Design and analysis of multi-level numerical experiments, with application to fire safety.

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Abstract:

Fire Dynamics Simulator (FDS) is a highly predictive state-of-the-art stochastic simulator of fire propagation in buildings. It is used in fire safety studies, in particular to assess the conformity of a building. This conformity is evaluated as the probability that several quantities of interest (temperature, visibility, etc.) remain below or above prescribed thresholds. These quantities are obtained by post-processing the output of an FDS simulation run. Accurate modeling, however, comes at the price of high computation times. In the case of FDS, the accuracy mainly depends on the number of mesh cells used to discretize the geometry of the building (Guillaume, 2013), with simulation times ranging from a few minutes for a very rough grid to several weeks for the kind of discretization that is routinely used by fire safety experts.

In this work, the objective is to predict the behavior of FDS at very high accuracies, and particularly the probability of failure of the smoke extraction system, using simulation results obtained at coarser levels (corresponding to larger mesh sizes). Dealing with multiple levels of accuracy is usually called *multi-fidelity*, and several Bayesian multi-fidelity models have been proposed in the literature for the case of deterministic simulators.

Kennedy and O'Hagan (2000) proposed to link the responses of the simulator at two successive levels using an autoregressive model:

$$Z_t\left(x\right) = \rho_{t-1} Z_{t-1}\left(x\right) + \delta_t\left(x\right), \qquad \delta_t \perp\!\!\!\perp \left\{Z_{t-1}, Z_{t-2}, \ldots\right\}, \tag{1}$$

where Z_t is a Gaussian random process modeling the response of the simulator at level t, x is the vector of input parameters, $\rho_t \in \mathbb{R}$, and δ_t is another Gaussian random process, independent of lower levels. This model is in principle applicable to any finite number of levels, with a significant reduction of computational complexity when input designs are nested (Le Gratiet, 2013, Section 3.6). However, each additional level brings additional parameters to be estimated, and a sufficiently high number of evaluations must therefore be available, even at the most expensive levels, to make their estimation possible. This limitation most certainly explains why published results are restricted to case studies with two levels of fidelity, sometimes three (Le Gratiet, 2013, Section 3.7).

Another approach, that makes it possible to deal with a large—even infinite—number of levels with a manageable number of parameters, has been proposed by Picheny and Ginsbourger (2013) and Tuo et al. (2014). Their model describes the response $Z(x,t)$ of the simulator as the sum of an ideal simulator $Z_\infty(x)$ and a numerical error $\varepsilon(x,t)$, which goes to zero when accuracy increases:

$$Z\left(x,t\right) = Z_\infty\left(x\right) + \varepsilon\left(x,t\right), \qquad \varepsilon \perp\!\!\!\perp Z_\infty, \quad \lim_{t\to 0} \varepsilon\left(x,t\right) = 0. \tag{2}$$

Again, both Z_∞ and ε are taken to be Gaussian processes for tractability, and the property of vanishingly small error for large t is achieved through an appropriate non-stationary covariance

structure for ε. This model has been shown to be useful in particular when accuracy is controled by a continuous tuning parameter such as mesh size (Tuo et al., 2014) or computation time (Picheny and Ginsbourger, 2013). An especially appealing feature is the ability to make predictions at level $t = \infty$, corresponding to an ideal level of simulation which is numerically out of reach.

We present in this communication our first results of multi-fidelity modeling for FDS, using as an example a simple fire scenario in a box-shaped building, with $d = 8$ input factors. Because the number of levels is potentially large (five levels in our example, corresponding to mesh sizes $100\,\mathrm{cm}/m$, $1 \leq m \leq 5$) and the number of simulation results at the finest level very limited (typically, considering the very large simulation times, less than ten), we decide to focus on the second family of models, where the number of parameters is not related to the number of levels. We assume for simplicity that all output distributions—remember that FDS is a stochastic simulator—are Gaussian, with a constant variance on each level, and compare several covariance structures for the mean response functions. Special attention is paid to the impact of the parameter estimation method, fully Bayesian or not, on the quantification of uncertainty for pointwise response predictions and for estimates of the probability of non-conformity.

References

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Short biography – Rémi Stroh received the Engineer's Degree (equivalent to a master's degree in Electrical Engineering) from Supelec in 2014, with a specialization in applied mathematics. Since February 2015, he is a full-time PhD student at Laboratoire National de métrologie et d'Essais and Laboratoire des Signaux et Systèmes (CentraleSupelec/CNRS/Univ. Paris Sud).