CIROQUO

Consortium Industrie Recherche pour l'Optimisation et la QUantification d'incertitude pour les données Onéreuses

Optimization under Uncertainty for Multi-Fidelity Black-box Simulators

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Guillaume Perrin Delphine Sinoquet

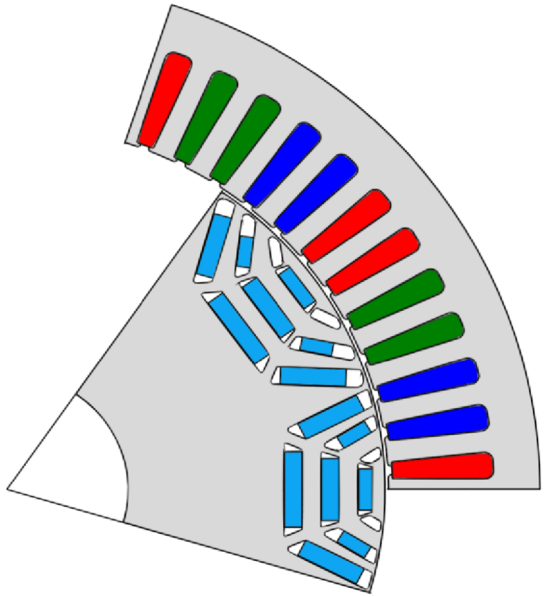
ETICS 2026 - Amboise

CENTRALE LYON

IFP Energies nouvelles

Université Gustave Eiffel

Motivation : Design of an electrical machine



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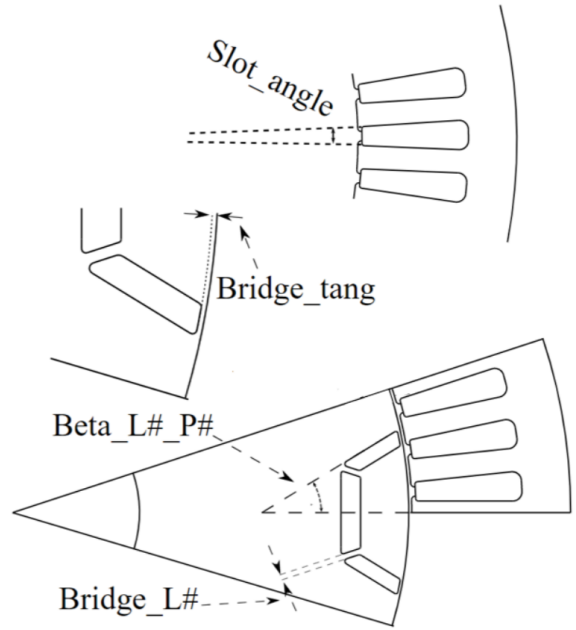

Figure: Design parameters to tune

Reyes-Reyes et al. [2022, 2024]

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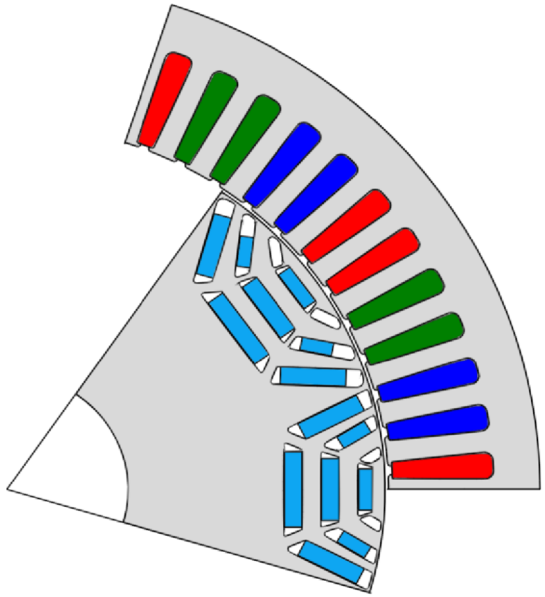

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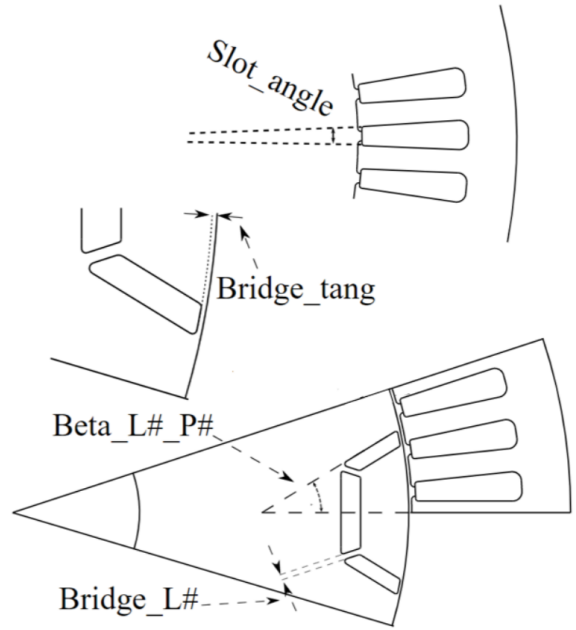

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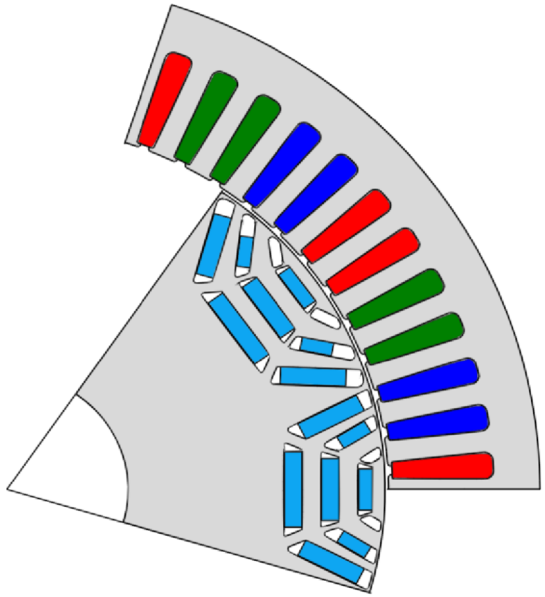

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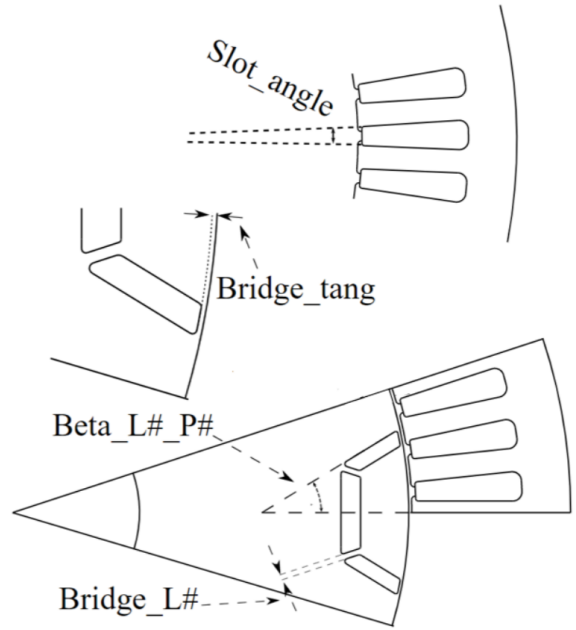

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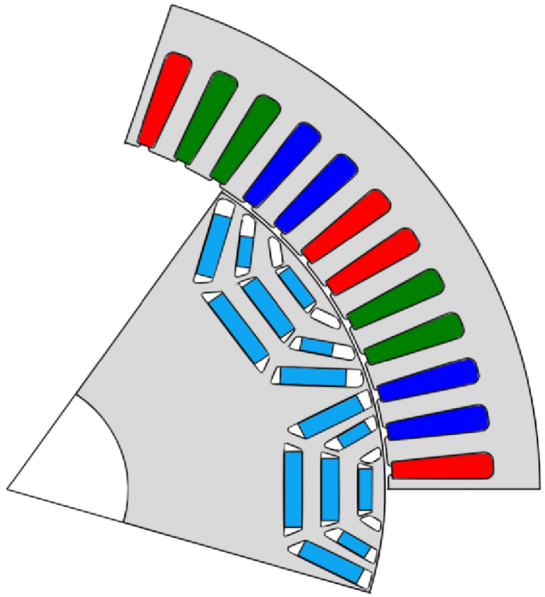

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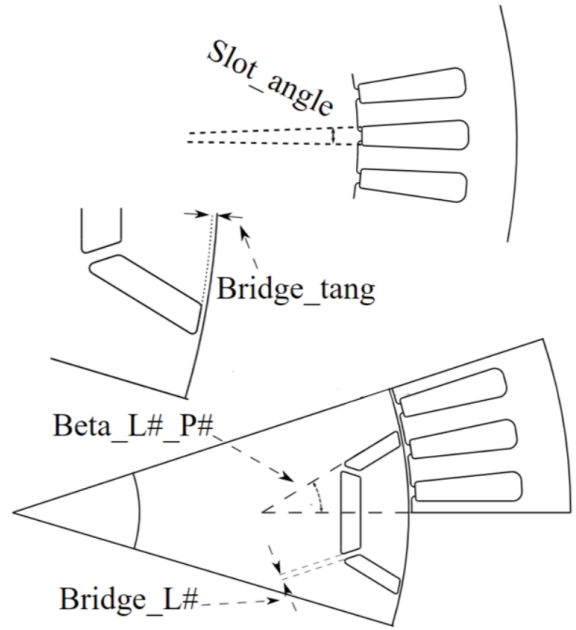

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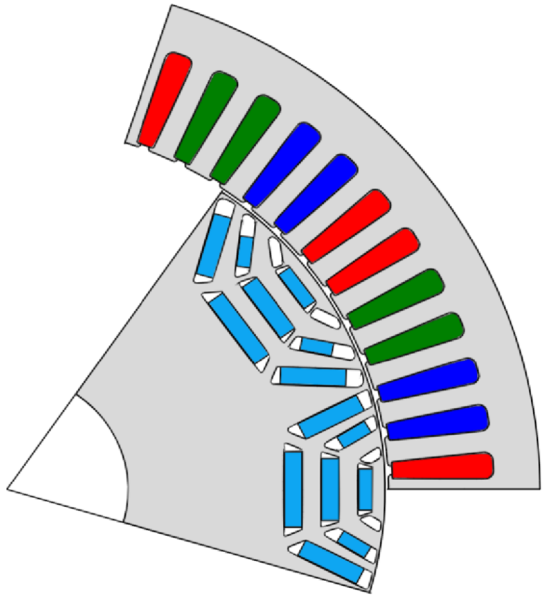

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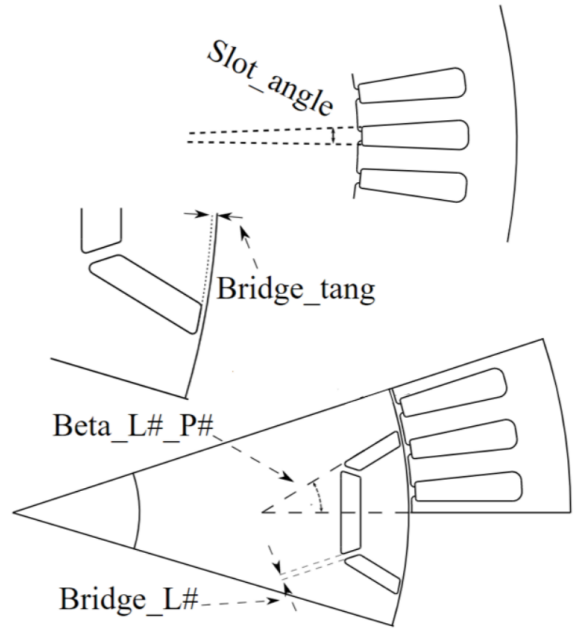

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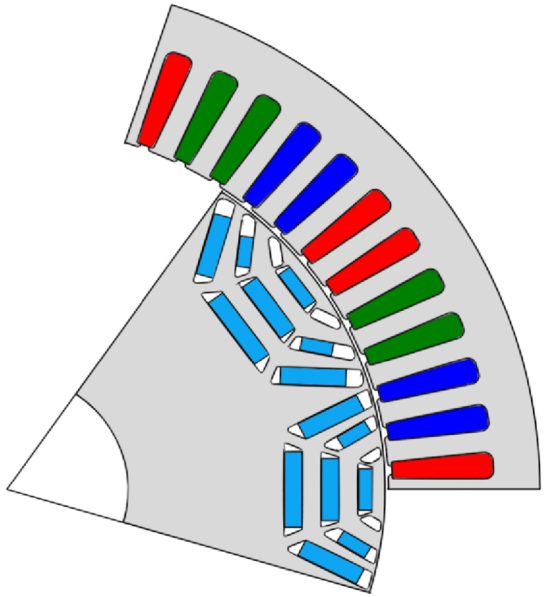

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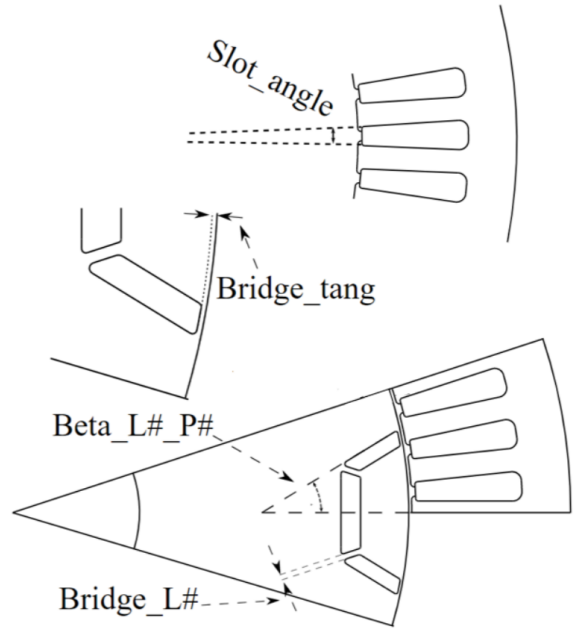

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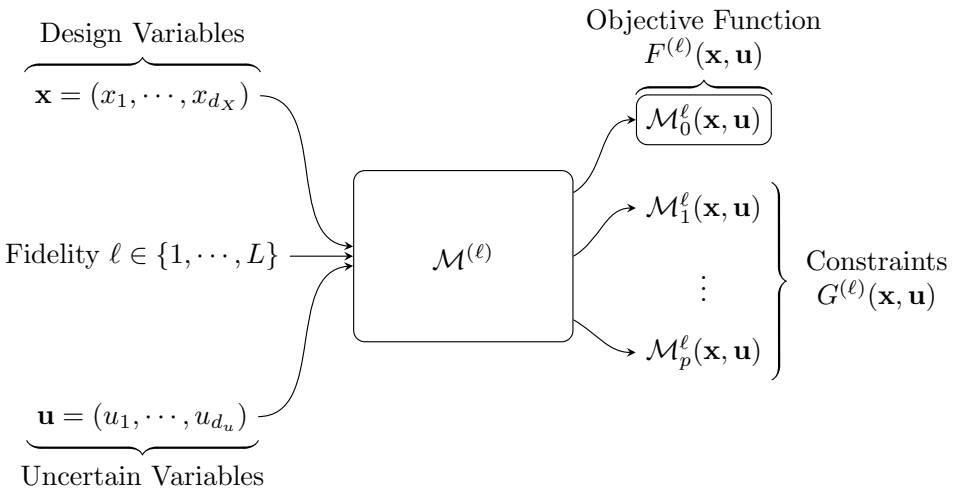
Our Optimization setting



- $\mathcal{M}$: A set of expensive codes

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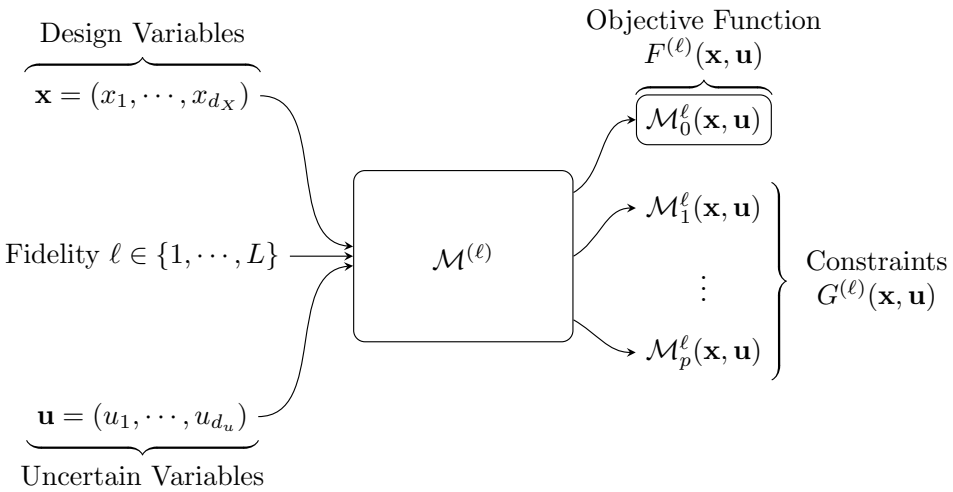

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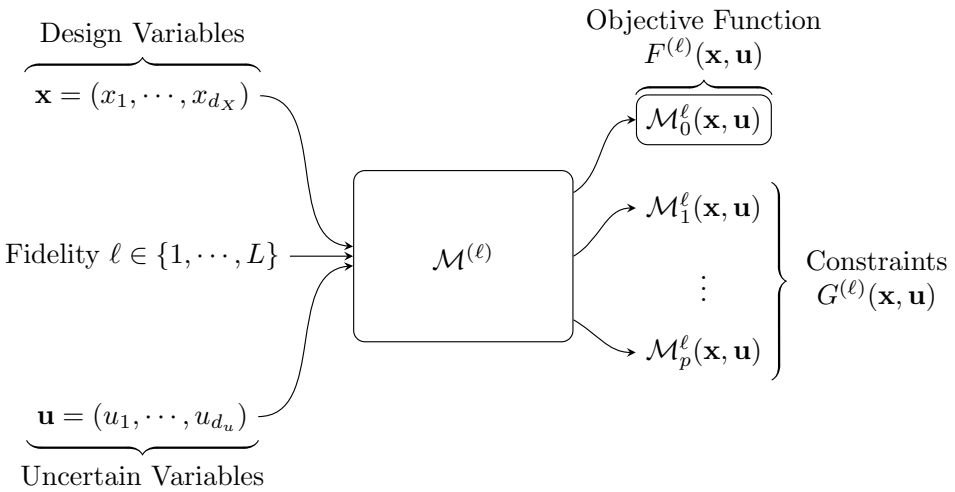

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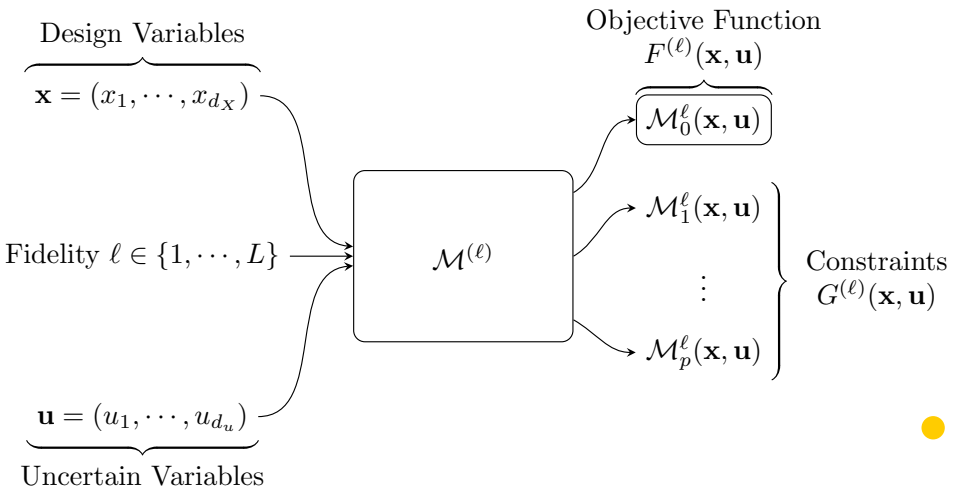

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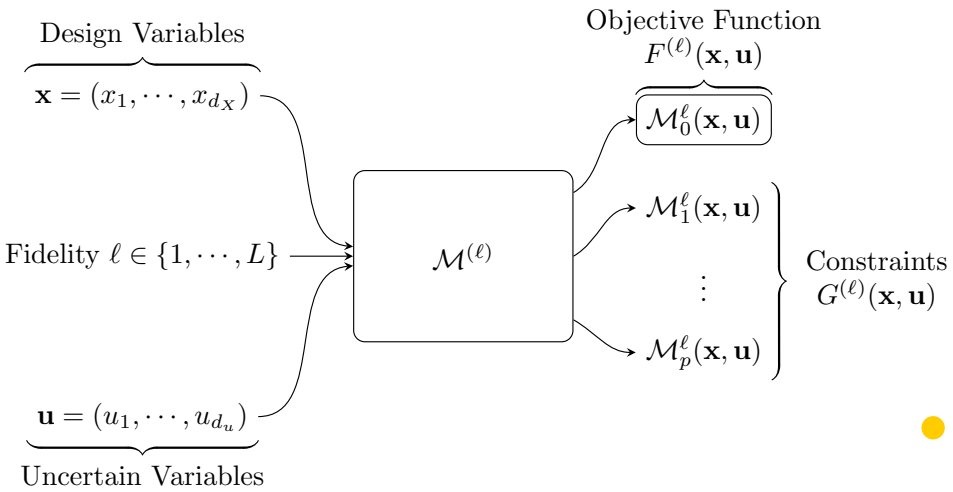

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Problem to solve

$$\underset{\boldsymbol{x} \in D_X}{\arg\min}\ \mathbb{E}\left[F^{(L)}(\boldsymbol{x}, \boldsymbol{U})\right] s.t. \mathbb{P}\left[G^{(L)}(\boldsymbol{x}, \boldsymbol{U}) \leq 0\right] \geq 1 - \alpha$$

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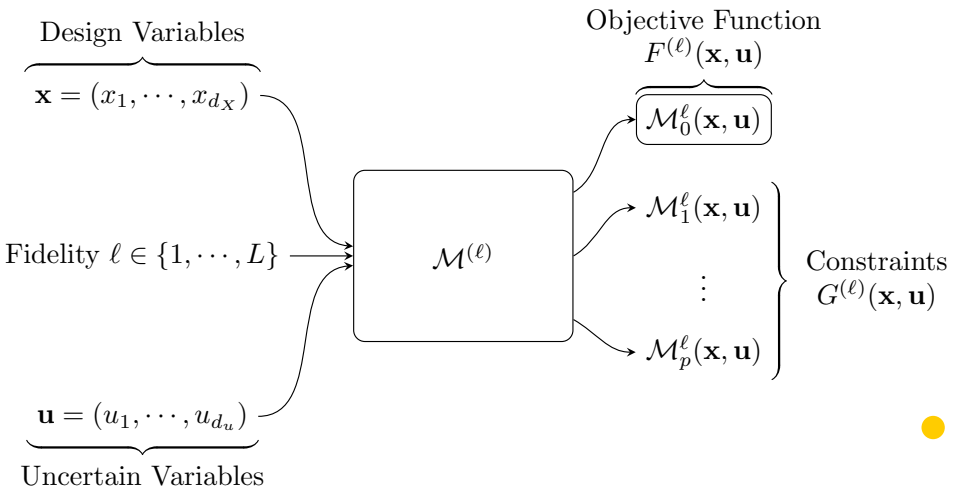

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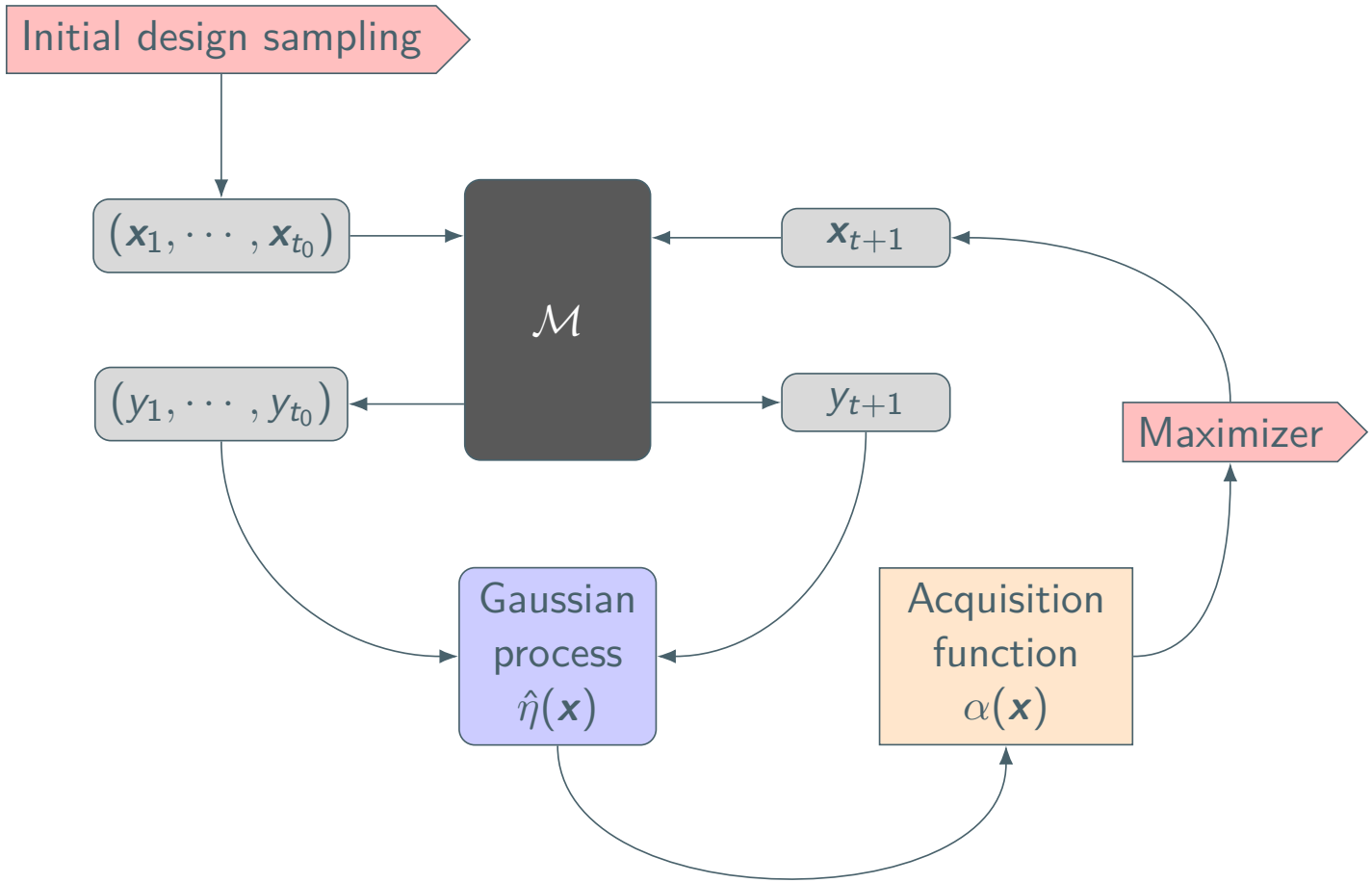
Problem to solve for now

$$\arg\min_{\boldsymbol{x} \in D_X} \mathbb{E}\left[F^{(L)}(\boldsymbol{x}, \boldsymbol{U})\right]$$

Recall : Bayesian optimization



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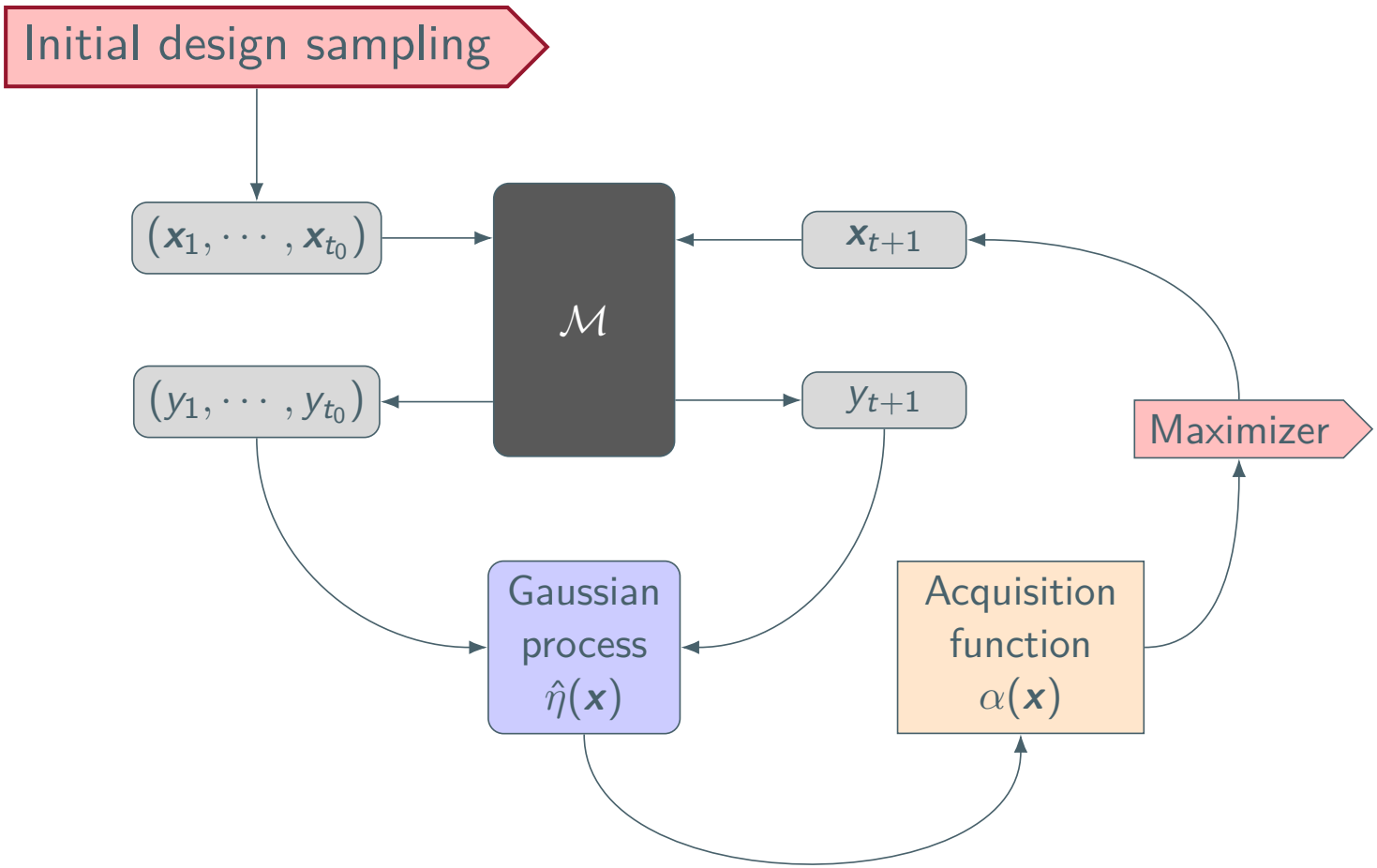

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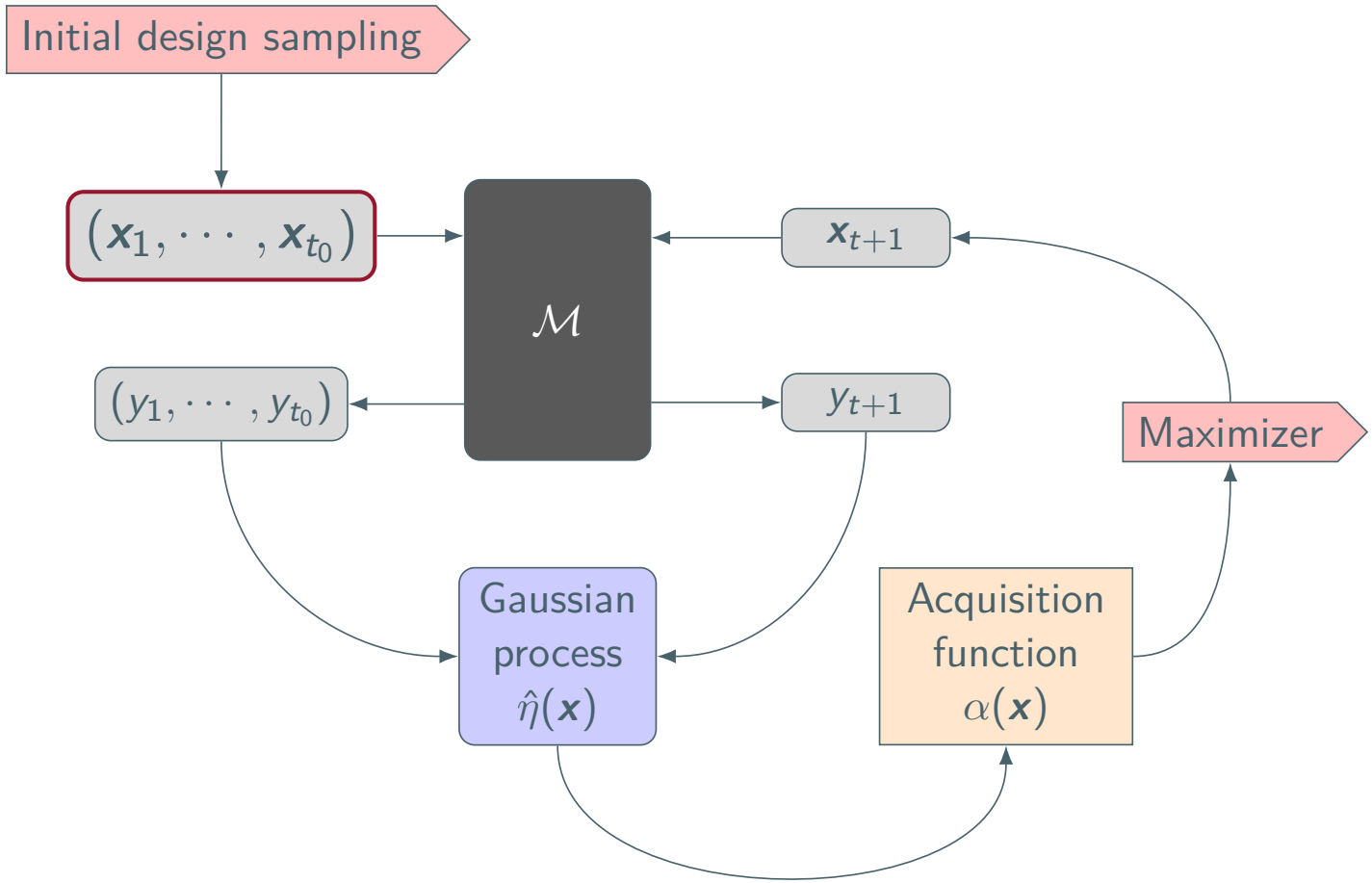

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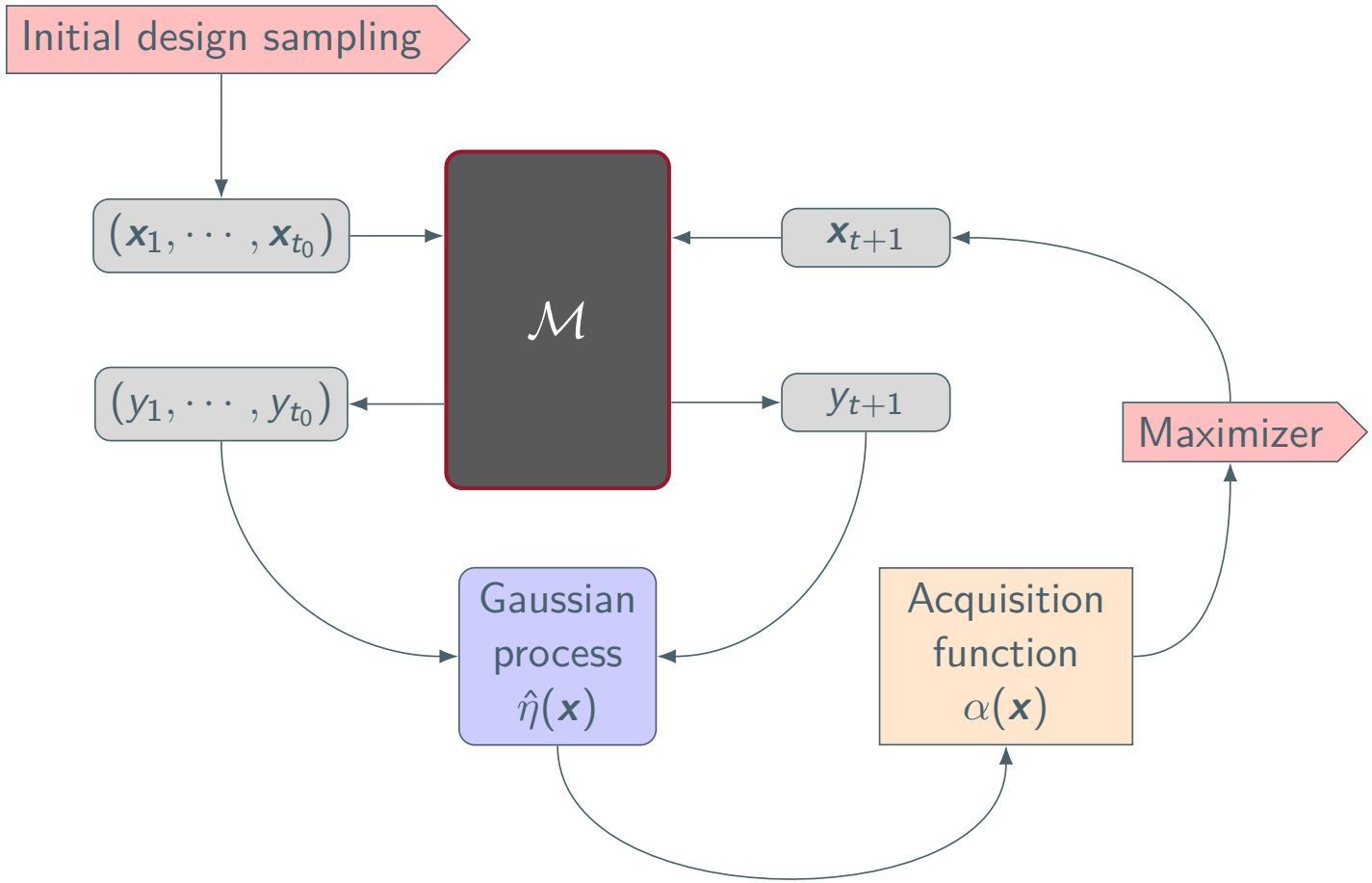

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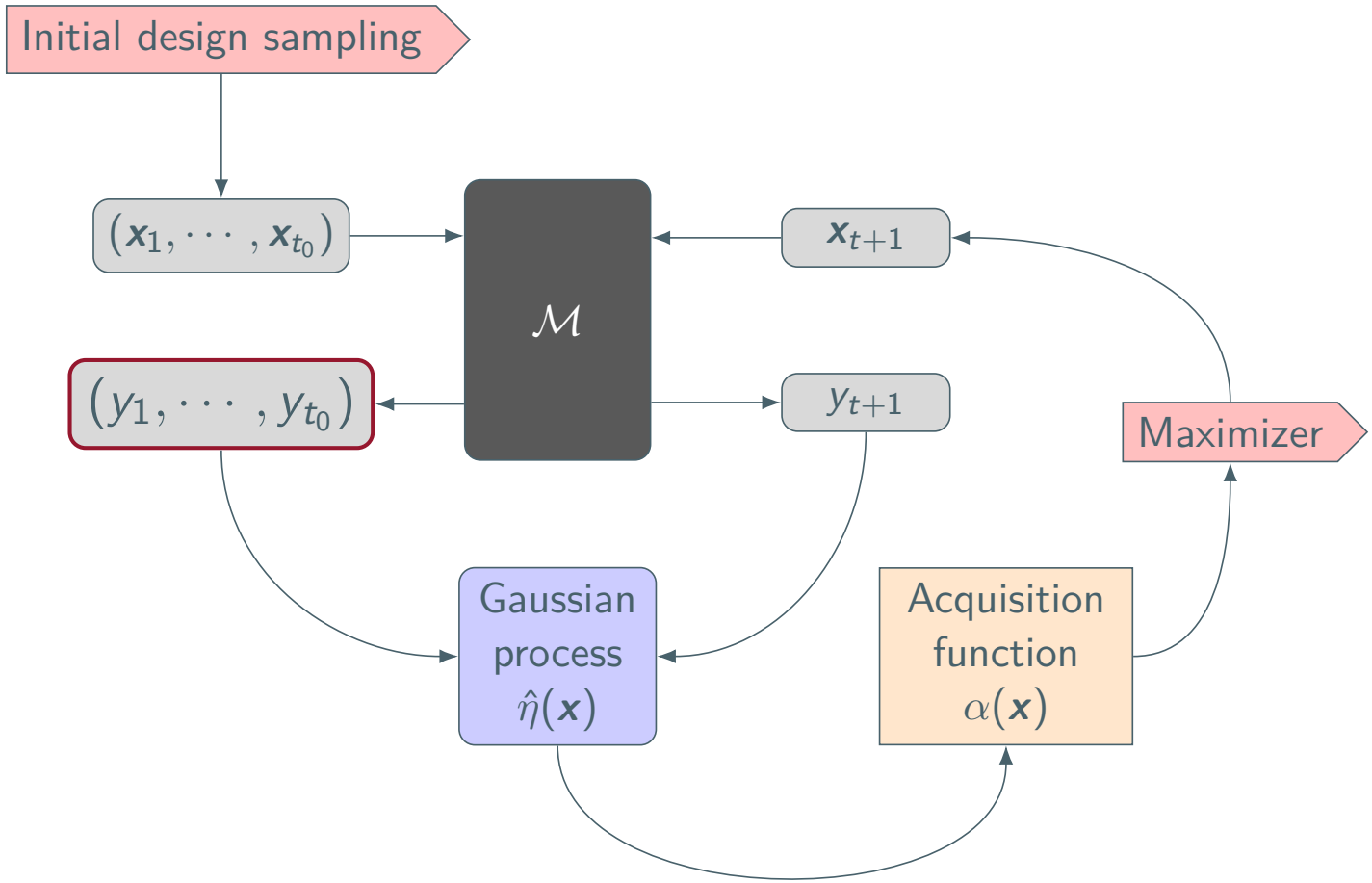

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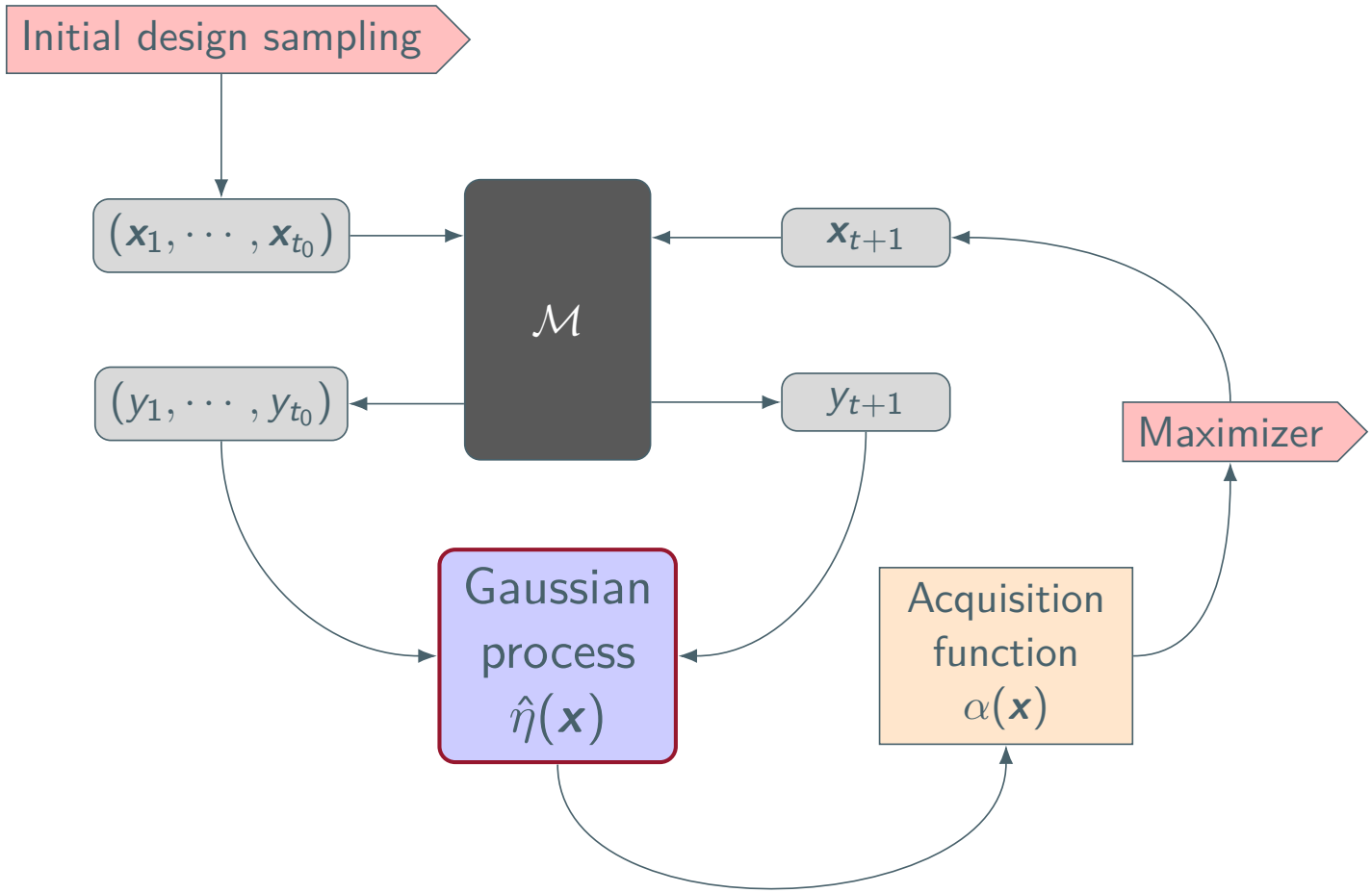

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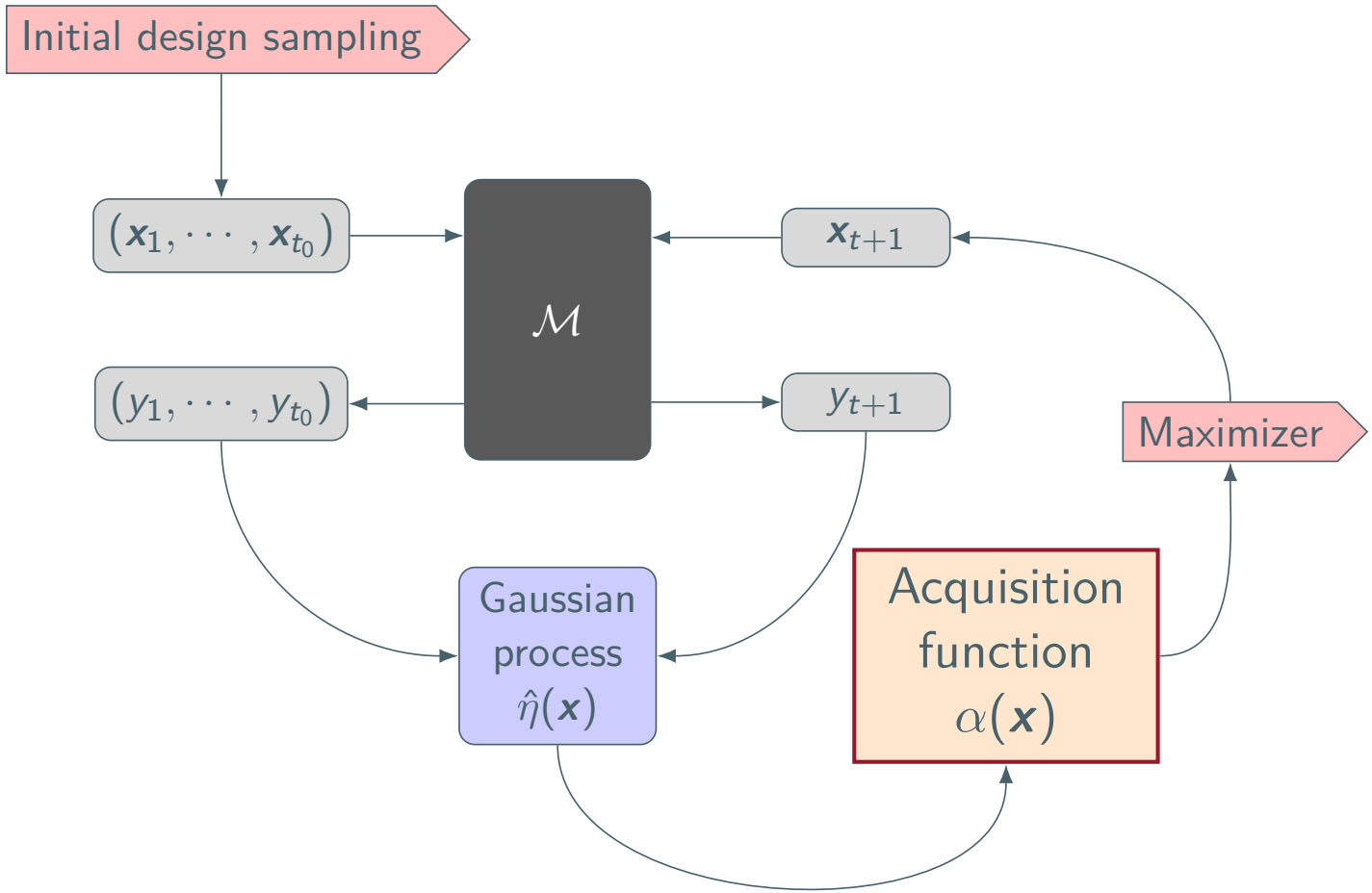

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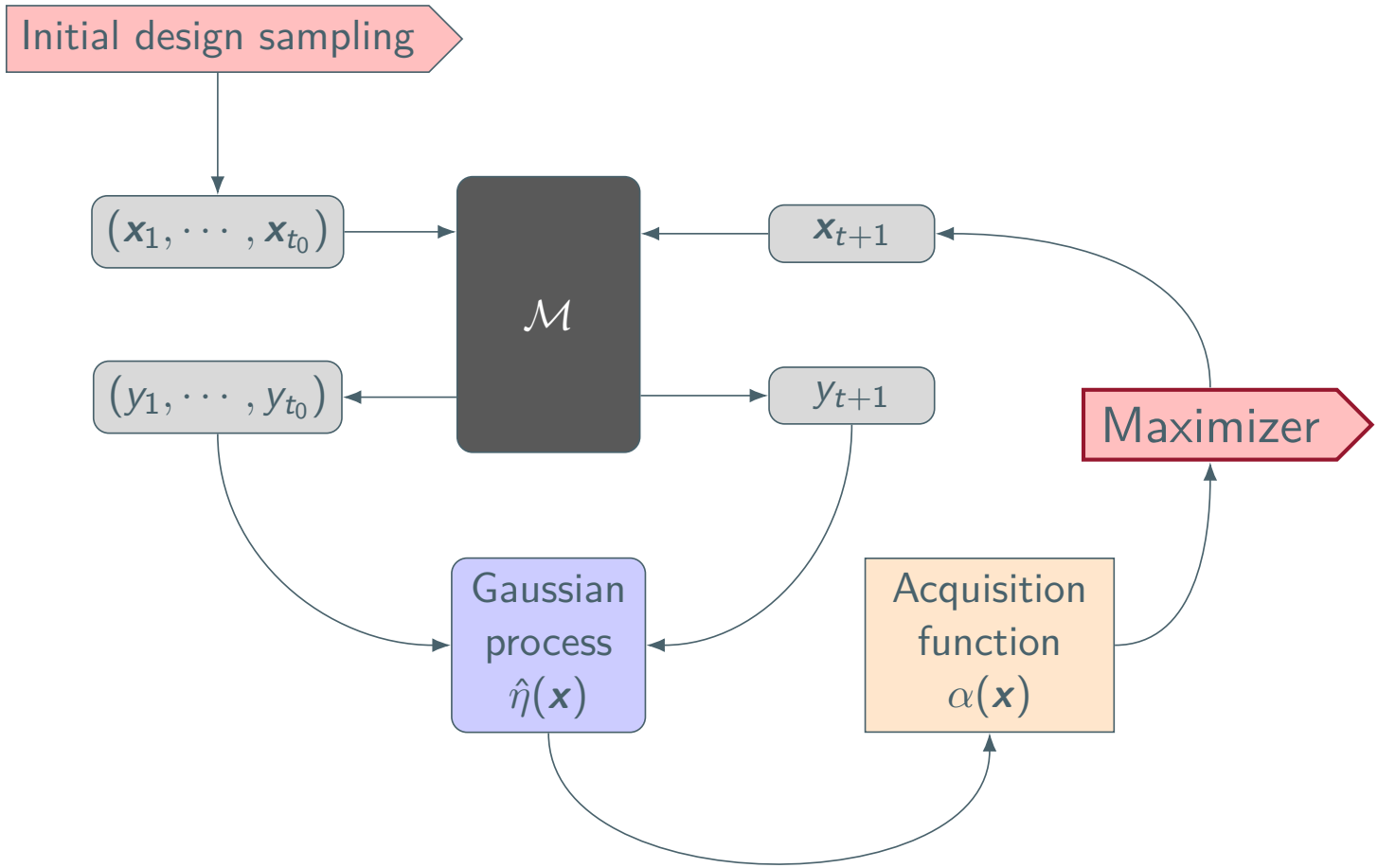

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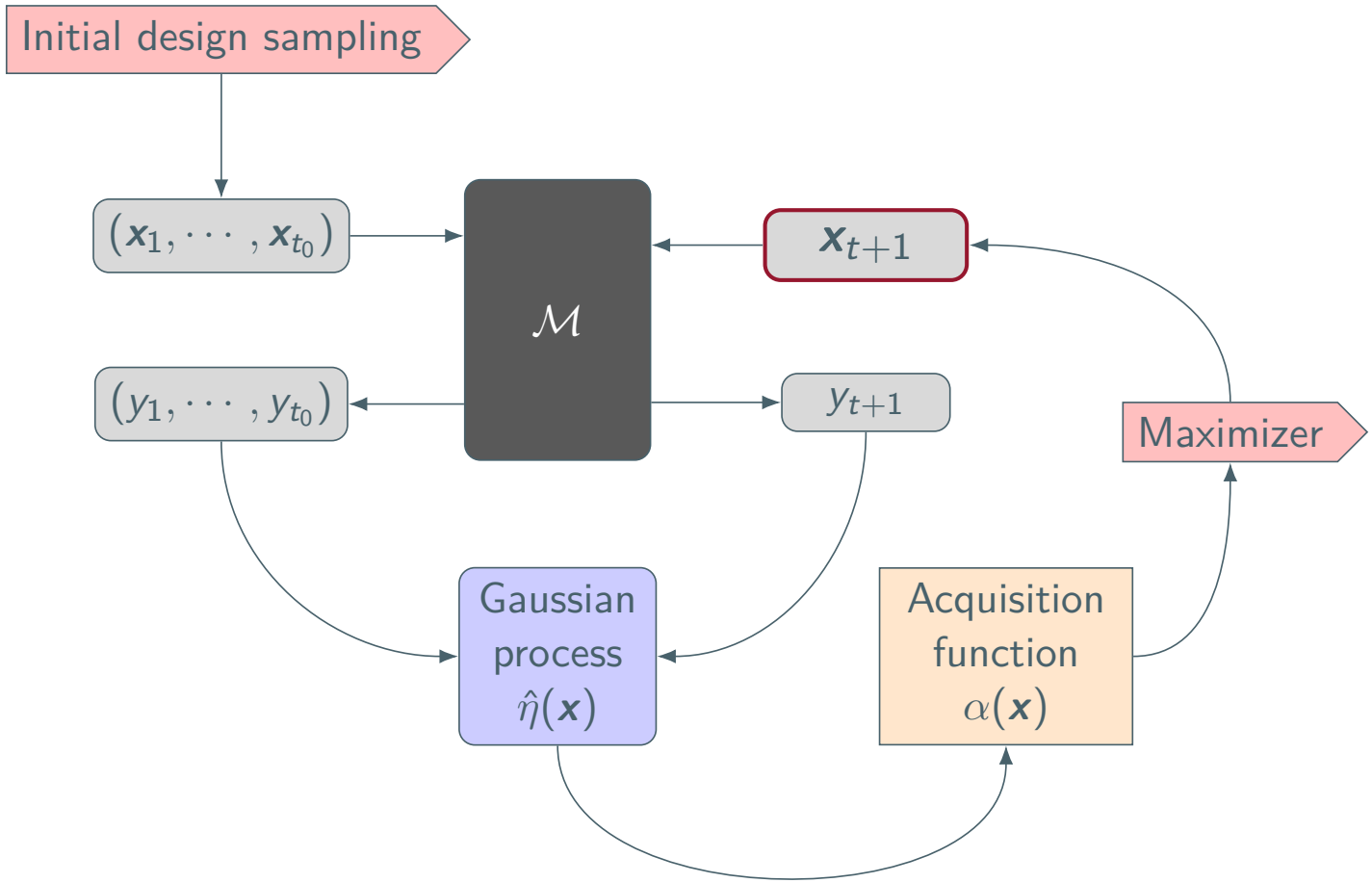

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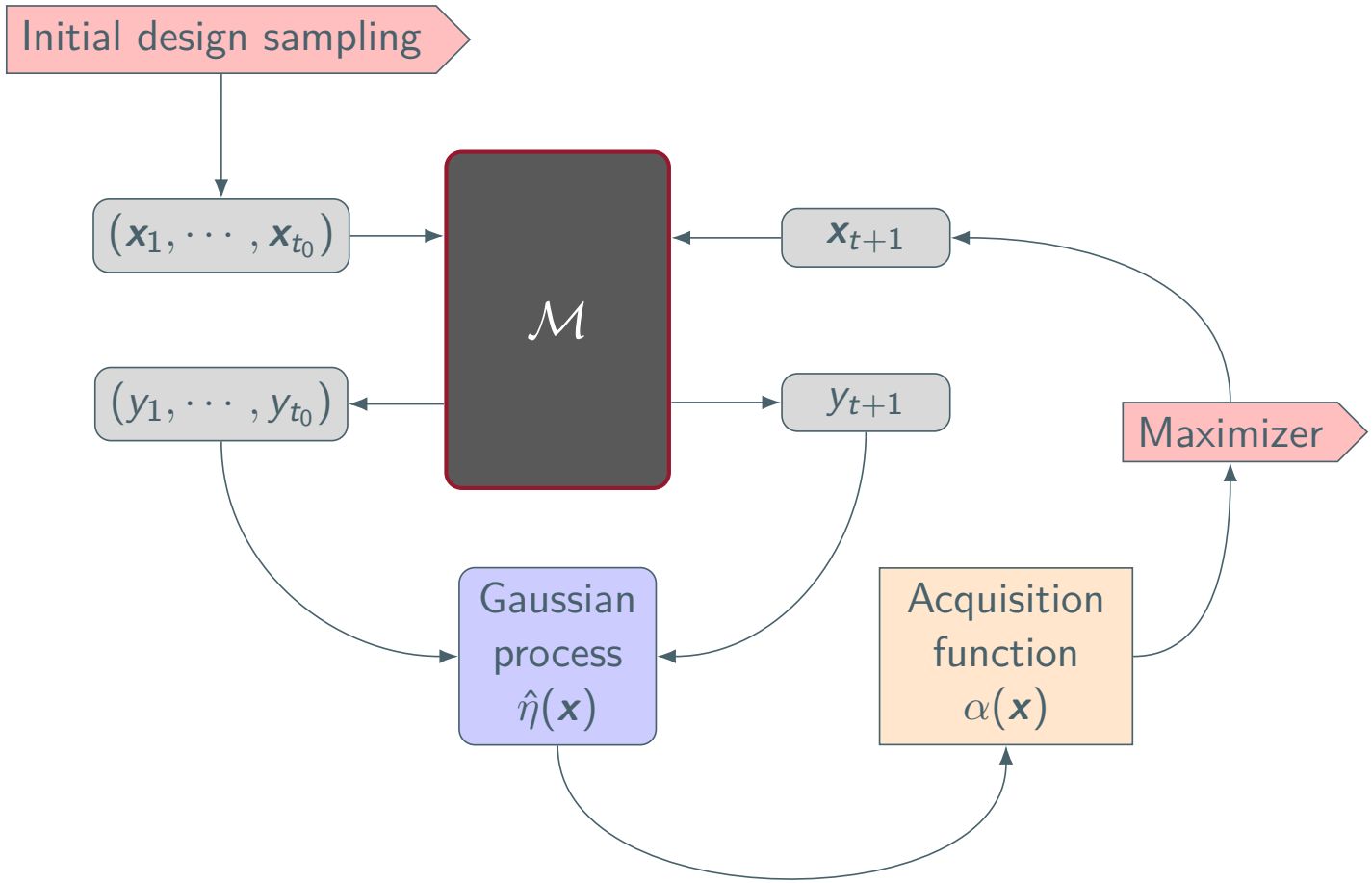

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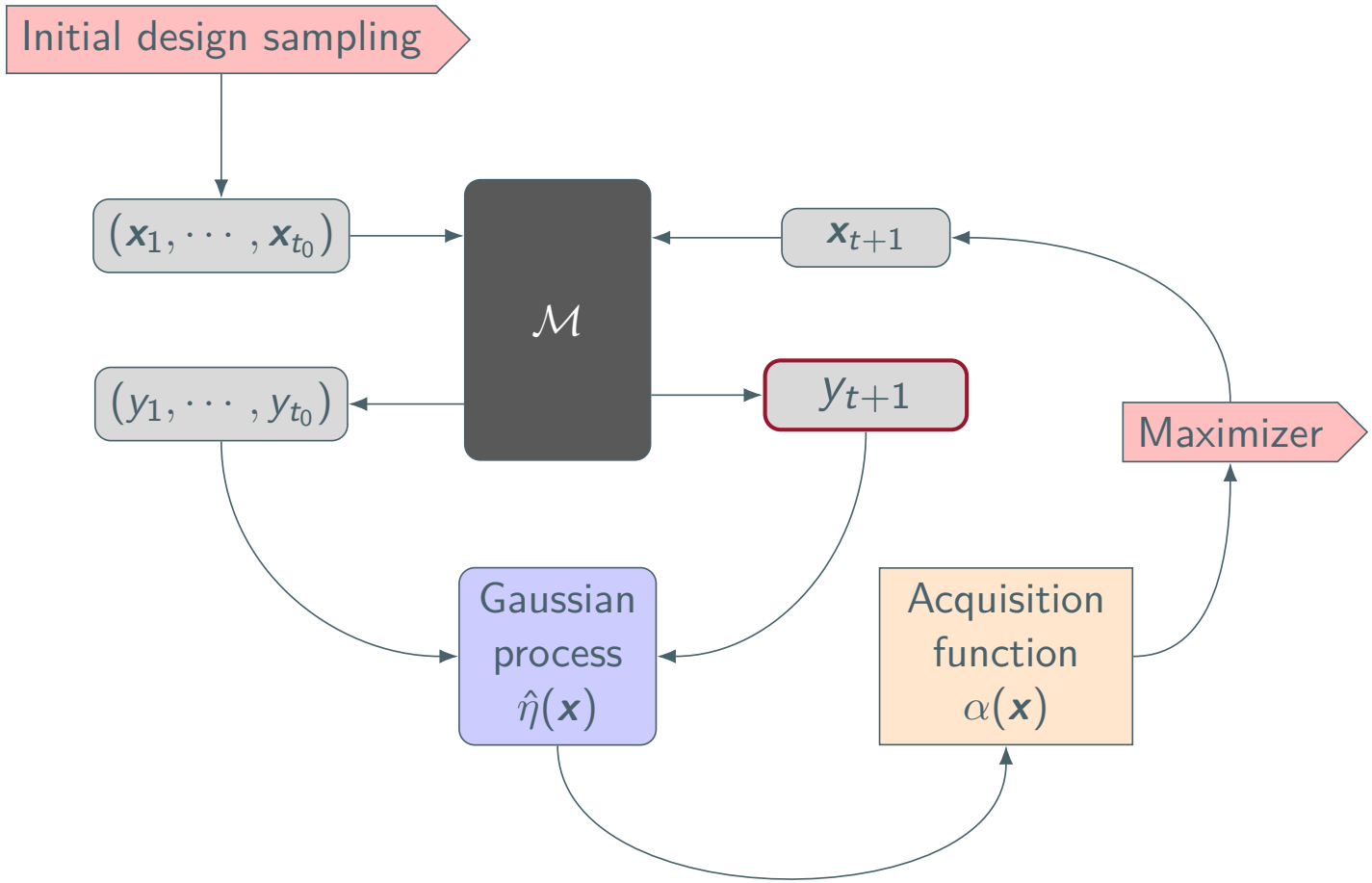

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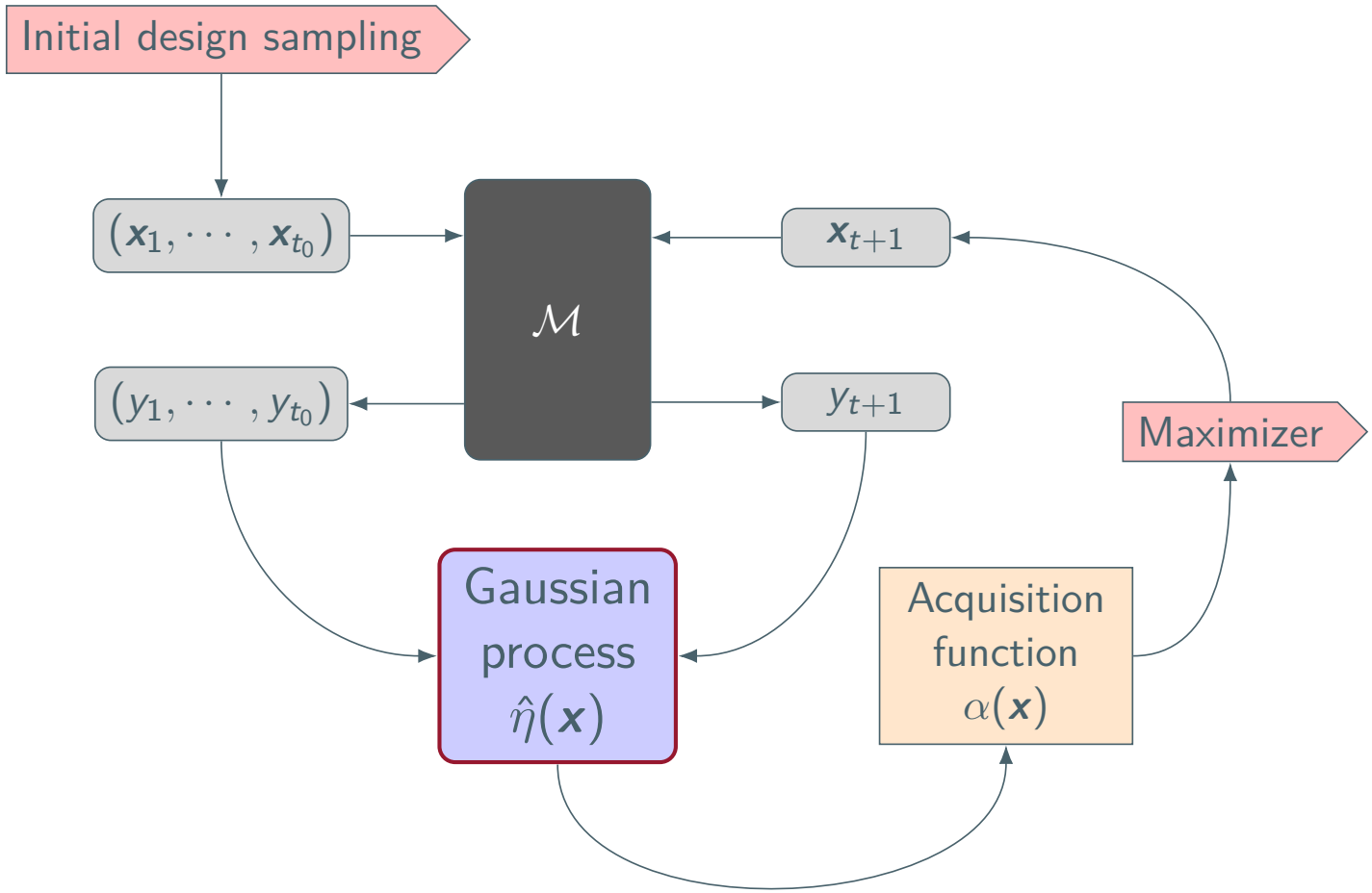

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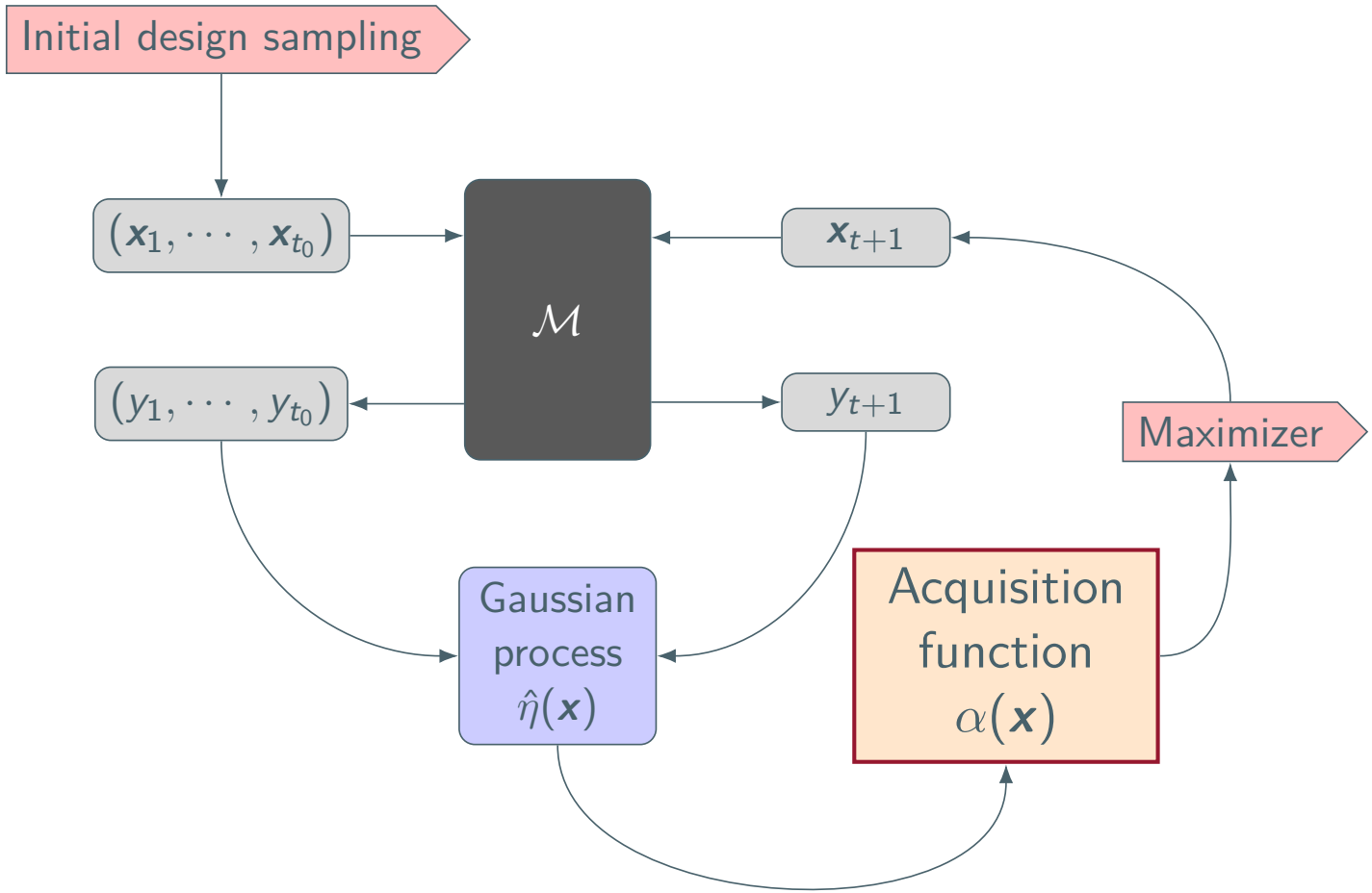

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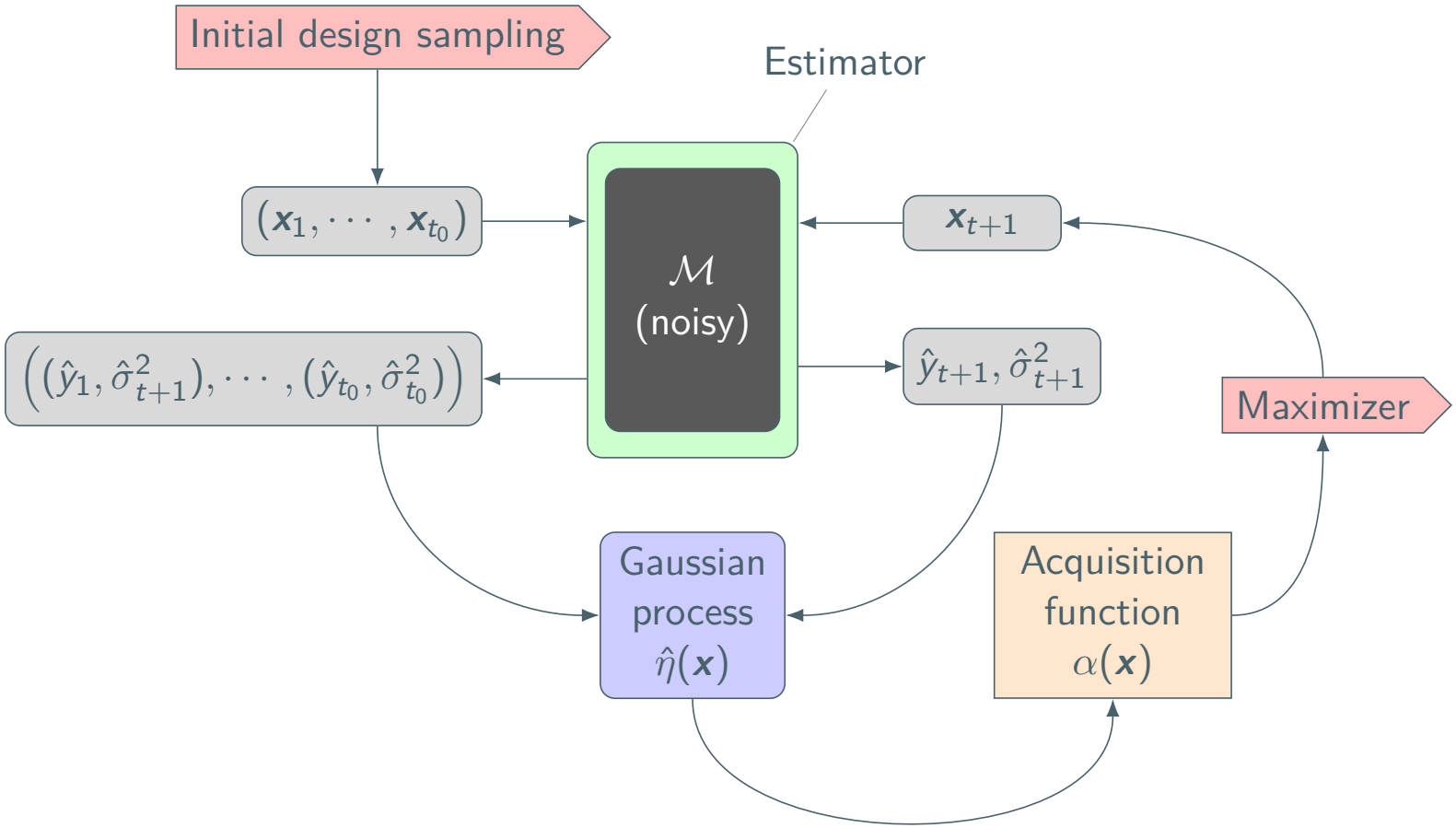

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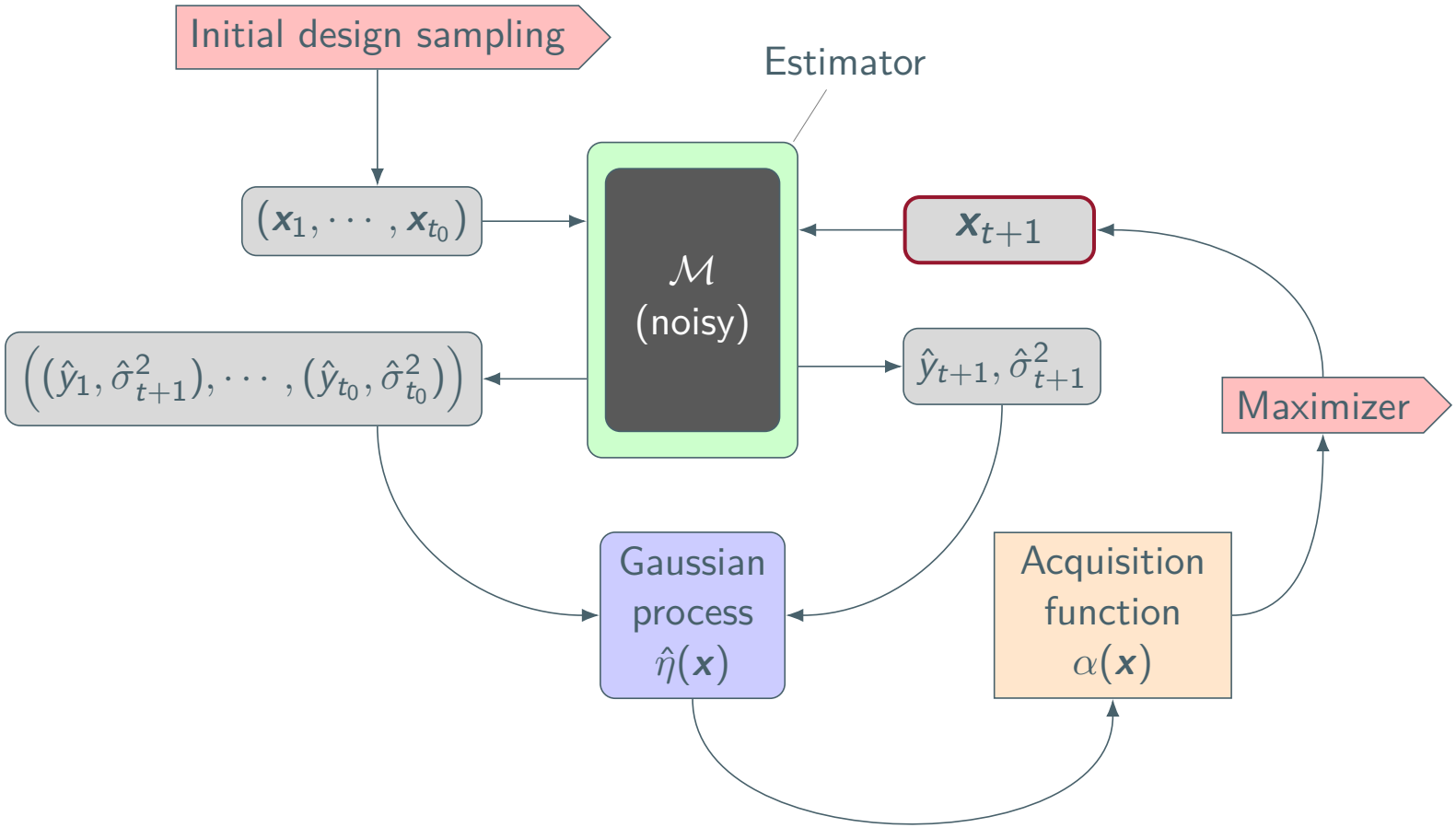

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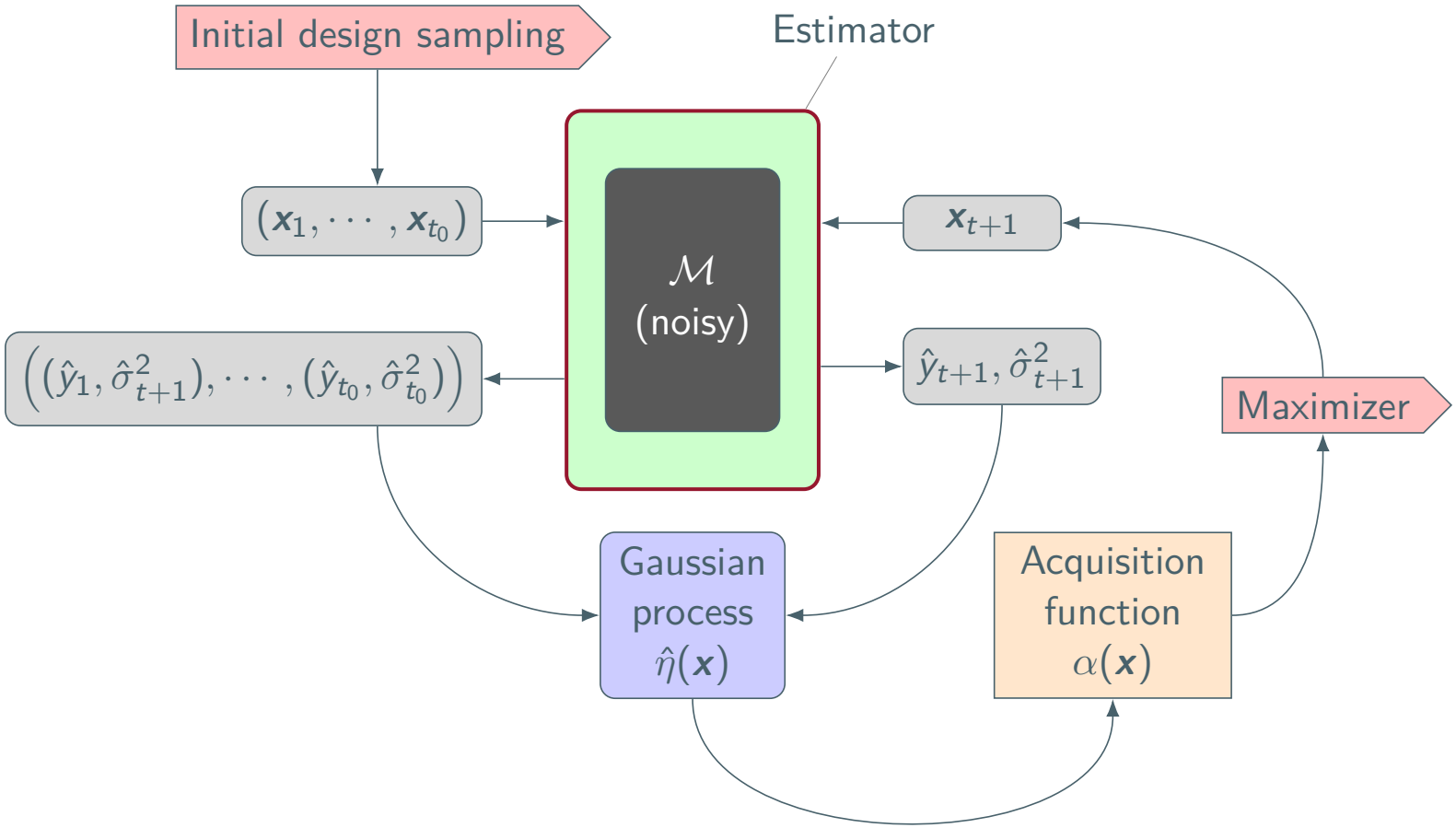

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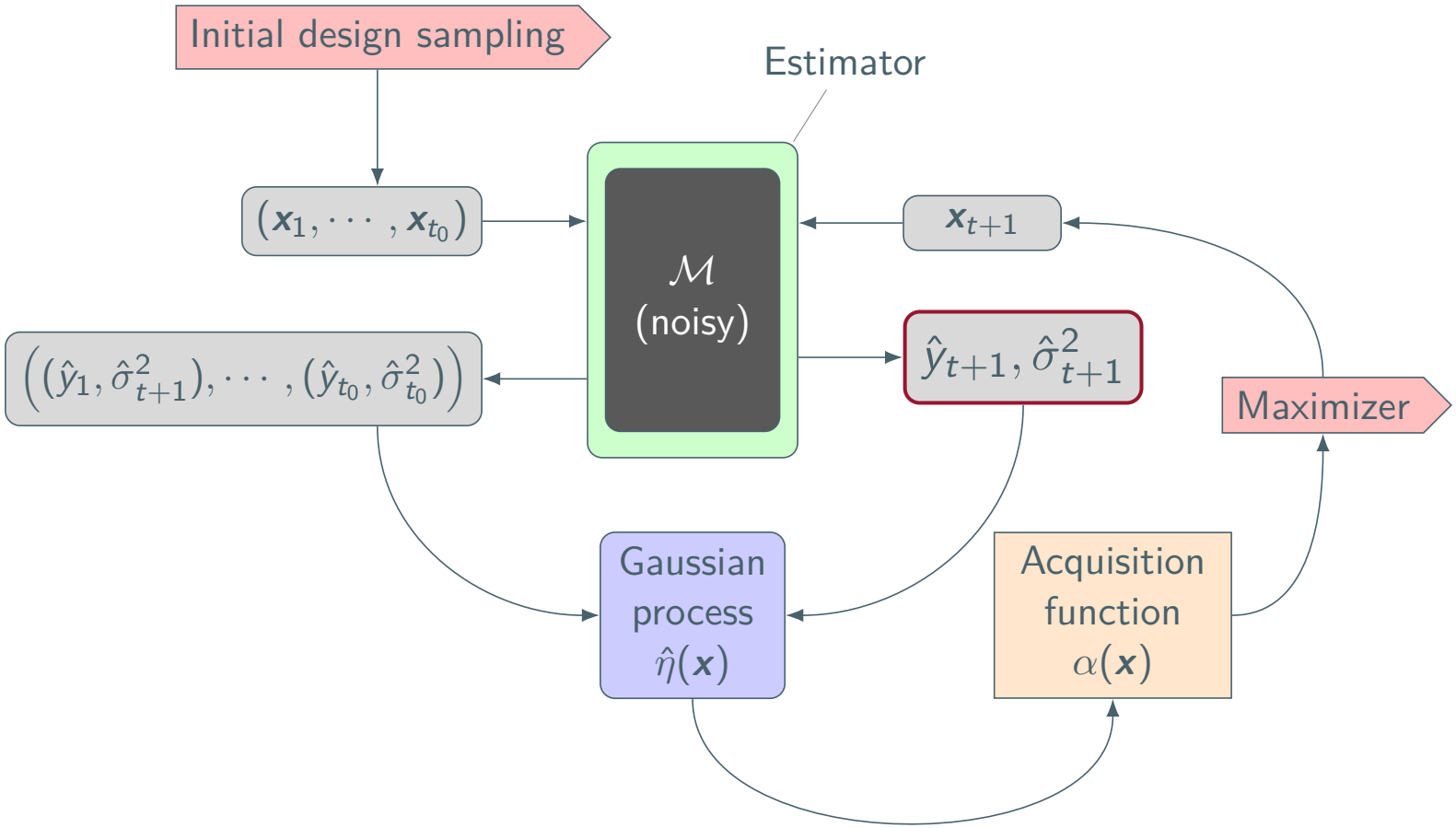

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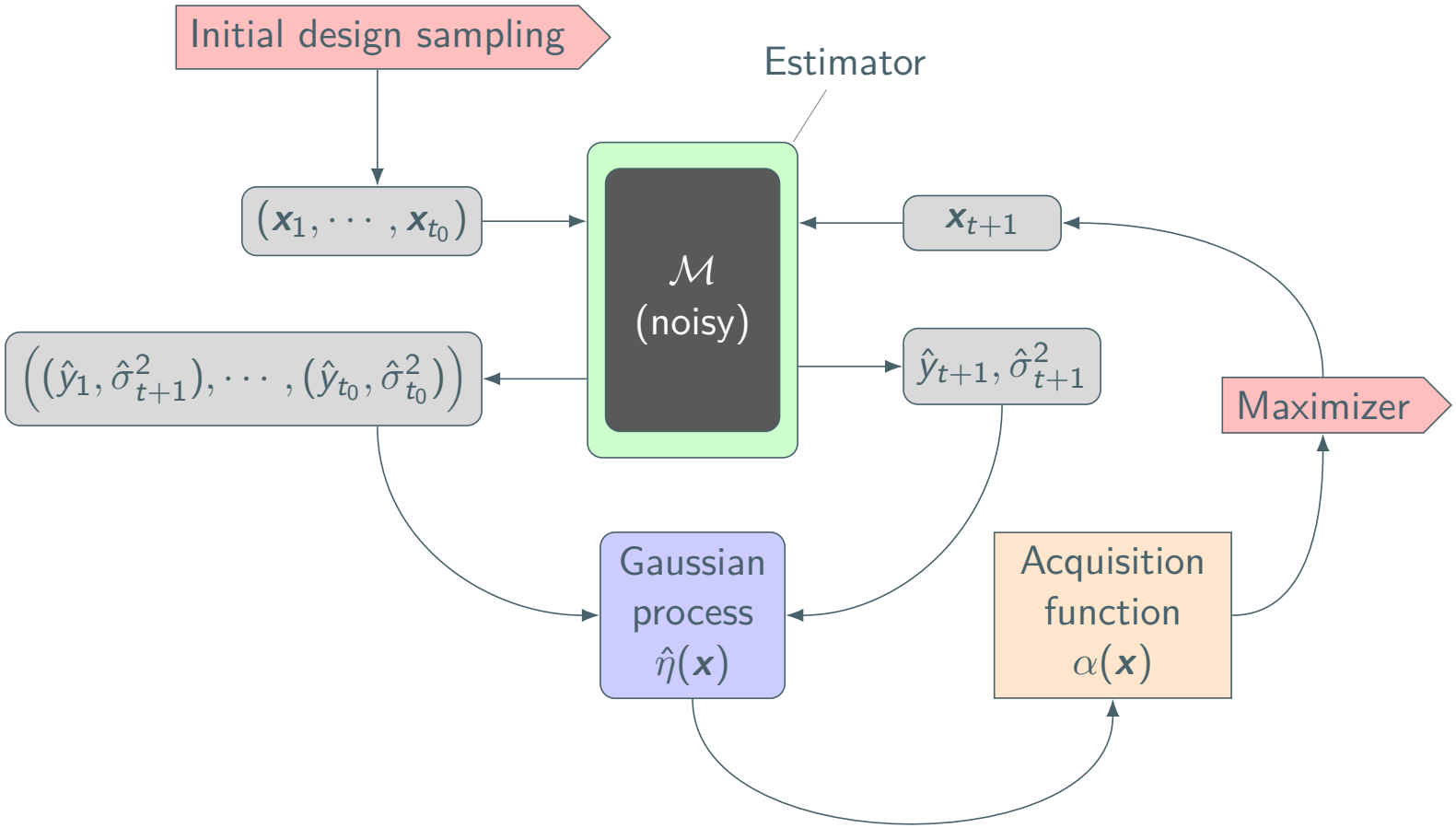

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Multi-Fidelity Monte-Carlo

Peherstorfer et al. [2016, 2018] El Amri et al. [2024]

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- One way to estimate $f^{(L)}$ in our case is MFMC:

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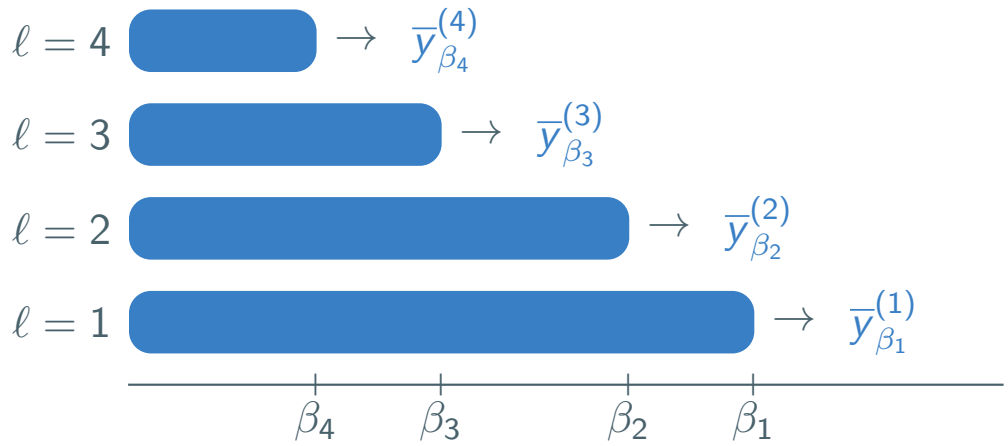

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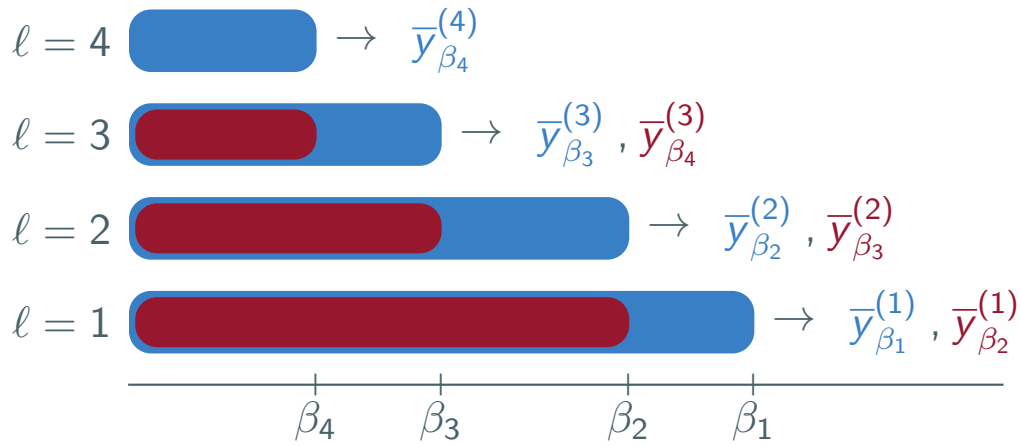

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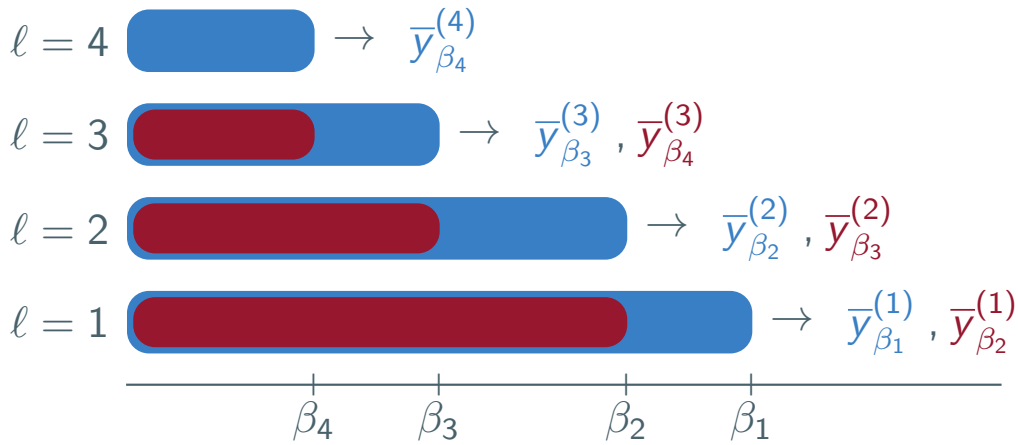

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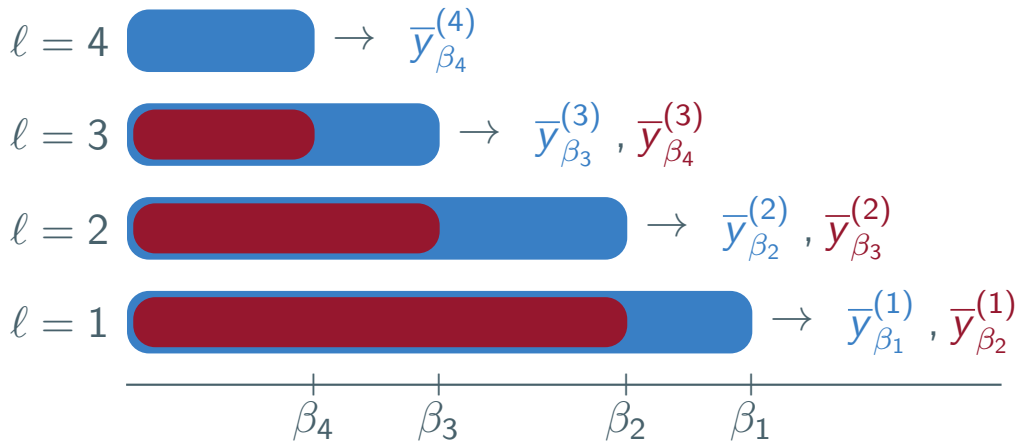

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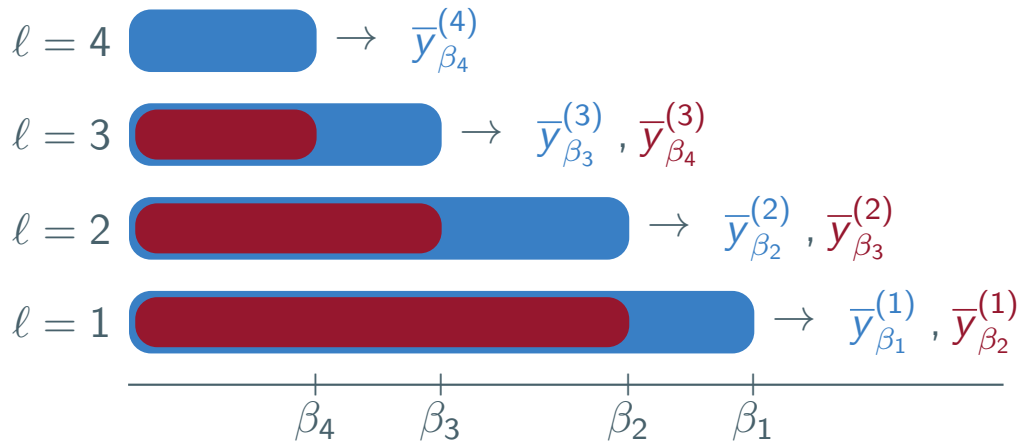

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Multi-Fidelity Monte-Carlo

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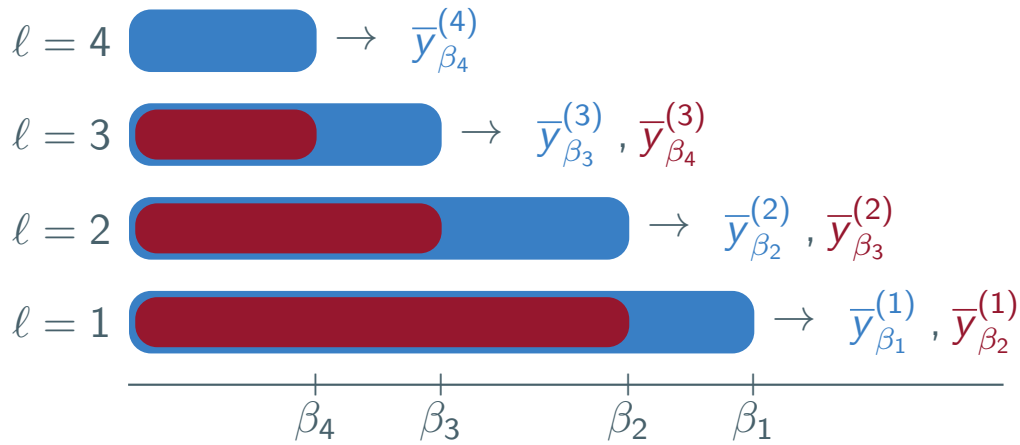

$$\hat{s}(\boldsymbol{\alpha}, \boldsymbol{\beta}) = \overline{y}^{(L)}_{\beta_L} + \sum_{\ell=1}^{L-1} \alpha_\ell \left(\overline{y}^{(\ell)}_{\beta_\ell} - \overline{y}^{(\ell)}_{\beta_{\ell+1}} \right)$$

- If we know $\Sigma = \mathbb{V}\left[\left(F^{(\ell)}(\boldsymbol{x}, \boldsymbol{U}) \right)_{1 \leq \ell \leq L} \right]$, then we can compute the variance of $\hat{s}$ in closed form, and optimize it w.r.t. $\boldsymbol{\alpha}, \boldsymbol{\beta}$

$$\mathbb{V}\left[\hat{s}\right] = \frac{\Sigma_{L,L}}{\beta_L} + \sum_{\ell=1}^{L-1} \left(\frac{1}{\beta_\ell} - \frac{1}{\beta_{\ell+1}} \right) \left(\alpha_\ell^2 \Sigma_{\ell,\ell} - 2\alpha_\ell \Sigma_{\ell,L} \right)$$

Peherstorfer et al. [2016, 2018] El Amri et al. [2024]

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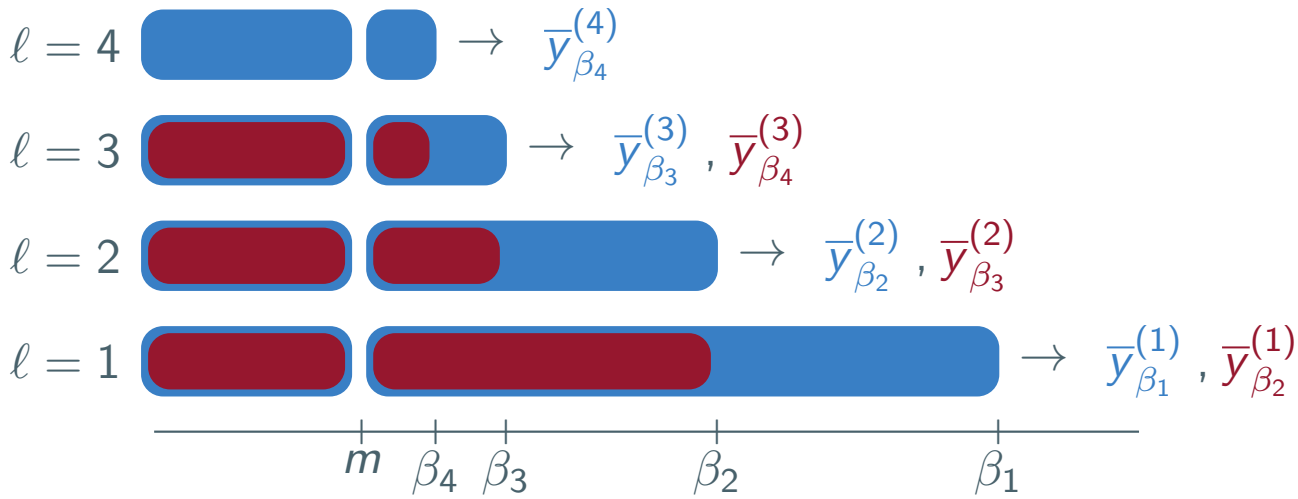

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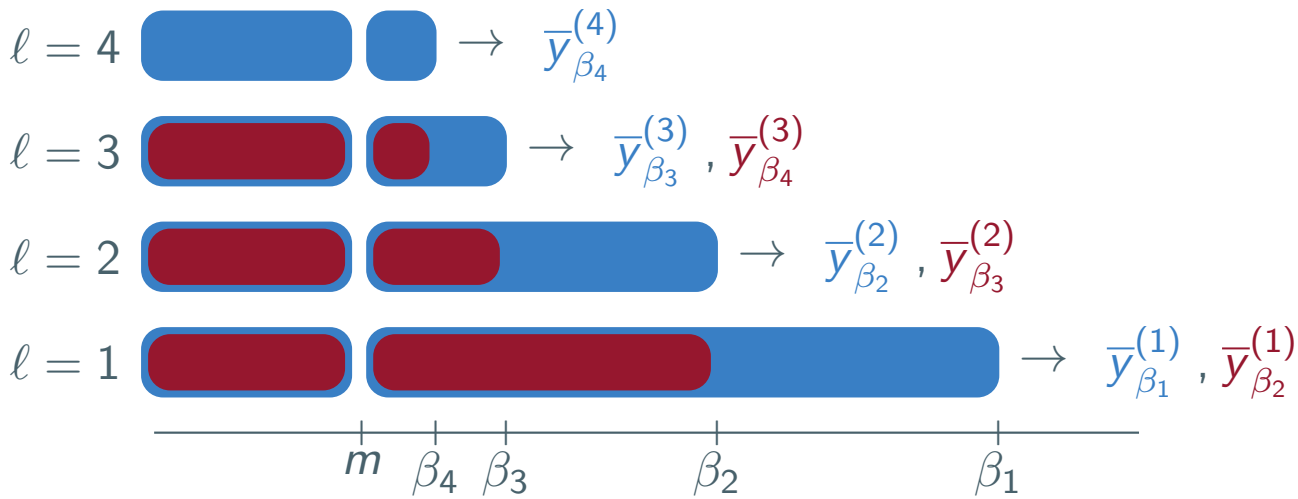

- Then we hope to be able to use the common $m \times L$ part to estimate the covariance matrix Σ.

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Kuhn and Tucker [1951]

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- Then $\boldsymbol{\beta}^\star$ checks $\beta_L^\star = \beta_L^\star(p)$, and recursively $\beta_{\ell-1}^\star = \beta_{\ell-1}^\star(p - \boldsymbol{\beta}_{\geq\ell}^{\star\intercal} \boldsymbol{w}_{\geq\ell})$

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Goal : learn jointly $f^{(L)}(\boldsymbol{x})$ and $\Sigma(\boldsymbol{x})$!

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Warning

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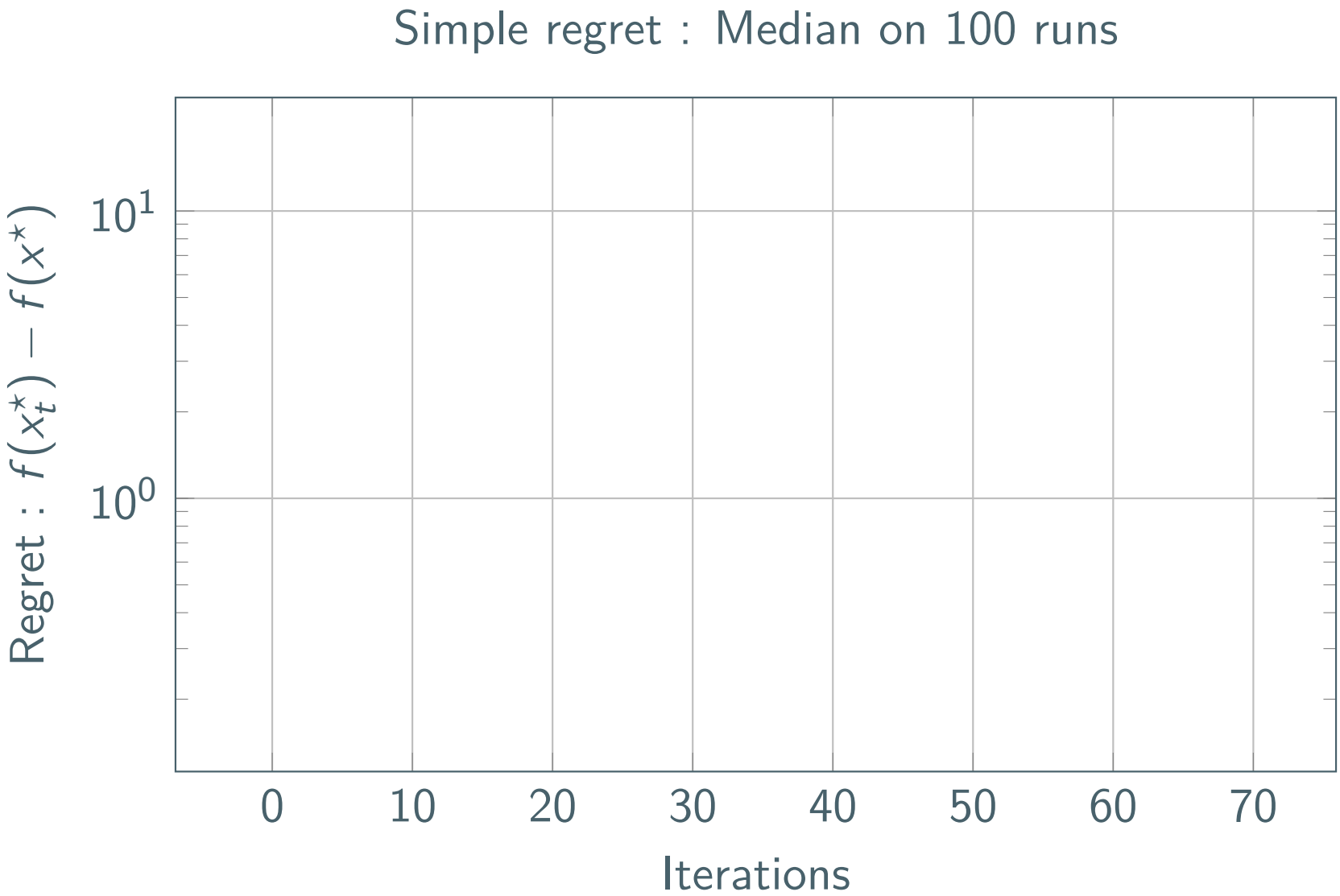
First numerical results on a toy function

With 3 dims in $\boldsymbol{U}$, 2 in $\boldsymbol{x}$, and $L = 3$.



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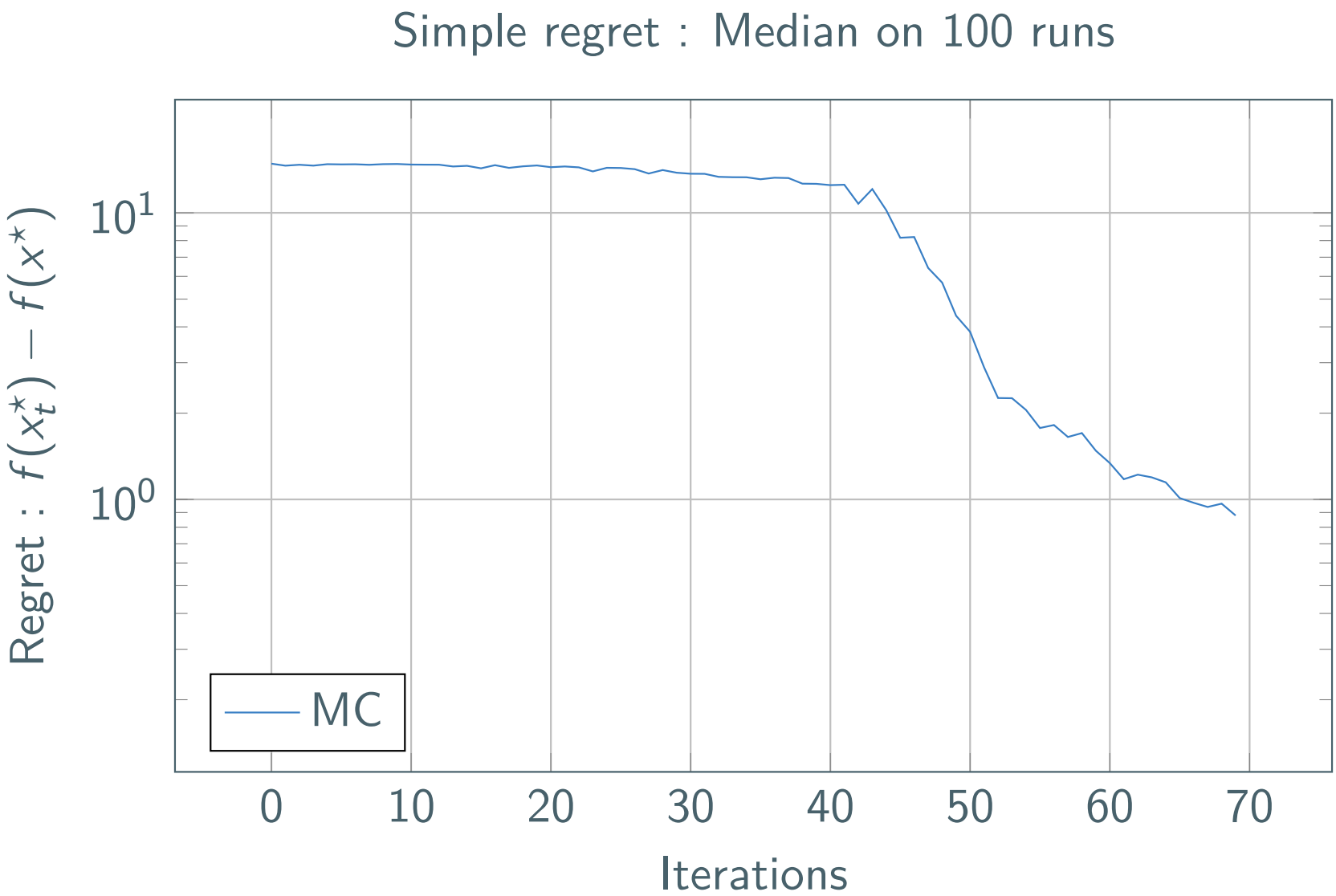

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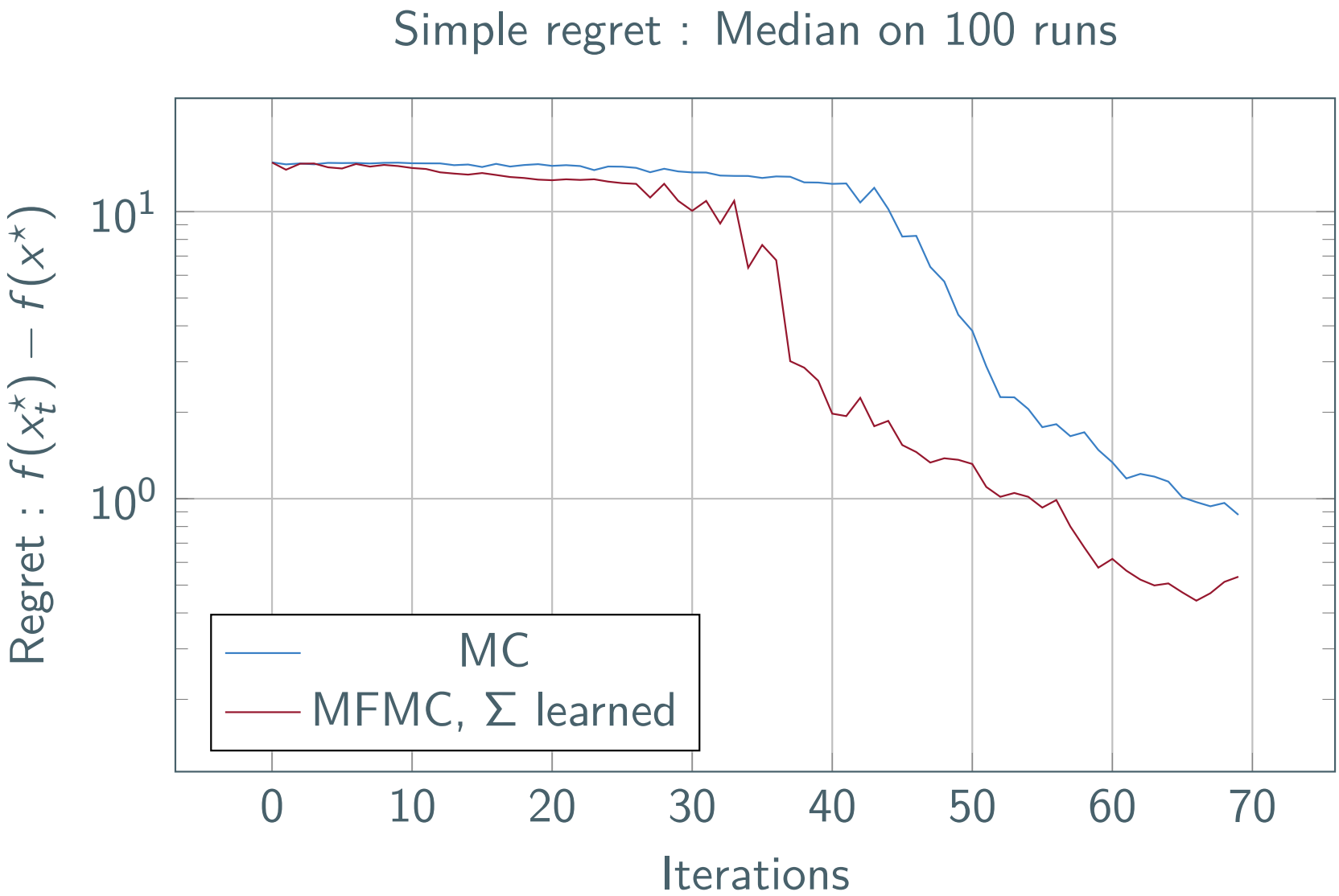

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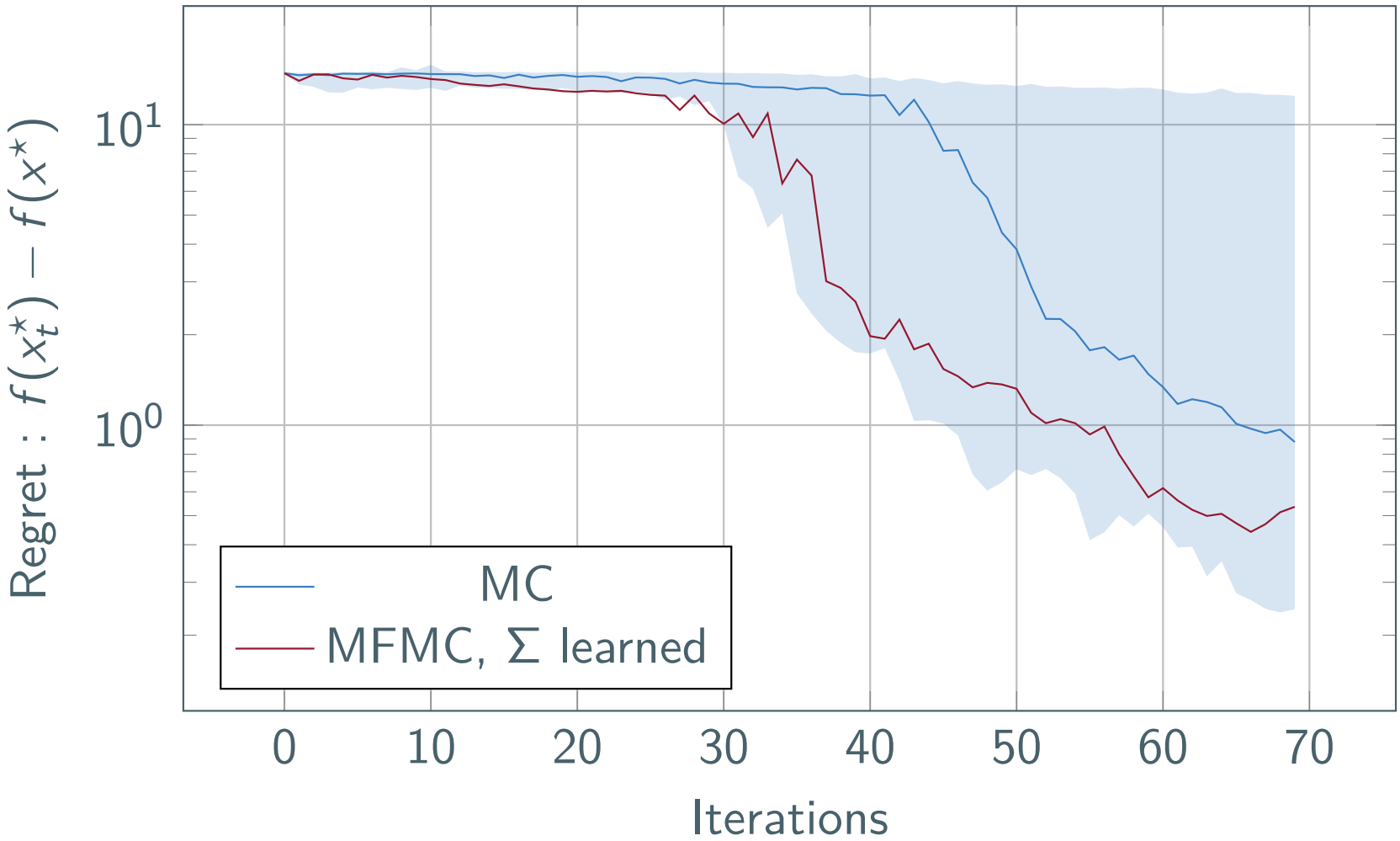
Simple regret : Median and quartiles on 100 runs



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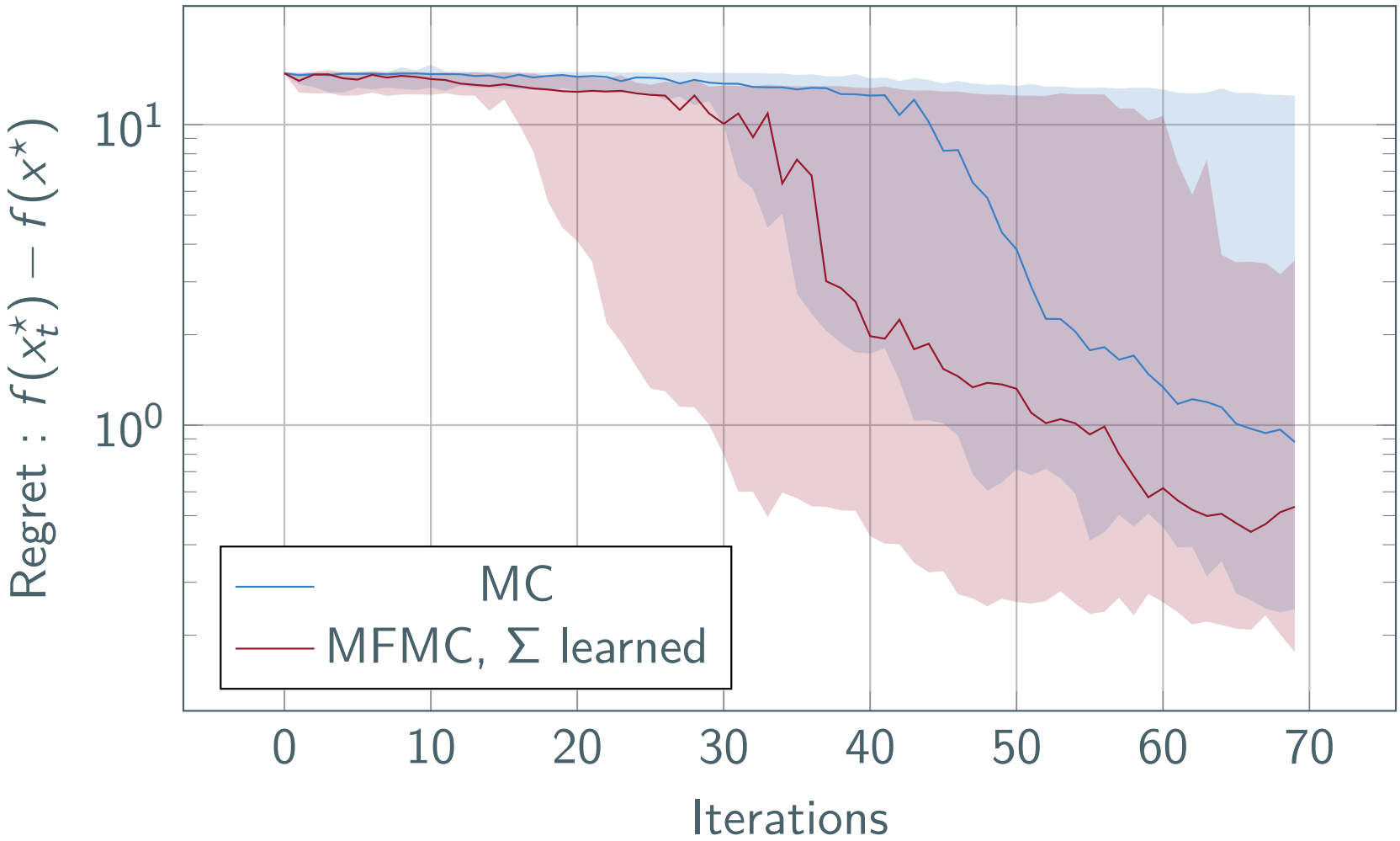

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Thank you for your attention !

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