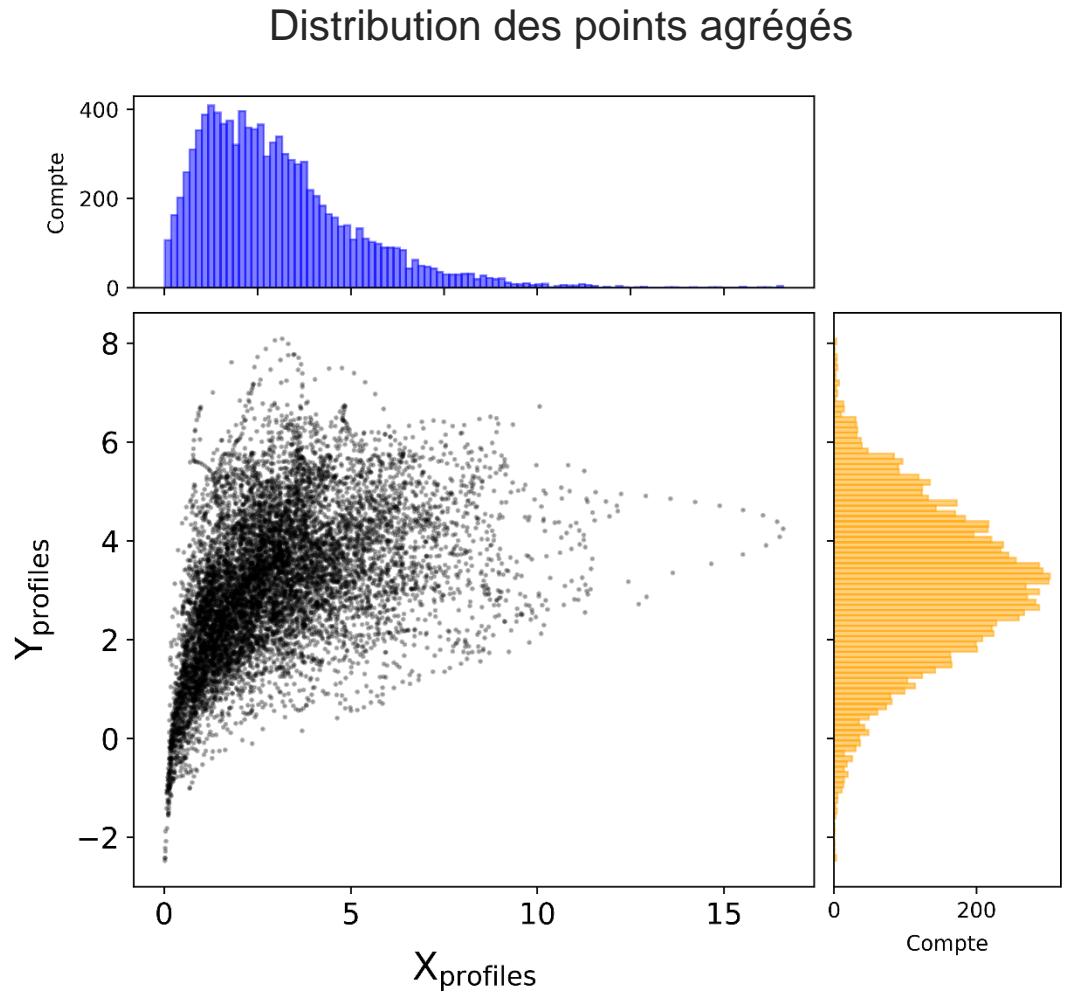
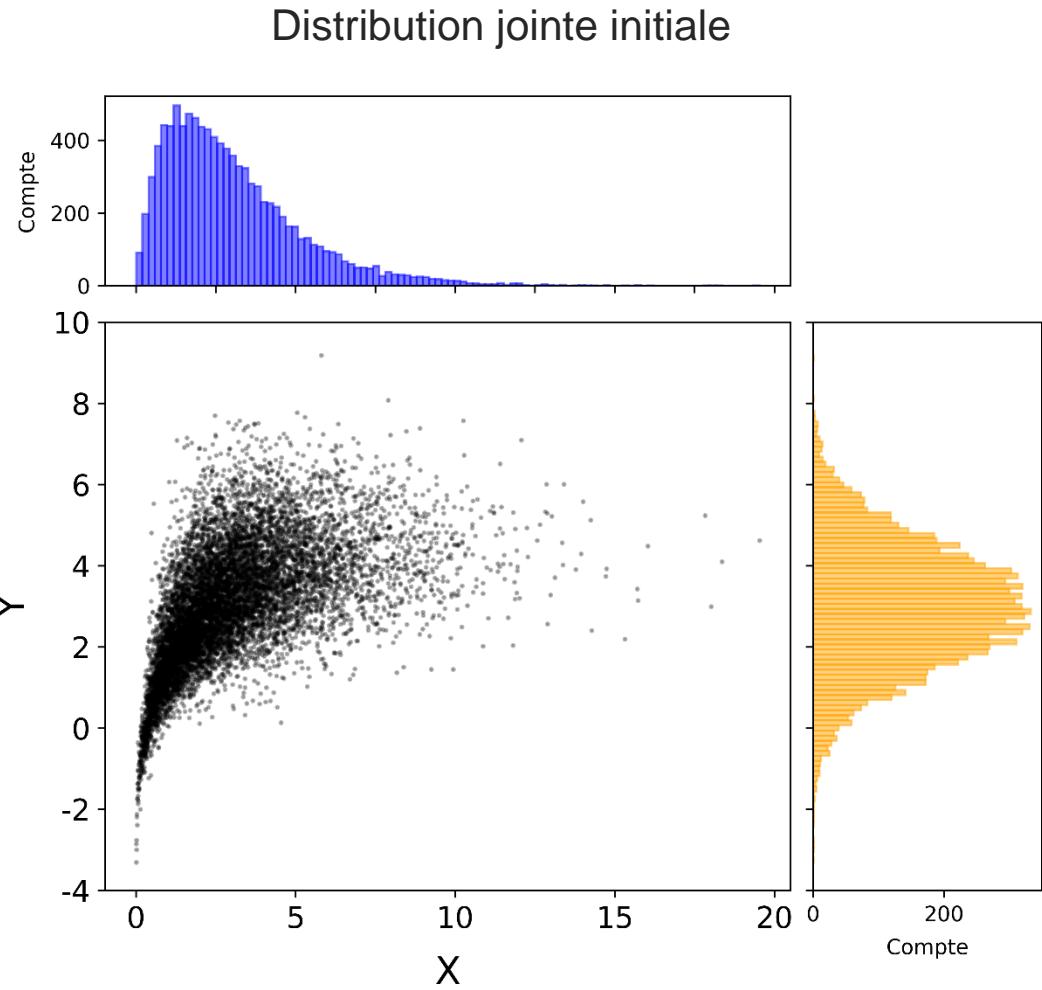
Distribution des points agrégés

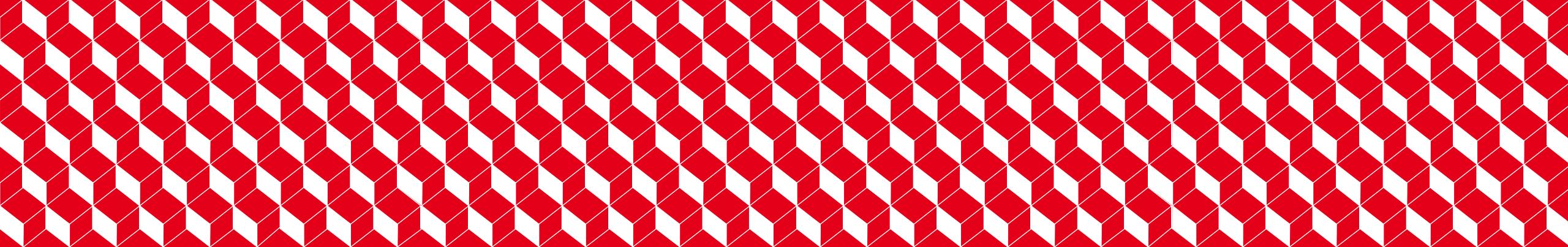




Génération des entrées fonctionnelles

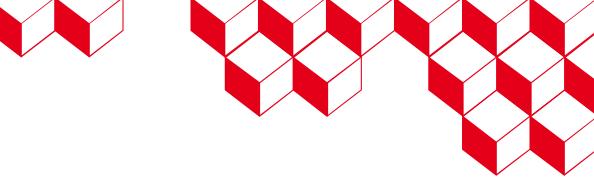
Comparaison des nuages de points



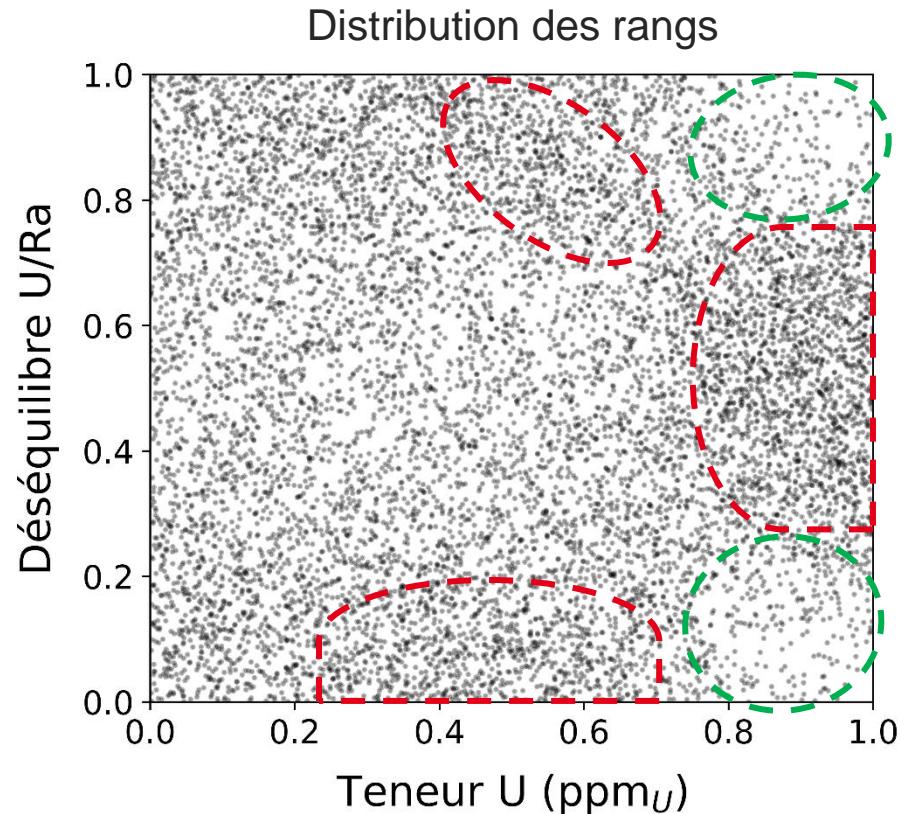
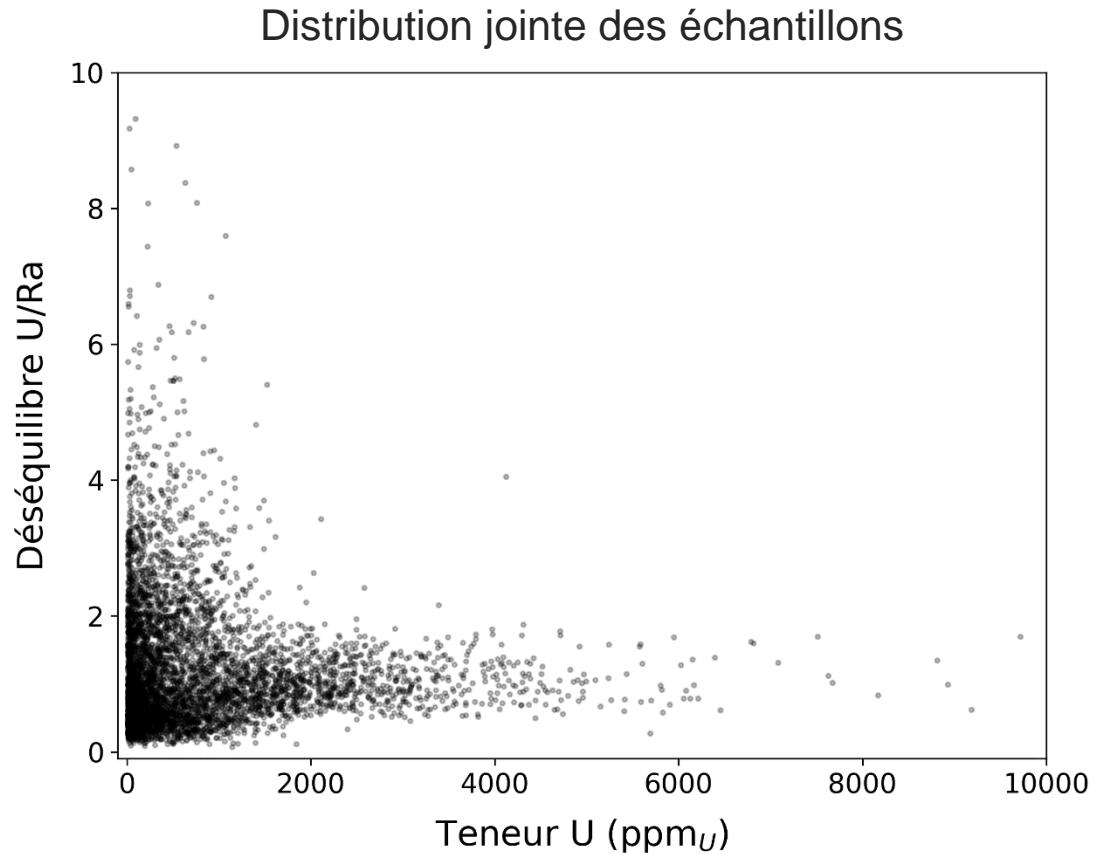


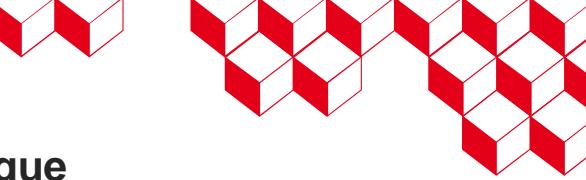
3

Génération d'entrée fonctionnelles liées par une copule de Bernstein



Relation Teneur U – Déséquilibre U/Ra





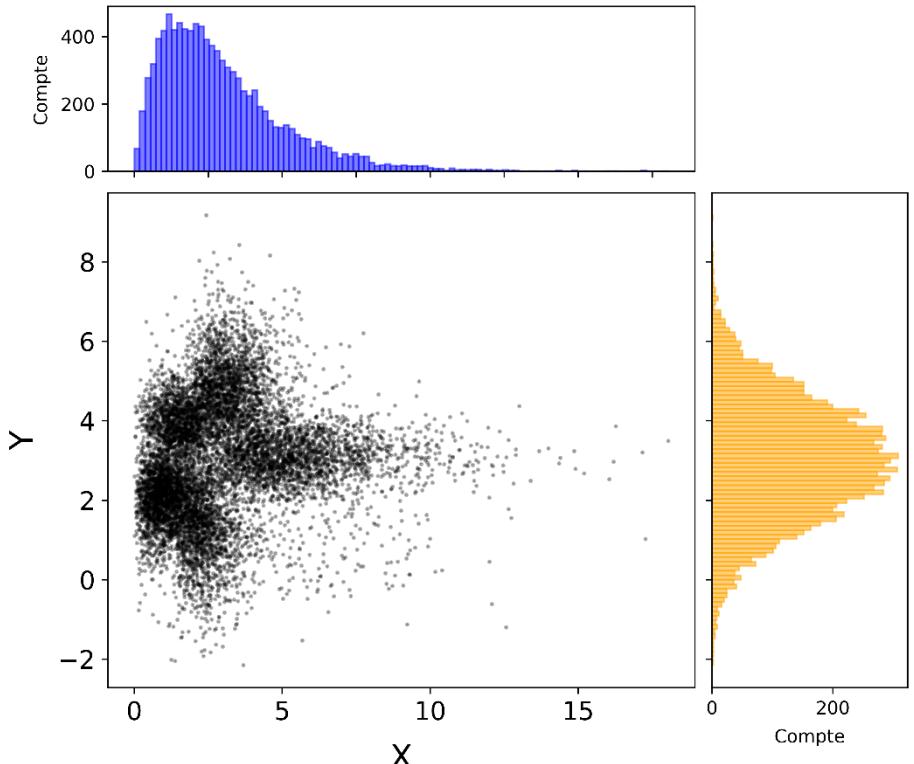
Copule de Bernstein

Copule non-paramétrique et lisse construite à partir de la copule empirique

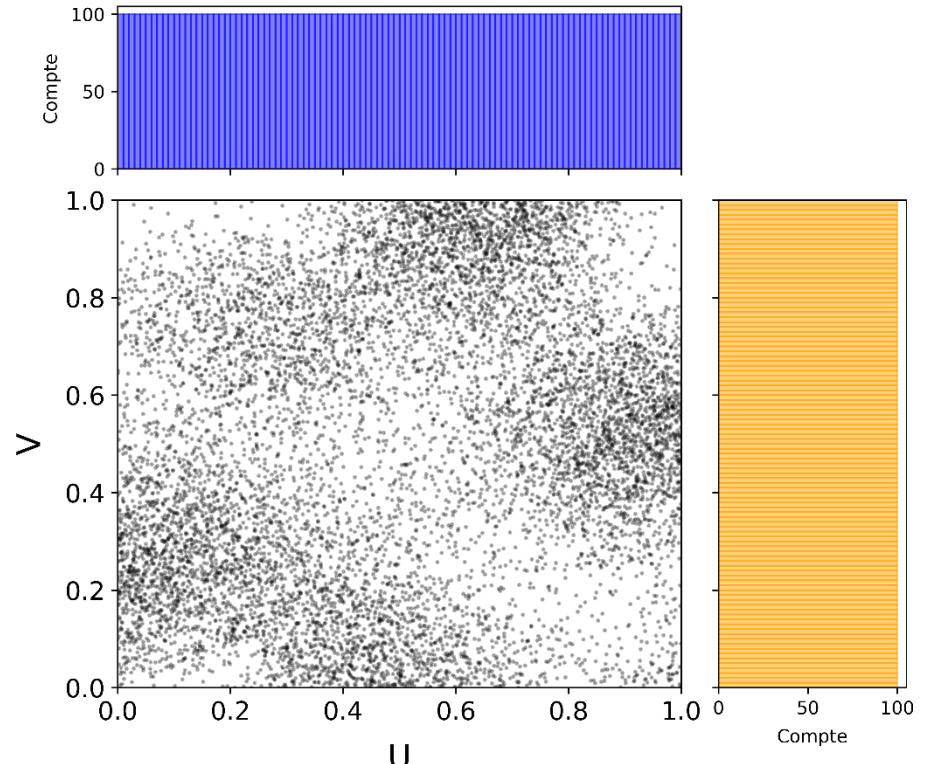
$$\forall (u, v) \in [0,1]^2, C_m(u, v) = \sum_{i=1}^m \sum_{j=1}^m \pi_{ij} B_{i,m}(u) B_{j,m}(v)$$

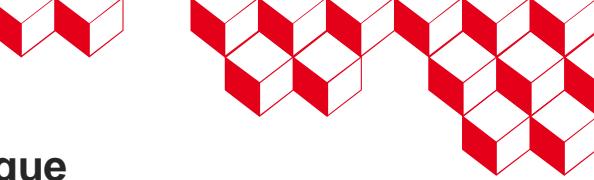
- avec :
- $B_{i,m}(u) = F_{Beta(i,m-i+1)}(u)$
 - $\sum_{i=1}^m \pi_{ij} = \frac{1}{m}, \sum_{j=1}^m \pi_{ij} = \frac{1}{m}, \sum_{i=1}^m \sum_{j=1}^m \pi_{ij} = 1$

Distribution jointe



Distribution des rangs





Copule de Bernstein

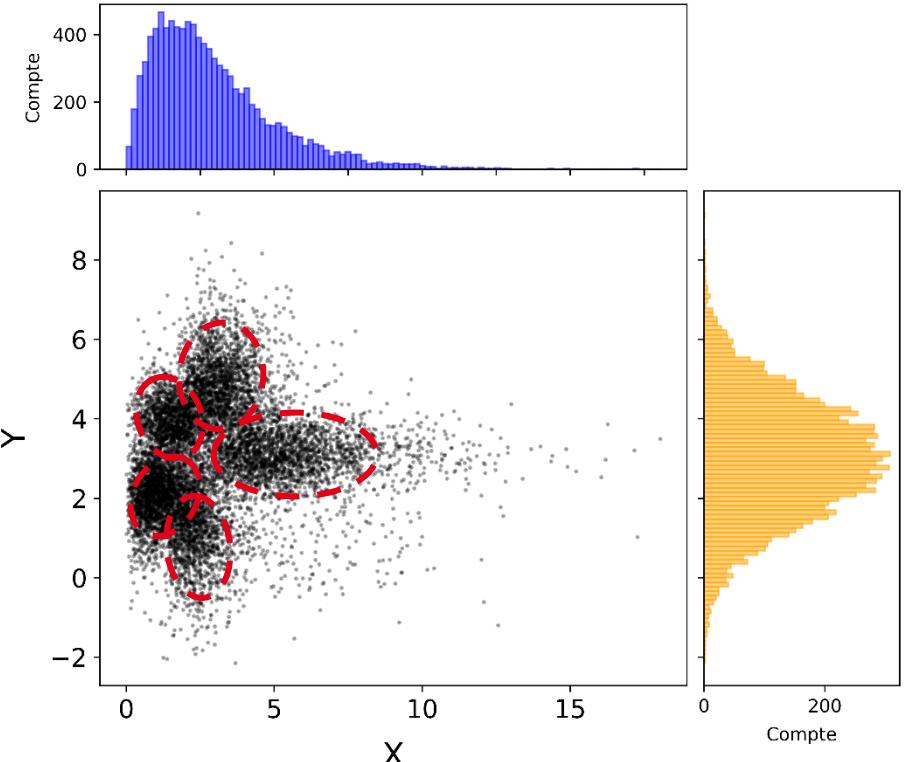
Copule non-paramétrique et lisse construite à partir de la copule empirique

$$\forall (u, v) \in [0,1]^2, C_m(u, v) = \sum_{i=1}^m \sum_{j=1}^m \pi_{ij} B_{i,m}(u) B_{j,m}(v)$$

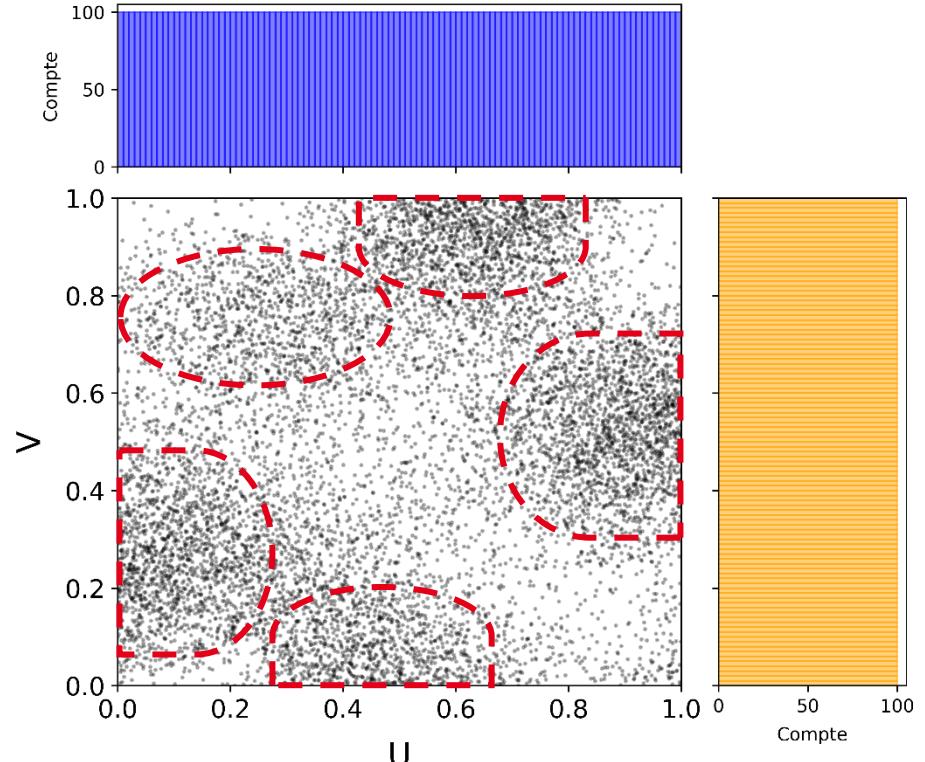
avec :

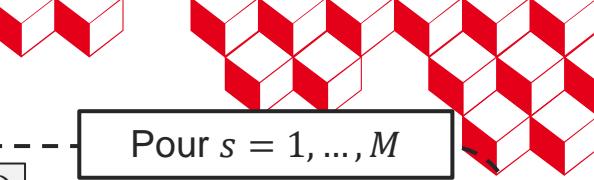
- $B_{i,m}(u) = F_{Beta(i,m-i+1)}(u)$
- $\sum_{i=1}^m \pi_{ij} = \frac{1}{m}, \sum_{j=1}^m \pi_{ij} = \frac{1}{m}, \sum_{i=1}^m \sum_{j=1}^m \pi_{ij} = 1$

Distribution jointe



Distribution des rangs



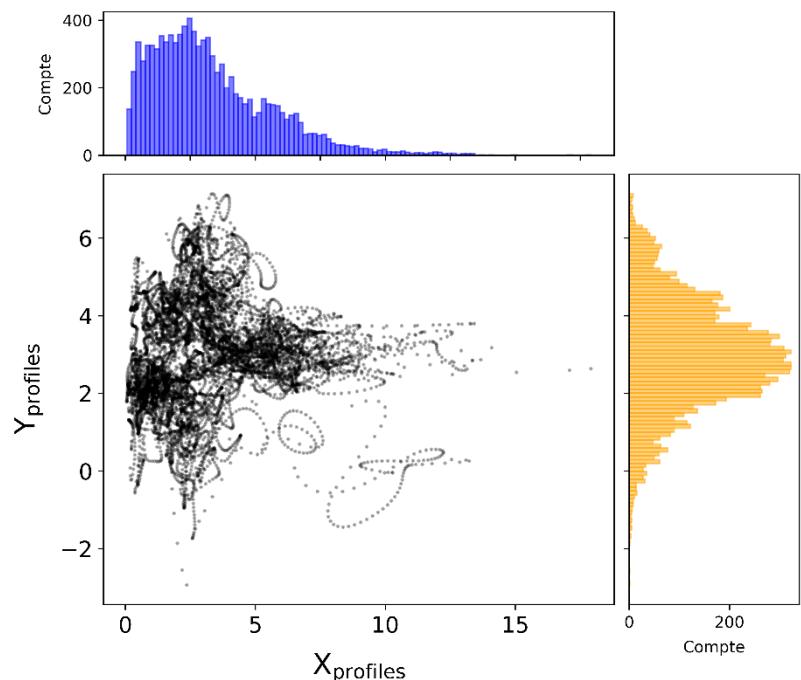
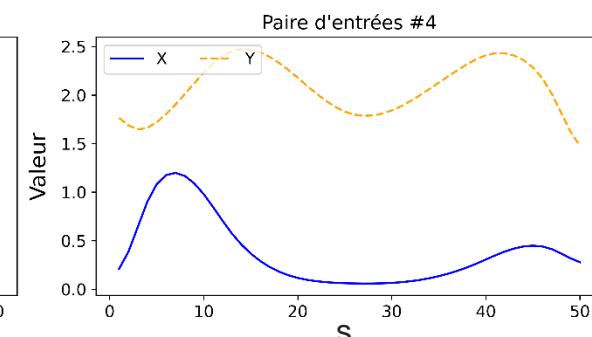
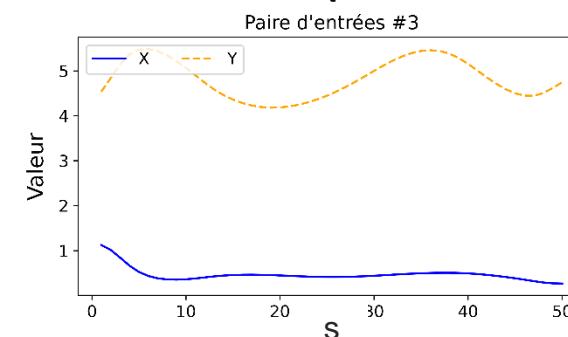
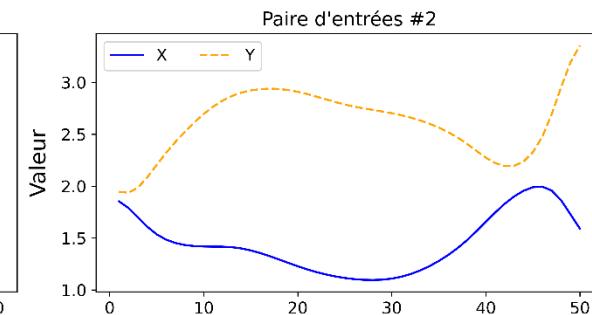
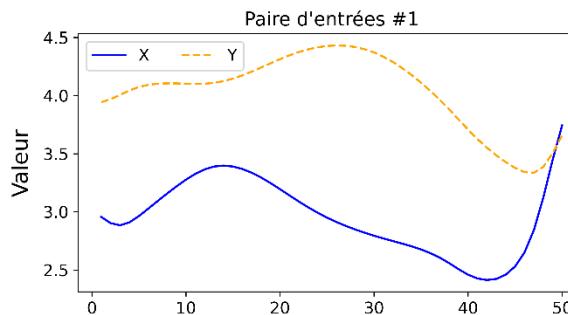
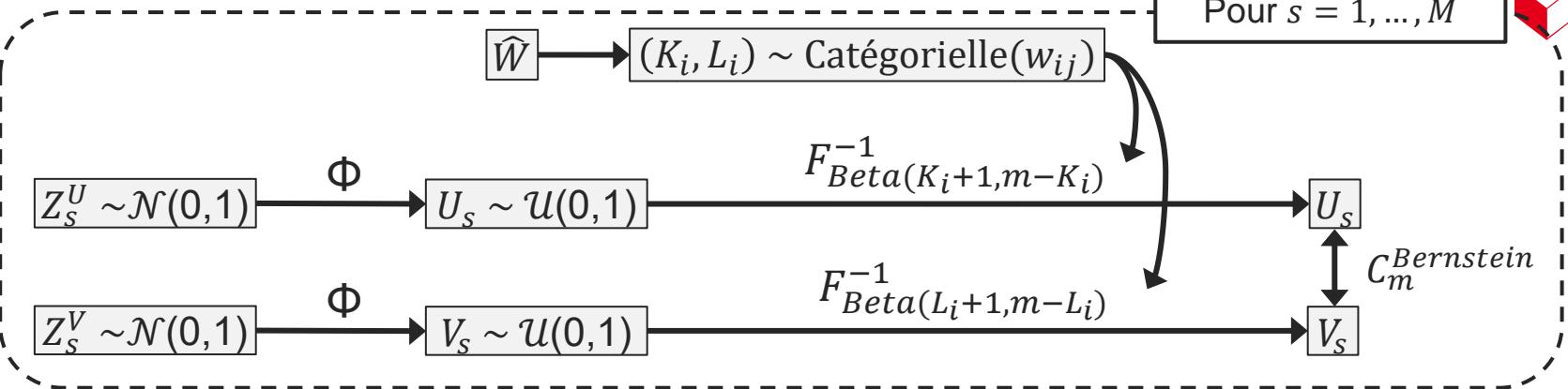


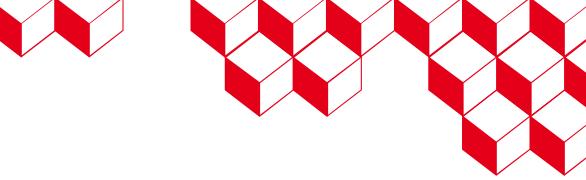
Génération des entrées fonctionnelles

Tirage aléatoire

$$\boldsymbol{\beta}^U \sim \mathcal{N}(0, I_m) \rightarrow Z^W \sim \mathcal{N}(0, \tilde{\mathbf{B}}^T \tilde{\mathbf{B}})$$

$$\boldsymbol{\beta}^V \sim \mathcal{N}(0, I_m) \rightarrow Z^{E_1} \sim \mathcal{N}(0, \tilde{\mathbf{B}}^T \tilde{\mathbf{B}})$$

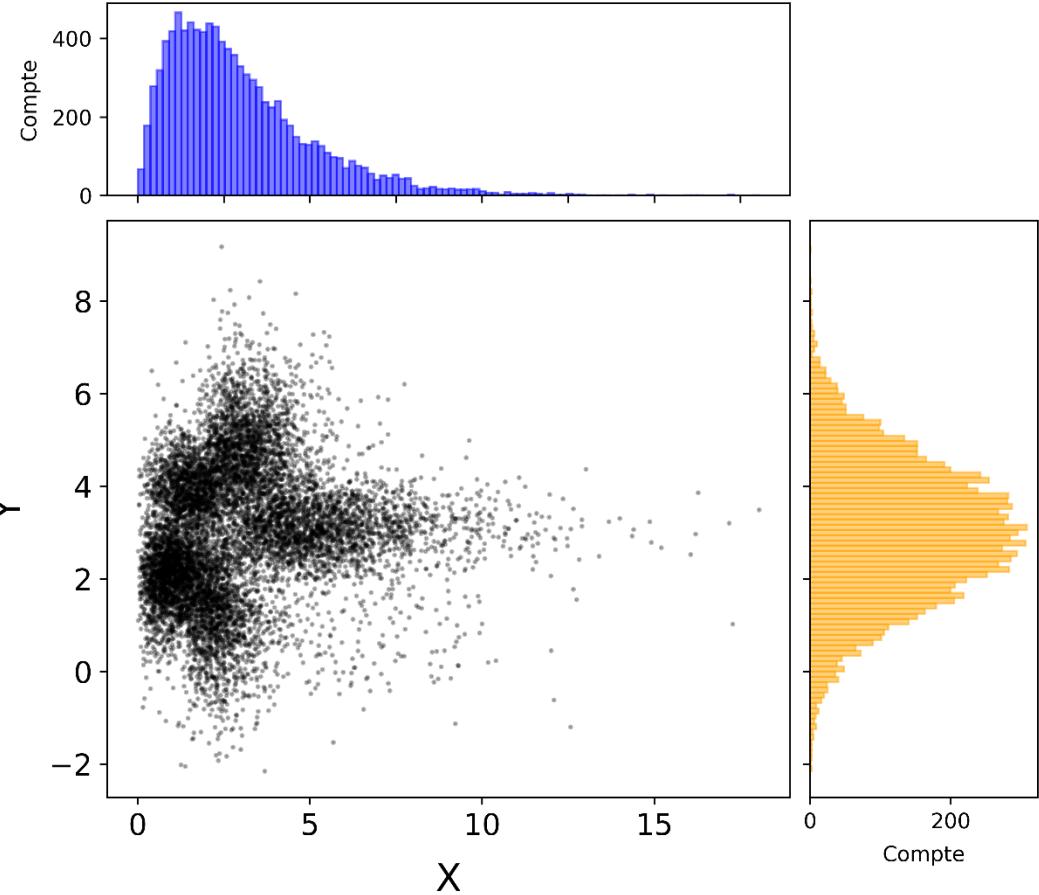




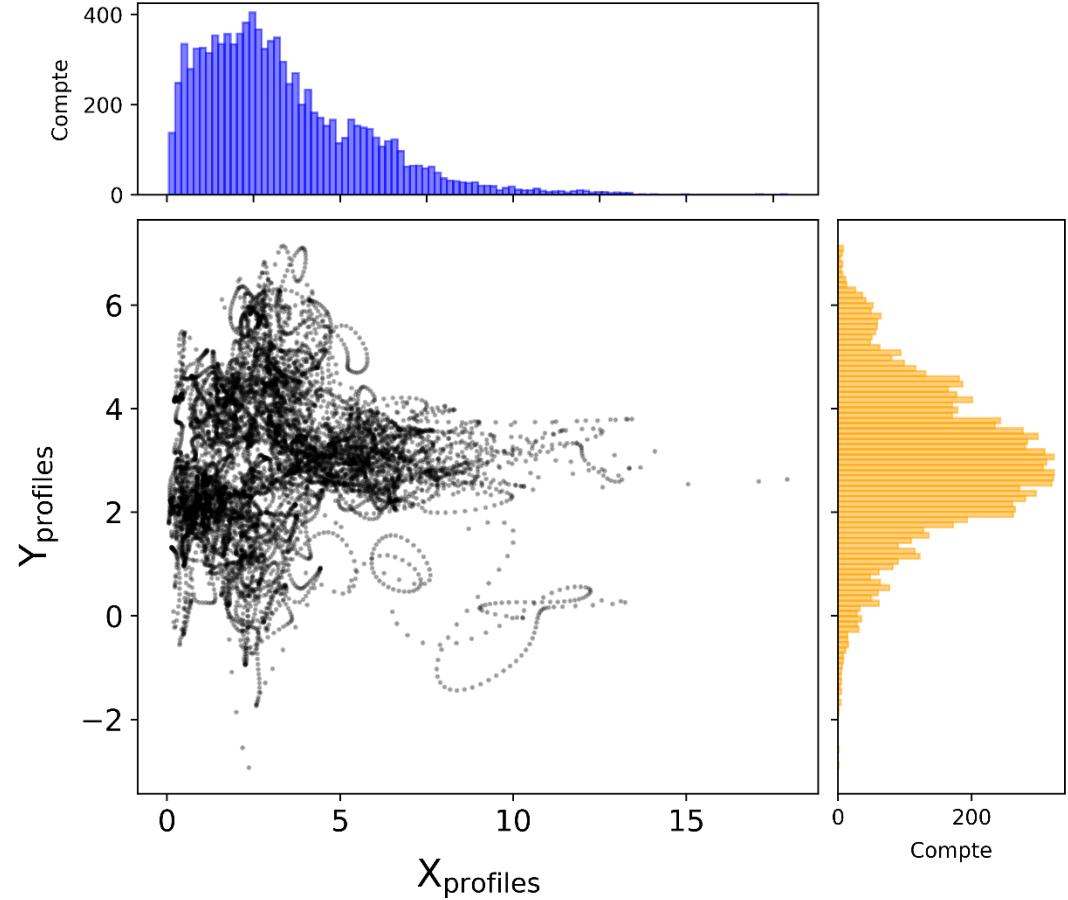
Génération des entrées fonctionnelles

Comparaison des nuages de points

Distribution jointe initiale



Distribution des points agrégés

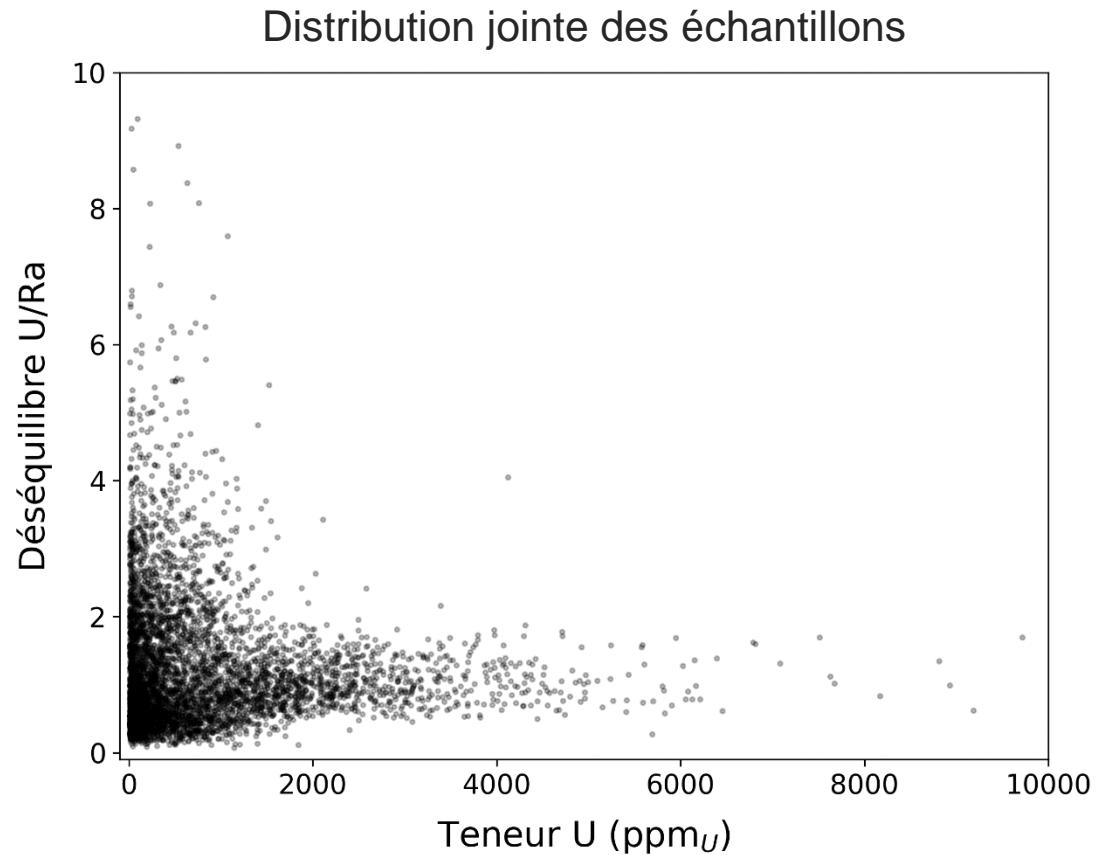


5

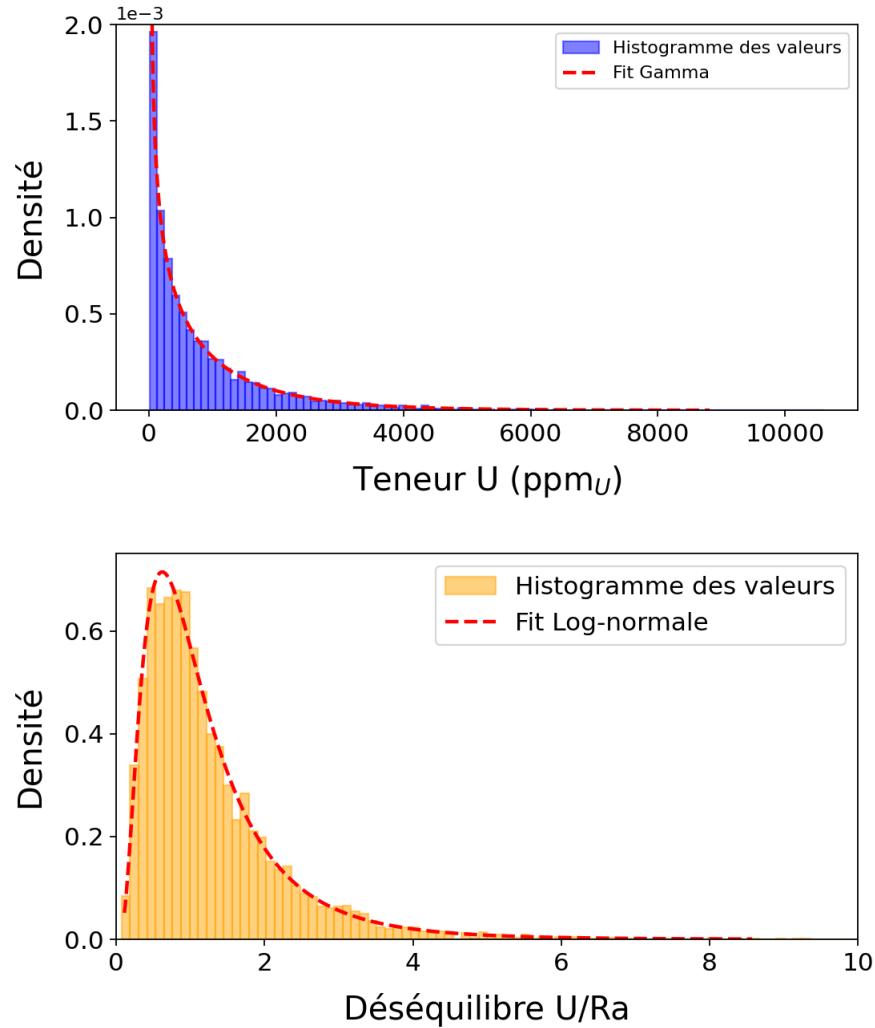
■ Plan d'expérience pour la génération de spectres gamma en puits de forage

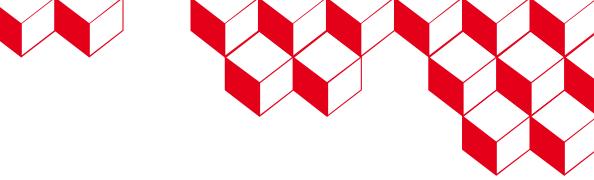


Relation Teneur U – Déséquilibre U/Ra

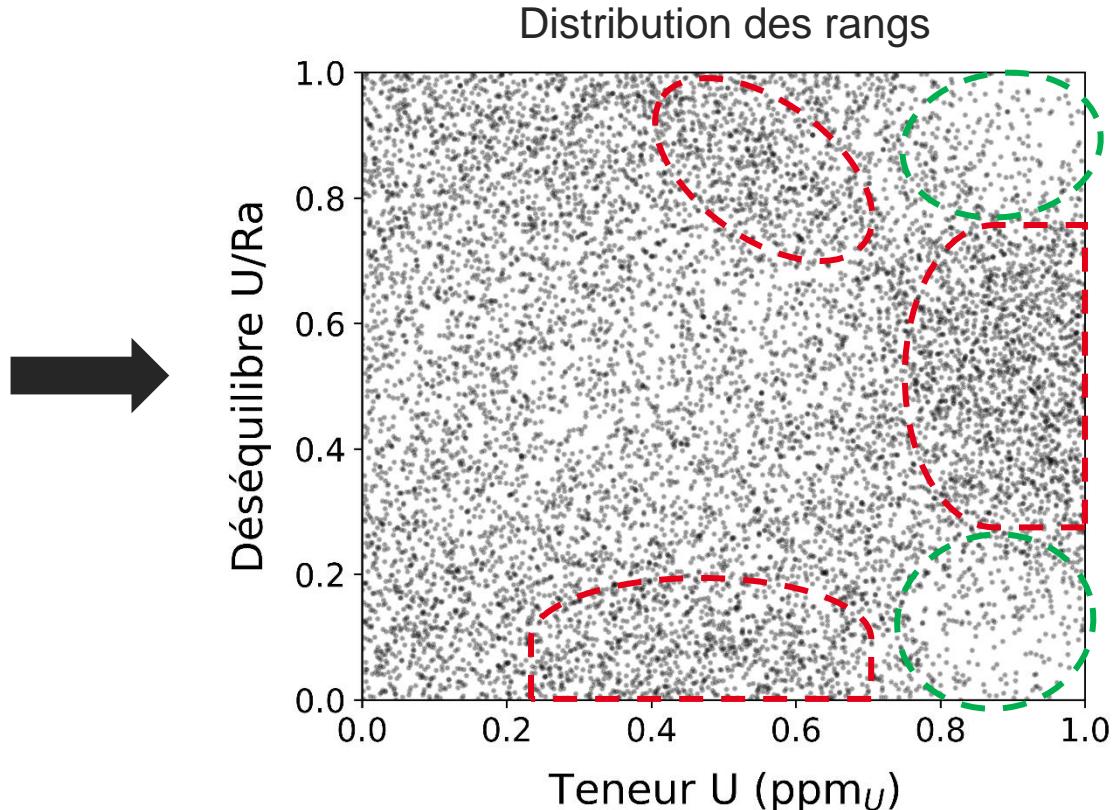
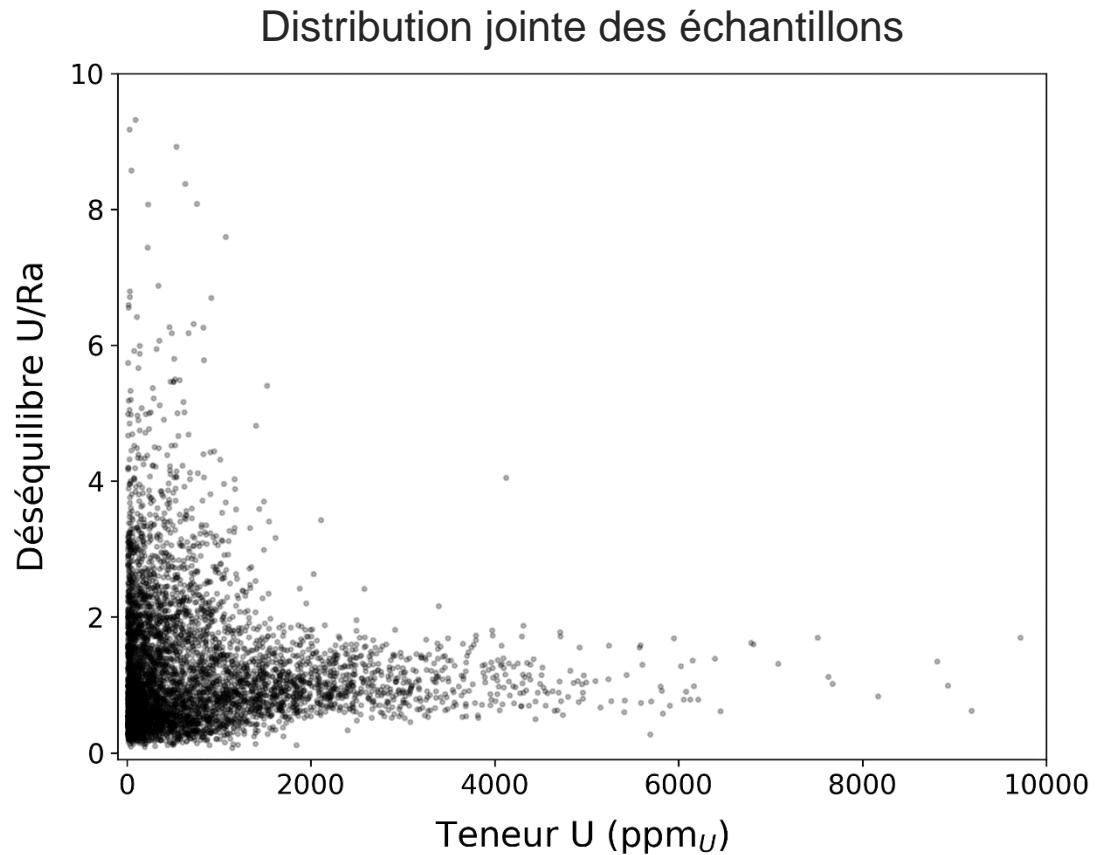


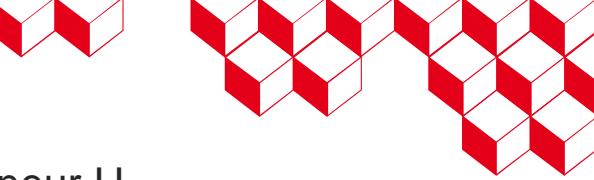
Distributions marginales empiriques et lois ajustées



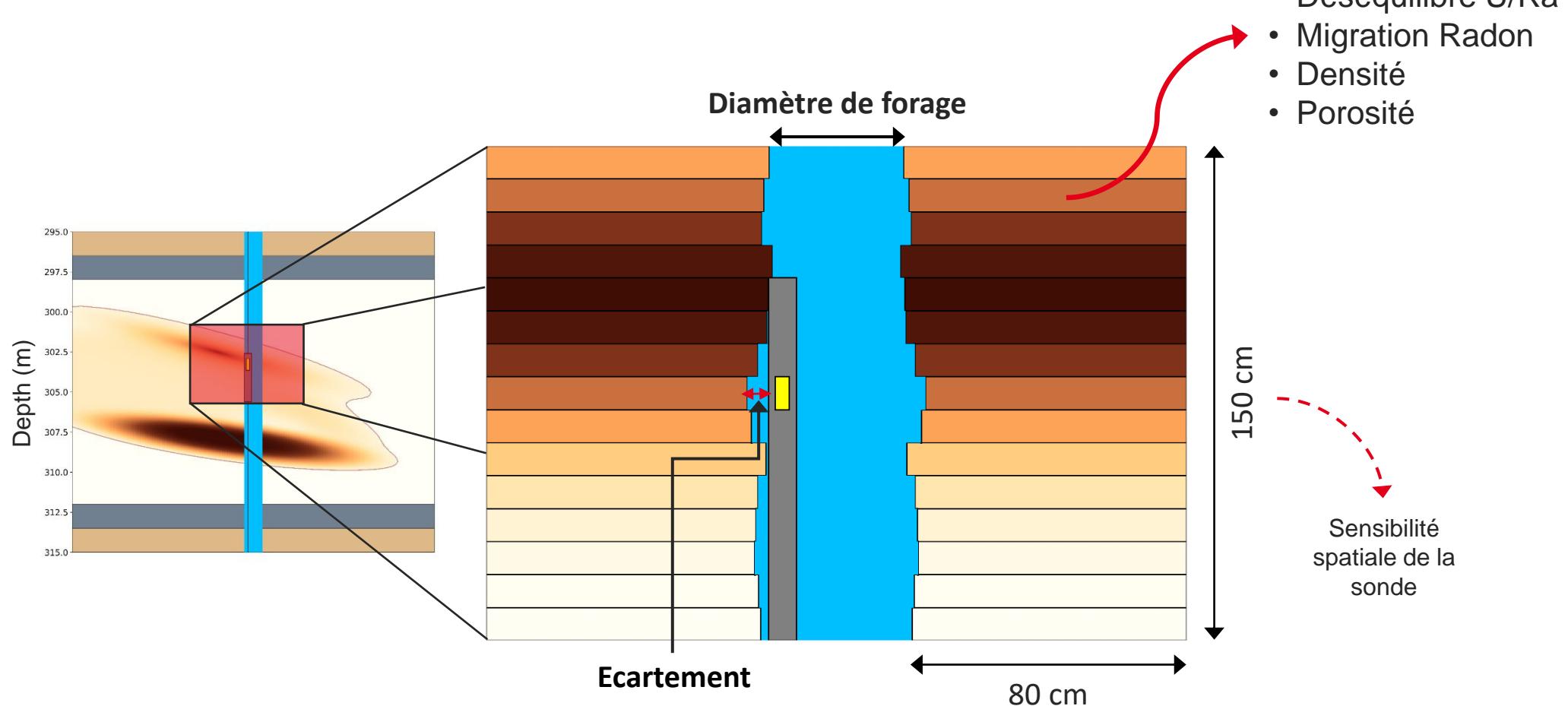


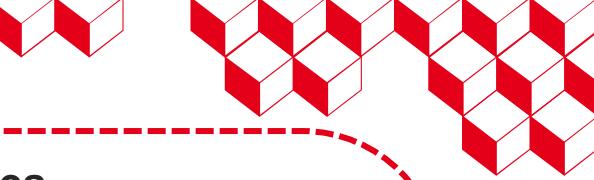
Relation Teneur U – Déséquilibre U/Ra





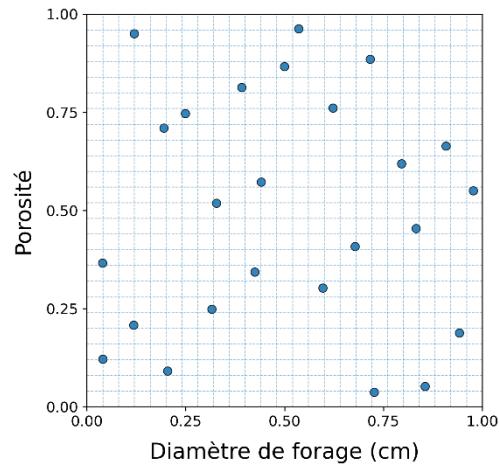
Autres variables d'entrée



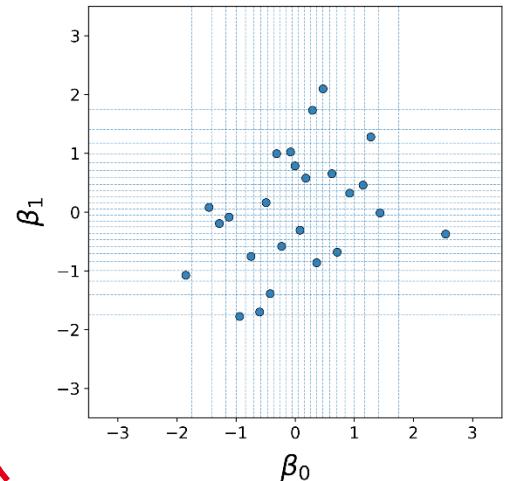


Plan d'expérience

Entrées scalaires

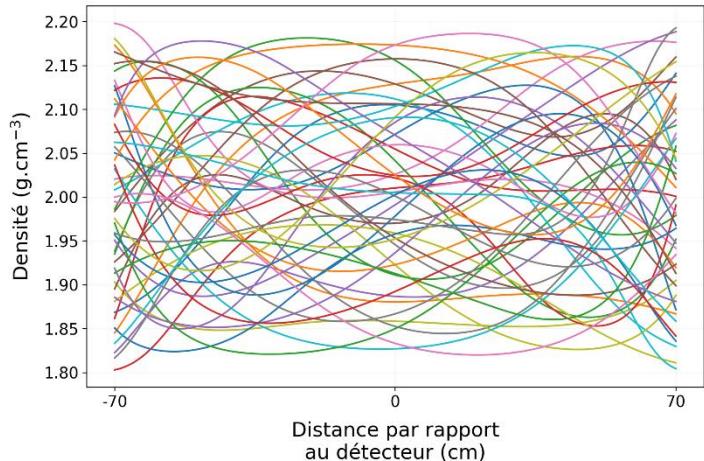


Optimisation de l'hypercube latin sur les coefficients β par recuit simulé



$$d_{ij} = \|f(\boldsymbol{\beta}^i) - f(\boldsymbol{\beta}^j)\|_{L_2}$$

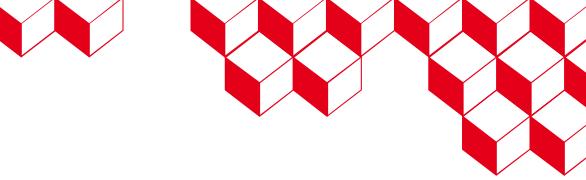
$$\phi_q = (\sum_{ij} (d_{ij})^{-q})^{1/q} [1,2]$$



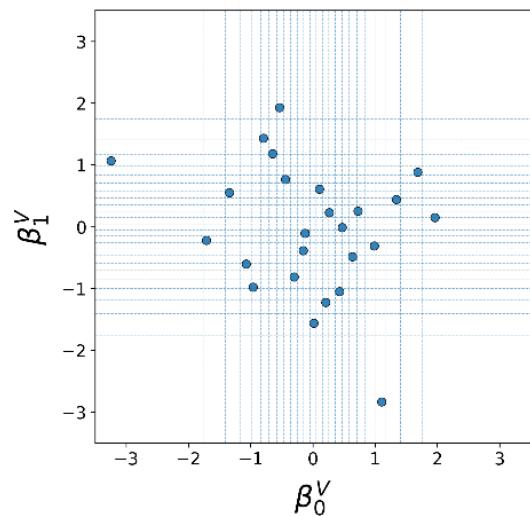
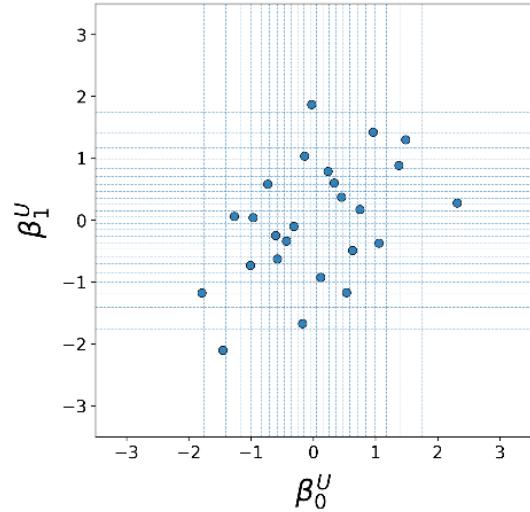
Appariement optimal avec optimisation de l'ensemble joint par recuit simulé

$$d_{ij}^{joint} = \left(\|x^{\pi(i)} - x^{\pi(j)}\|_2^2 + \|f(\boldsymbol{\beta}^{\pi(i)}) - f(\boldsymbol{\beta}^{\pi(j)})\|_{L_2}^2 \right)^{1/2}$$

$$\phi_q = (\sum_{ij} (d_{ij}^{joint})^{-q})^{1/q} [1,2]$$



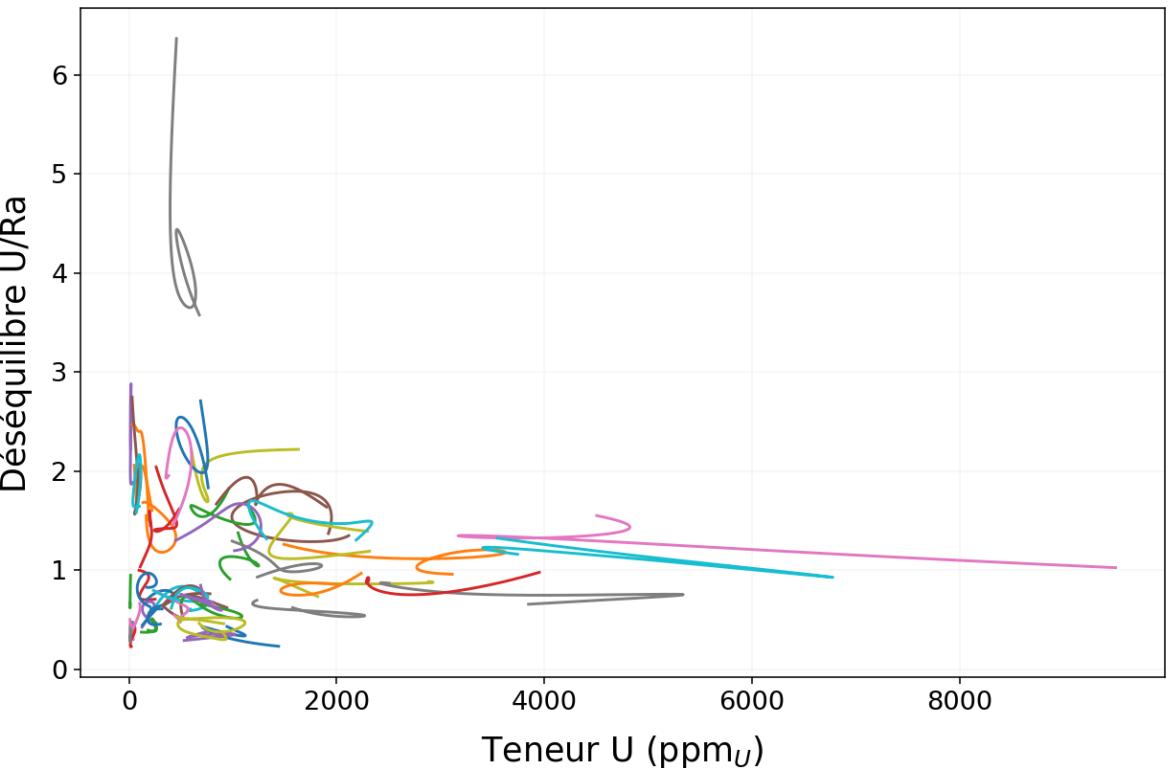
Plan d'expérience

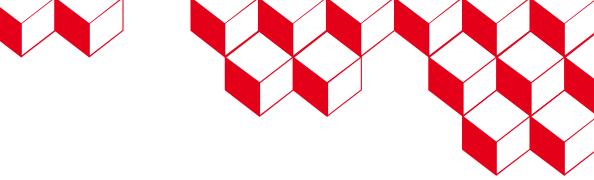


Optimisation séparée des hypercubes latins selon le même principe

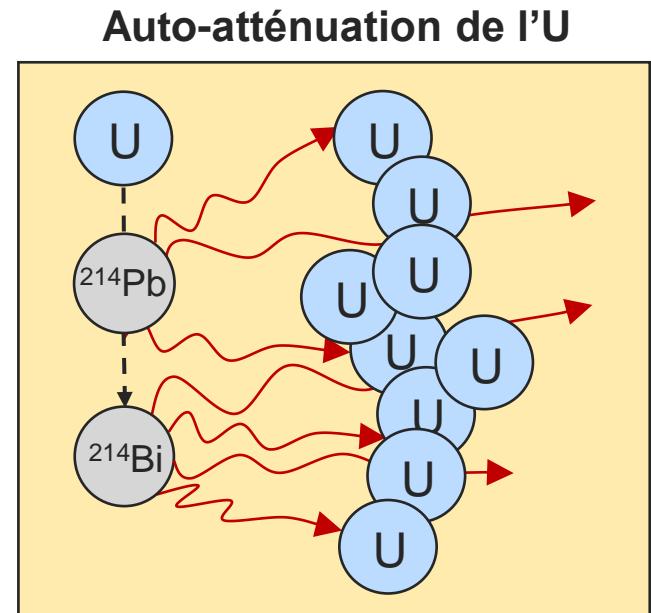
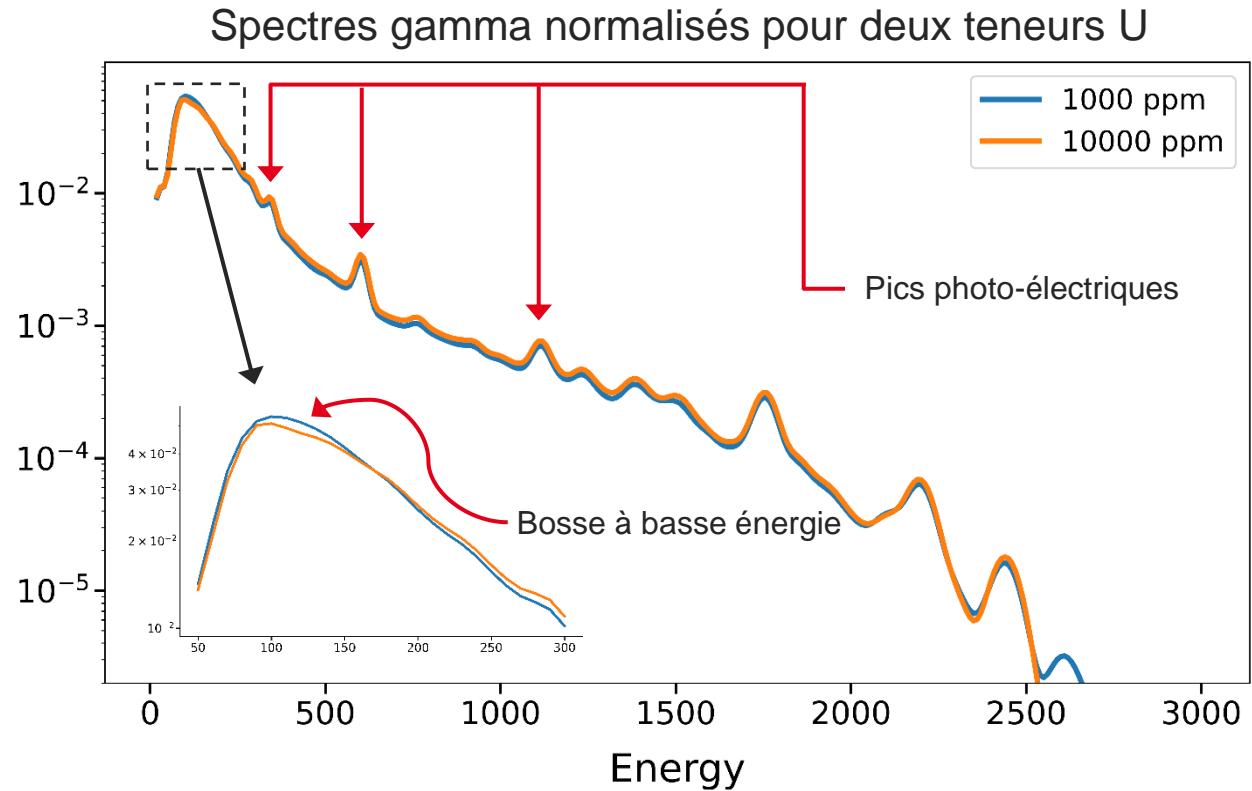


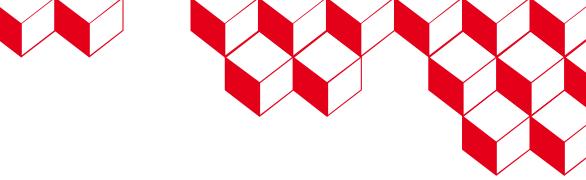
Entrées fonctionnelles dépendantes





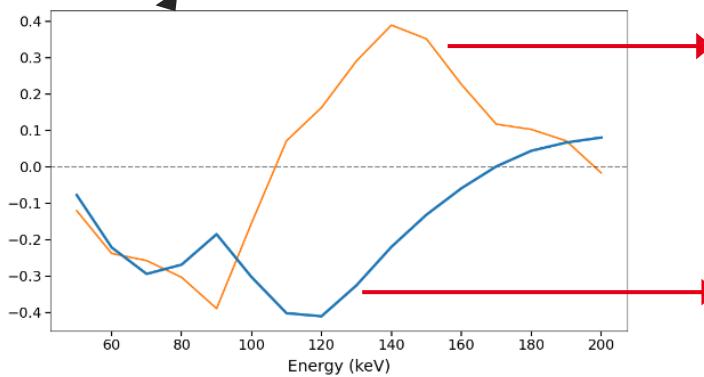
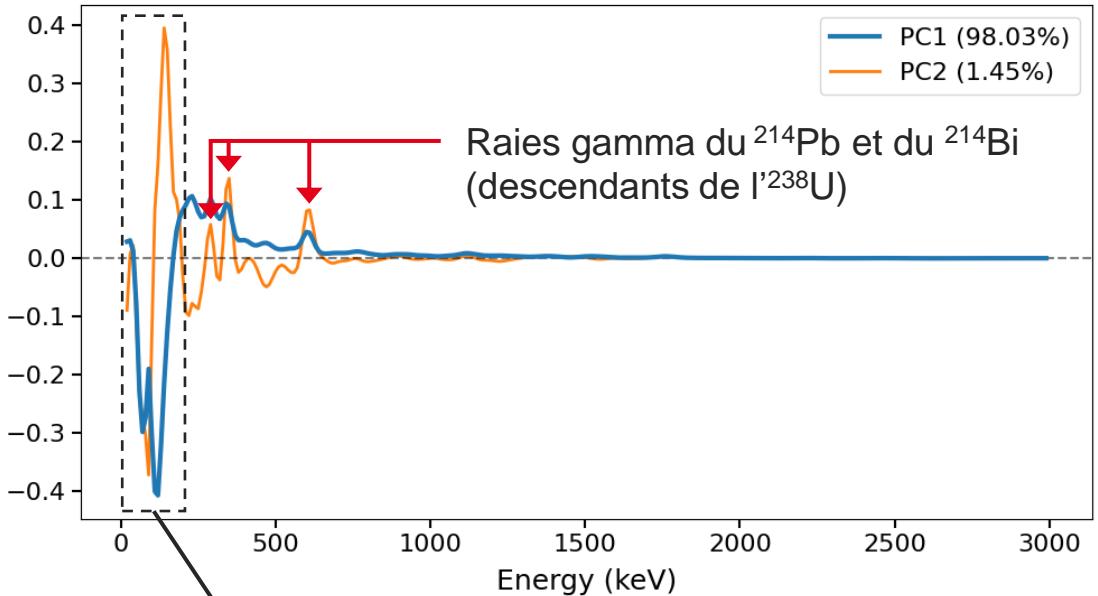
Sorties fonctionnelles : spectres gamma





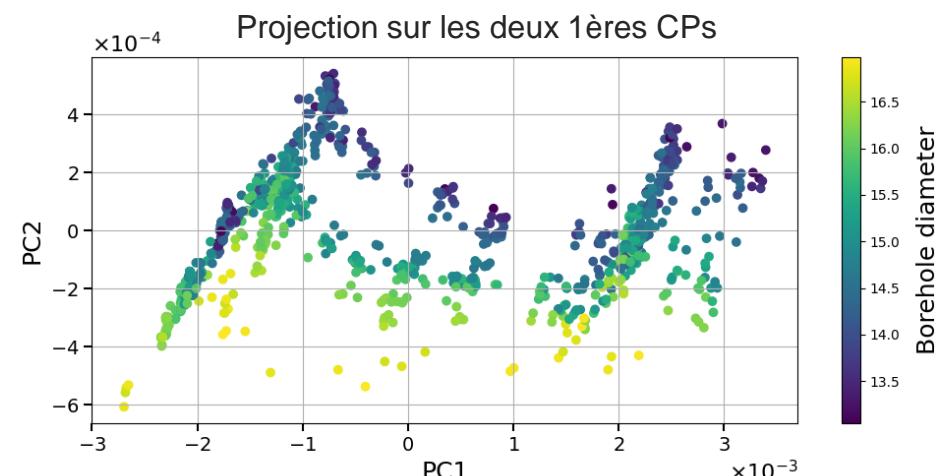
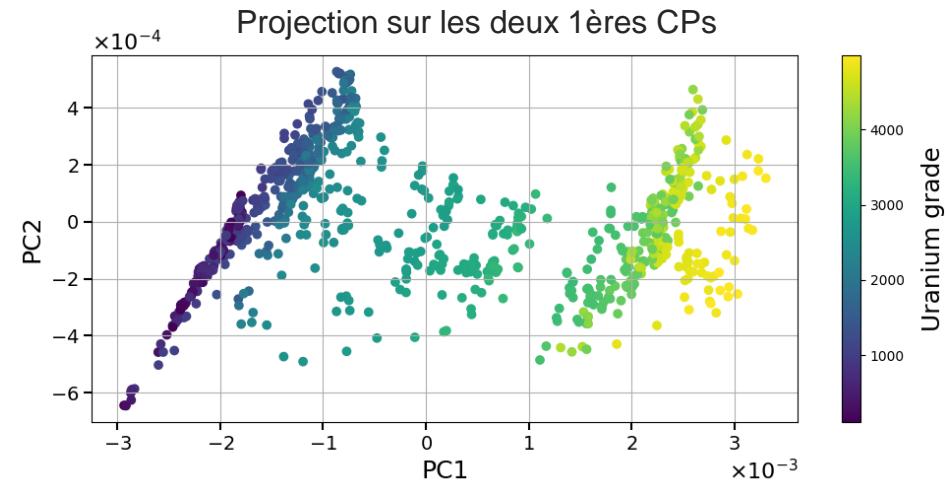
Sorties fonctionnelles : décomposition par ACP

Deux premières composantes principales



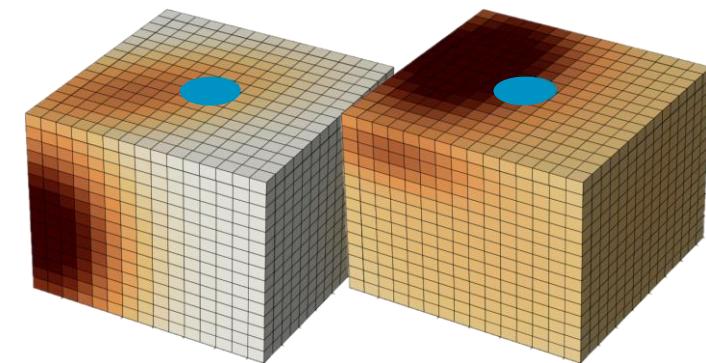
Déformation de la bosse à basse énergie

Auto-atténuation de l'uranium à basse énergie



Conclusions et perspectives

- Méthode de génération d'entrées fonctionnelles dépendantes pour différentes copules
 - Décomposition sur une base de B-splines standardisées naturellement adaptée à la définition d'un plan d'expérience
 - Choix de la dimension de la base fonctionnelle : complexité des courbes vs taille du plan d'expérience
- Plan d'expérience pour la génération de spectres gamma
 - Incorporation de la relation teneur U – déséquilibre U/Ra, modélisée par une copule de Bernstein
 - Limite 1 : loi jointe estimée sur un ensemble fini d'échantillons \Rightarrow propager l'incertitude liée à l'estimation des paramètres ?
 - Limite 2 : tendance des trajectoires à n'occuper qu'une partie restreinte de l'espace
 - Perspective : relation très haute teneur U – épaisseur du dépôt
- Perspective : entrées fonctionnelles dépendantes 3D



Merci pour votre attention