Speed-up reliability assessment for multi-component systems: importance sampling adapted to piecewise deterministic Markovian processes

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Abstract:

The failure of a dynamical system often corresponds to a physical variable of the system (temperature, pressure, water level) entering a critical region. Reliability assessment of such a system requires an accurate model of the physical variable's trajectory. These physical variables are often determined by simple differential equations but those equations depend on the status of the multiple components of the systems (on, off, or out-of-order). Thus the model should incorporate the dynamics of the status of components, which rely on deterministic feedback mechanisms and on random failures and repairs. One difficulty is that the time to the next change in components status can be random or deterministic, whether it is due to a failure/repair or to an automatic activation/deactivation, consequently its associated random variable has a continuous part and a discrete one. We propose to model the trajectories of the physical variables of such systems, by a piecewise deterministic Markovian process (PDMP) introduced by M.H.A. Davis [3]. PDMP allows to take into account the interplay between components status and the trajectory and the hybrid aspect of the jump times of the components status.

A PDMP is a general kind of Markovian process that does not include diffusion. Many Markovian systems used in reliability analysis can be seen as PDMPs (as for instance colored petri nets [4]). A PDMP describes the trajectory of a pair (X_t, m_t) that is called 'state'. X_t is an element of $\mathbb{R}^d$ and is referred to as the 'position', and m_t is referred to as the 'mode' and takes a countable number of values. In our cases the mode represents the current components status and the position represents the values of physical variables. In a PDMP the mode follows a discrete jump process, and between two jumps of the mode the position X_t follows a simple differential equation which depends on the current value of the mode. At a jump time, it is possible to allow position to jump simultaneously with the mode. The arrival state of the jump is given by a Markovian kernel which depends on the departure state. In a PDMP the position can be restricted to evolve in a bounded domain which depends on the mode. When the differential equation leads the position at the boundary of the domain, a jump called 'deterministic' is triggered. This is used to model automatic activations and deactivations of components in our model. 'Random' jumps can also occur before position hits the boundary of the domain, their law is given through a hazard function which depends on the current position and current mode. These are used to model failures and repairs. Note that with this model, failure times do not necessarily follow an exponential distribution, for instance it is easy to included aging effect of components by adding components ages in the position vector and choosing a suitable hazard function.

As an example of multicomponent system which can be modeled by a PDMP, one can consider a dam [5]: the position refers to the water level, system failure corresponds to water level going above a given threshold, components correspond to evacuation valves, which open and close automatically depending on water level, but may be stalled (out-of-order) in open or closed positions, and the water level is given by a debit equations which depends on valves opening status.

The probability of failure can rarely be calculated analytically and when system failure is a rare event, a precise estimation using a crude Monte-Carlo method is too intensive from a computational point of view. A well-known way to reduce this computational burden is to decrease the necessary number of simulations by using a variance reduction technique. Amongst variance reduction techniques we may think of the multilevel techniques and the importance sampling techniques. Multilevel techniques assume that it is possible to build a nested sequence of sets of states $(A_i)_{0\leq i\leq n}$ such that $A_{i+1} \subset A_i$, with a final A_n corresponding to the critical region. These techniques work well if the probability of having a trajectory passing through A_{i+1} knowing it passes through A_i is not too small, but this practical condition is not always easy to fulfill when PDMP involve boundaries. Indeed if one considers the probability that the position passes below a threshold as a function of this threshold, then this function can present discontinuities near boundaries. If these discontinuities are large then the choice of A_i's is difficult. For this reason we focus on importance sampling.

Importance sampling consists in simulating from a more fragile system (with higher failure rates) while eliminating the induced bias by weighting each simulation by a likelihood ratio. To define a likelihood ratio for PDMP trajectories it is necessary to dispose of a measure dominating both the law of the trajectories of our system and the law of the importance system (i.e. the weaker system used for simulations). We first define the law of PDMP trajectories as an image law of the embedded Markov chain of a PDMP. Then, using this definition we show a way to construct a dominant measure on the trajectory space by changing the reference measure of the jump times. We deduce from this dominant measure the possible kinds of importance processes and their densities. We then apply our adaptation of importance sampling to PDMP on a two-component system, where we use the cross entropy method[1] to optimize the variance reduction. A comparison between our importance sampling estimate and a crude Monte-Carlo estimate shows we gain a factor ten in terms of standard deviation. The simulation of the PDMP is carried on by the python library “PytCATSHOO” developed by EDF R&D [2].

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Short biography – Master degree in statistics at ”Ecole nationale de la statistique et de l'analyse de l'information” (ENSAI) in 2014. Currently doing a PhD at the Industrial risk management departement of EDF R&D and the university Paris 7. This PhD is funded by EDF R&D, its goals are to develop techniques to speed-up the reliability assessment of dynamic hybrid systems, and to implement them in the ”PyCATSHOO” simulation tool developed by EDF.