SUMMER SCHOOL CEA-EDF-INRIA 2011
UNCERTAINTY QUANTIFICATION FOR NUMERICAL MODEL VALIDATION

COURSE ON "RARE EVENT ESTIMATION IN OUTPUT OF COMPUTER MODEL"

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Synopsis

The design of a system or a technological product has to take into account two different sources of uncertainties: uncertainty due to intrinsic randomness in the system and its environment, and epistemic uncertainty stemming from a lack of knowledge about some underlying phenomena, in particular when those phenomena result from abnormal and dangerous operating conditions of the system, never observed before. Due to these uncertainties, a failure of the system has always a nonzero probability to happen. Such a failure may trigger other failures with increasing levels of severity. Therefore, it is of utmost importance to be able to assess the probability of undesirable events that can occur in a system.

The operation of a system and the chain of events leading to a failure are generally simulated using computer models of the critical parts of the system. Then, the probability of a failure event can be estimated in simulation by varying randomly the operating conditions of the system. This approach is referred to as a Monte Carlo (MC) method. While the MC method makes it possible to obtain an accurate assessment of the probability of failure of a complex system, it often requires high computational efforts. Indeed, when the probability of failure is small, a large number of simulations of the complex system must be carried out to observe a few failure events. For instance, if the probability of failure is 10^{-6}, it will take about 3.10^6 simulations to have a probability 0.95 of observing at least one failure event. However, a single simulation of a complex system can take hours or days to run. Thus, alternative methods must be sought to avoid the computational burden associated to the MC method.

Three directions of research have been considered in the literature. A first direction aims at modifying the MC method to obtain good estimators with a limited number of simulations. Techniques such as Importance Sampling (Au and Beck, 2003; Schueller 2007), Stratified Sampling (Helton and Davis, 2003; Cacuci and Ionescu-Bujor, 2004) have been widely used in reliability analysis and risk assessment (Helton 1998). Recently, advanced sampling methods such as Subset Simulation (SS) (Au and Beck, 2001; Au and Beck, 2003) and Line Sampling (Koutsporelakis et al., 2004; Pradlwarter et al., 2005) have been proposed for structural reliability assessment. These methods can decrease significantly the computational cost of the estimation of a probability of failure with respect to an MC approach. However, these methods generally do not make it possible to reduce the number of simulations below a few hundreds, which may be mandatory in the presence of computer codes requiring several hours to run a single simulation.

A second direction, which is very popular in the domain of structural reliability, is to determine the factors that can bring the system to a failure, and then try to approximate by a simple geometrical shape the frontier between the regions of safe and abnormal operations, in the space of influent factors. The popular first-order reliability method (FORM) uses an hyperplane as the approximating shape and the second-order reliability method (SORM) uses

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a paraboloid (Bjerager et al., 1990). Compared to MC methods, this class of methods does achieve a large reduction in the required number of simulations. Nevertheless, this class of methods does not provide statistically consistent estimators of the probability of failure.

A third direction is to build a cheap-to-evaluate surrogate model, also called response surface or meta-model, of the expensive-to-evaluate model, and then to use this surrogate for failure analysis. The construction of such a surrogate model entails running a small number of expensive simulations for specified values of the uncertain input factors/variables and collecting the corresponding values of the output of interest; then, statistical techniques are employed for fitting the response surface of the regression model to the input/output data generated in the previous step. In (Bucher and Most, 2008; Gavin and Yau, 2008; Liel et al., 2008), polynomial Response Surfaces (RSs) are employed to evaluate the probability of failure of structural systems; in (Deng, 2006; Hurtado, 2007; Cardoso et al., 2008; Cheng et al., 2008), learning statistical models such as Artificial Neural Networks (ANNs), Radial Basis Functions (RBFs) and Support Vector Machines (SVMs) are trained to provide local approximations of the failure domain in structural reliability problems. In this same line of research, several strategies based on Gaussian process models and kriging look very promising. These strategies correspond to sampling criteria proposed by Ranjan et al. (2008), Bichon et al. (2008), Vazquez and Bect (2009), Picheny et al. (2010) and Echard et al. (2010), which have their roots in the field of design and analysis of computer experiments (see, e.g., Sacks et al., 1989; Currin et al., 1991; Welch et al., 1992; Oakley and O'Hagan, 2002, 2004; Oakley, 2004; Bayarri et al., 2007; Volkova et al., 2008; Marrel et al., 2009). More specifically, kriging-based sequential strategies for the estimation of a probability of failure are closely related to the field of Bayesian global optimization (Mockus et al., 1978; Mockus, 1989; Jones et al., 1998; Villemonteix, 2008; Villemonteix et al., 2009; Ginsbourger, 2009).

Outline of lectures (DRAFT)

The course lasts 7h30, divided in 5 sessions (5 × 1h30)

1. Modeling probability laws and distribution tails (1h30)
   - 1.1 Taking into account uncertainties during the design of a system
     - (i) MISO-MIMO systems
     - (ii) Different formulations of the problem
       - Probability of failure
       - Quantile
       - Worst case
   - 1.2 The Monte Carlo approach and its limitations
   - 1.3 Extreme Value Modeling
     - (i) Introduction to the problem of modeling extreme values problem
     - (ii) Tail modeling: elements on extreme value theory
     - (iii) Examples
2. Reliability methods (FORM/SORM) (1h30)
   - 2.1 Approximation of the failure domain
   - 2.2 Factor space transformations
   - 2.3 Implementation issues
   - 2.4 Sensitivity analysis
3. Advanced Monte Carlo methods (1h30)
   - 3.1 Statement of the objective
   - 3.2 Importance sampling
   - 3.3 Stratified sampling
   - 3.4 Conditional sampling
   - 3.5 Subset simulation
4. Use of metamodels (2 × 1h30)
   - 4.1 Controlled Monte Carlo

4.2 Sequential approximation using a Gaussian process metamodel
- Optimal statistical decisions in a Bayesian setting
- Gaussian models
- Review of some strategies in the literature
- Implementation
- Current limitations

4.3 Optimization + quantile estimation

Training sessions

Practical computer works (use of Matlab software): 4h30 divided in 3 sessions

(1) Estimation of a small probability of exceedence using Monte Carlo and extreme value theory
(2) FORM/SORM, application, sensitivity analysis, vs MC
(3) Bayesian (Gaussian process) model

Annex

Programme.

- 4 July
  - Morning (beginning 11h) : Course 3.1 Vazquez
  - Afternoon : TDs B et 3.1
- 5 July
  - Morning : Course 3.2 Vazquez - Seminar 3
  - Afternoon : TD 3.2 - Seminar 4
- 6 July
  - Morning : Course 3.3 Vazquez - Seminar 5
  - Afternoon : Atelier logiciel OpenTURNS (3h)
- 7 July
  - Morning : Cours 3.4 Vazquez - TD 3.3
  - Afternoon : Atelier logiciel URANIE (3h)
- 8 July
  - Morning : Course 3.5 Vazquez - Seminar 6 - Conclusion/Infos sur le sujet (1h)
  - End at 13h00