EADS Innovation Works - Applied Mathematics Team

Calibration of a PDE system for thermal regulation of an aircraft cabin

SMAI 2013, 27 - 31 of May 2013, Seignosse Le Penon

EADS France - *Applied Mathematics Team*

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Outline

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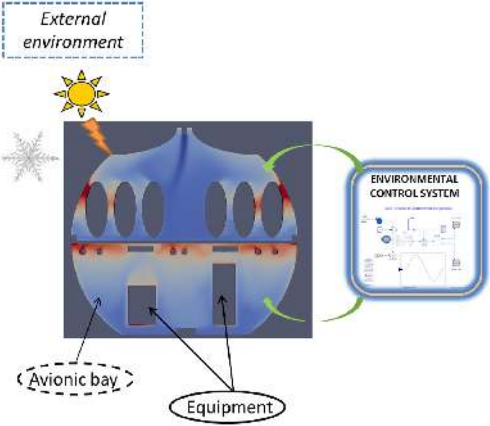
General context of thermal regulation



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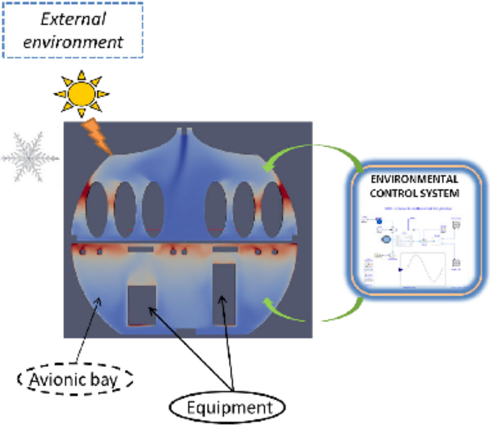

- Provide thermal comfort & cabin pressurization for crew / passengers
- Thermal control of electric cores or highly dissipative equipment of avionic bay

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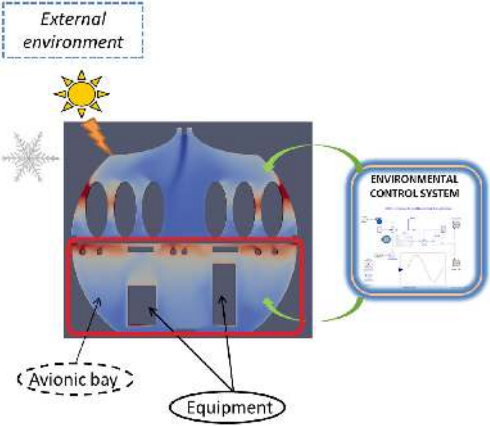

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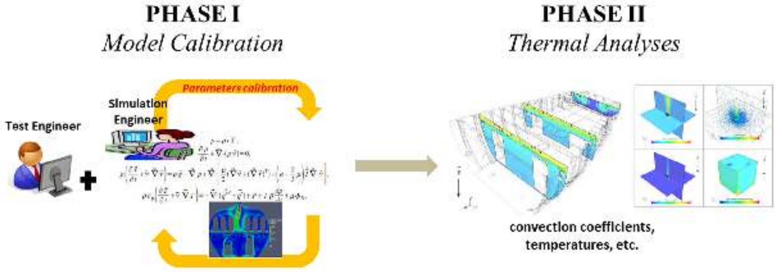
Topics of the presentation

Installation of equipment in avionic bay requires the specification of equipment thermal environment

Figure : Aircraft & Equipment - Avionic bay

- Need to provide convection coefficients around the equipment...
- ... For a robust equipment conception

Topics of the presentation



Two phases:

- 1/ PDE parameter estimation
- 2/ Phenomenon study with parametrized PDE

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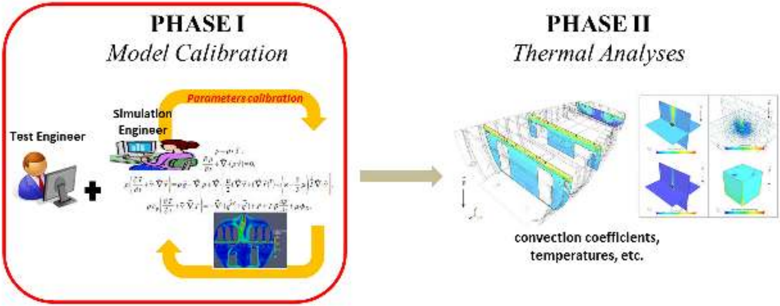

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Modelling

- **(simplified) Thermal exchange modelling** (Navier Stokes equations)

Equations :
$$\begin{cases} \frac{\partial \rho}{\partial t} + \nabla.(\rho\, u) &= 0 \\ \frac{\partial(\rho C_p T)}{\partial t} + \nabla.(u.\rho C_p T) &= \nabla.(k \nabla T) \\ \frac{\partial(\rho u)}{\partial t} + (u.\nabla)u + \nabla p &= \mu \Delta u + \rho g \end{cases}$$

Boundary Conditions :
$$\begin{cases} u &= u_0(M) \quad \text{with turbulence model RANS}(\tau) \\ \phi &= h_C\,(T - T_{Skin}) \end{cases}$$

ρ= air density, u=air speed, T=temperature, τ=turb. rate, h_C= heat transf. coef., T_{Skin}= skin temp.

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$\Rightarrow$ Lack of knowledge on τ, h_C and T_{Skin} ! $\Leftarrow$

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- **Input/Output model view**

Equation & Boundary Conditions induce an Input/Output system

$$\mathcal{H}\left((\tau, T_{Skin}), h_C\right) .$$

In particular, the post-processing providing convection coefficients is some function $h\left((\tau, T_{Skin}), h_C\right)$.

Question ?

How to estimate h_C in presence of uncertainties (τ, T_{Skin}) ?

- We need additional information (reference measures, experimental data, etc.)
- How to model the uncertainties ?
- How to take into account uncertainties in identification procedures ?

Outline

Experiments

Figure : Flight test - Chamber test

- **Principle:**
  At a fixed environmental condition, one can measure convection coefficients C_i^{obs} around the equipment.
  - Flight tests / Chamber tests
  - Few sensors are used

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  At a fixed environmental condition, one can measure convection coefficients C_i^{obs} around the equipment.
  - Flight tests / Chamber tests
  - Few sensors are used

  Finally, one gets a very precious database (C_i^{obs}) for $i = 1, ..., N$ with N limited !

Summary

We have two ingredients:

- We can compute convection coefficients of the equipment from Navier Stokes equations

$$C^{comp} = h\left((\tau, T_{Skin}), h_C\right)$$

- Experimental database

$$(C_i^{obs})_{i=1,\dots,N}$$

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Q : How to estimate h_C from the experimental database ?

Mathematical formalization

- **Variable of interest** (induced by a PDE system)
  We call a variable of interest any quantity obtained by a post-processing of some PDE equations resolution. It takes the form

$$h(\mathbf{X}, \boldsymbol{\theta}) \quad \text{field or scalar}$$

  where
  - $\mathbf{X} \in (\mathbb{R}^d, \mathcal{B}(\mathbb{R}^d), P_x)$ is a random vector representing the **uncertainties**
  - $\boldsymbol{\theta} \in \mathbb{R}^k$ is the vector of parameters to identify

  (in our application: $\mathbf{X} = (\tau, T_{Skin}) \in (\mathbb{R}^2, \mathcal{B}(\mathbb{R}^2), P_x)$ and $\boldsymbol{\theta} = h_C \in \mathbb{R}$)

- **Experimental/Reference data** (also called **Learning data**)
  It is a set of points:
  - $(z_i, Y_i)_{i=1,\dots,N} \rightarrow$ if $h(\mathbf{X}, \boldsymbol{\theta})$ is a field $z \mapsto h(\mathbf{X}, \boldsymbol{\theta})[z]$
  - $(Y_l)_{l=1,\dots,N} \rightarrow$ if $h(\mathbf{X}, \boldsymbol{\theta})$ is scalar
    (Remark: a priori, **There is not** a model linking the observation Y and the simulation $h(\mathbf{X}, \boldsymbol{\theta})$. For instance, we don't have the regression framework

$$Y = h(\mathbf{X}, \boldsymbol{\theta}) + \varepsilon$$

    where ε is the model error. Indeed, **we don't have** joint information $(\mathbf{X}_i, Y_i)$!)

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Calibration methods

There are two calibration methods depending on the nature of the variable of interest $h(\mathbf{X}, \boldsymbol{\theta})$, scalar or field.

Calibration method I (case for fields)

- **Least Squares principle:**
  Find parameters $\boldsymbol{\theta} \in \mathbb{R}^k$ which minimize the quantity

$$\mathcal{J}(\mathbf{X}, \boldsymbol{\theta}) = \sum_{i=1}^{N} (Y_i - h(\mathbf{X}, \boldsymbol{\theta})[z_i])^2$$

- **Remark !:**
  the function $\boldsymbol{\theta} \mapsto \mathcal{J}(\mathbf{X}, \boldsymbol{\theta})$ to minimize is random (due to uncertainties $\mathbf{X}$) !

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- **Issue: Stochastic Optimization**

  Principle: Minimize a quantity $\rho(\mathcal{J}(\mathbf{X}, \boldsymbol{\theta}))$ (deterministic)

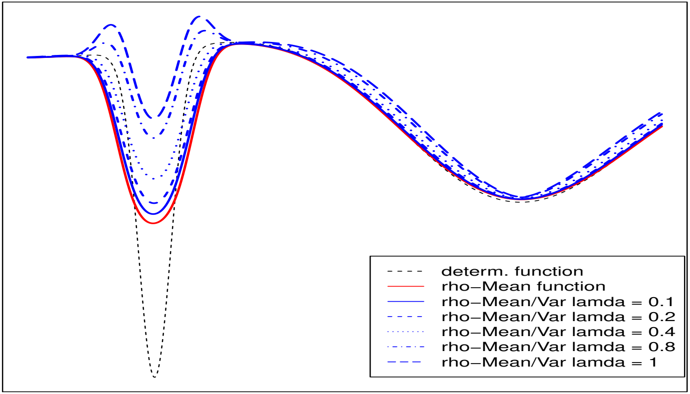
  - Mean : $\rho(\mathcal{J}(\mathbf{X}, \boldsymbol{\theta})) = \mathbb{E}_{\mathbf{X}}(\mathcal{J}(\mathbf{X}, \boldsymbol{\theta}))$
  - Variance : $\rho(\mathcal{J}(\mathbf{X}, \boldsymbol{\theta})) = \mathrm{Var}_{\mathbf{X}}(\mathcal{J}(\mathbf{X}, \boldsymbol{\theta}))$
  - Mixed : $\rho_\lambda(\mathcal{J}(\mathbf{X}, \boldsymbol{\theta})) = \mathbb{E}_{\mathbf{X}}(\mathcal{J}(\mathbf{X}, \boldsymbol{\theta})) + \lambda \sqrt{\mathrm{Var}_{\mathbf{X}}(\mathcal{J}(\mathbf{X}, \boldsymbol{\theta}))}$
  - etc.

Calibration method I (case for fields)

- **Illustration**

$$\boldsymbol{\theta} \mapsto \rho_\lambda(\mathcal{J}(\mathbf{X},\boldsymbol{\theta})) = \mathbb{E}_{\mathbf{X}}(\mathcal{J}(\mathbf{X},\boldsymbol{\theta})) + \lambda\sqrt{\mathbf{Var}_{\mathbf{X}}(\mathcal{J}(\mathbf{X},\boldsymbol{\theta}))}$$ **for different** $\lambda > 0$

(deterministic function $\Leftrightarrow \theta \mapsto \mathcal{J}(X_{nom}, \theta)$**, where** X_{nom} **is the nominal value of X)**



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Calibration method I (case for fields)

- **Stochastic/Robust Optimization**
  - **Large literature**

  - **Practical algorithms**
    Need practical and efficient algorithms ...

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Calibration method II (case for scalar outputs)

- **Recall the framework:**
  - We have observations $(Y_i)_{1,\ldots,N}$
  - We get a scalar output $h(\mathbf{X}, \theta)$ after a post-processing of a PDE system

Calibration method II (case for scalar outputs)

- **Recall the framework:**
  - We have observations $(Y_i)_{1,\ldots,N}$
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- **Estimation method:**

[Rachdi *et al* 2012] Risk bounds for new M-estimation problems, ESAIM:Probability & Statistics, 2012

Principle:

Find parameters $\boldsymbol{\theta} \in \mathbb{R}^k$ which minimize "a distance" between the **empirical distribution** of the Y_i's and the **simulated distribution** of the random variable $h(\mathbf{X}, \boldsymbol{\theta})$ (based on a simulated sample $h(\mathbf{X}_1, \boldsymbol{\theta}), \ldots, h(\mathbf{X}_m, \boldsymbol{\theta})$, where $\mathbf{X}_1, \ldots, \mathbf{X}_m$ are m simulations of the uncertainty $\mathbf{X}$).

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- **Example:** Maximum-Likelihood based method

[Rachdi *et al* 2012] Stochastic inverse problem with noisy simulator, Ann. Fac. Sc. Toulouse, 2012

Find $\boldsymbol{\theta}$ minimizing

$$\mathcal{J}(\boldsymbol{\theta}) = -\sum_{i=1}^{N} \log\left(\sum_{j=1}^{m} K_b(Y_i - h(\mathbf{X}_j, \boldsymbol{\theta}))\right), \quad \text{with} \quad K_b(y) = \frac{1}{\sqrt{2\pi}\, b} e^{-y^2/2b^2}$$

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Calibration method II (case for scalar outputs)

Theoretical results of the estimator $\widehat{\boldsymbol{\theta}}_{N,m}$ where

$$\widehat{\boldsymbol{\theta}}_{N,m} = \operatorname*{Argmin}_{\boldsymbol{\theta}\in\Theta} -\sum_{i=1}^{N} \log\left(\sum_{j=1}^{m} K_b(Y_i - h(\mathbf{X}_j, \boldsymbol{\theta}))\right)$$

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Theorem (Consistency) [Rachdi2012]

Denote by $f_{\theta}^{\mathbf{x}}$ the density function of $h(\mathbf{X}, \boldsymbol{\theta})$ and $\boldsymbol{\theta}^*$ by

$$\boldsymbol{\theta}^* = \underset{\boldsymbol{\theta}\in\Theta}{\text{Argmin}} -\mathbb{E}\left(\log(f_{\theta}^{\mathbf{x}})(Y)\right) \quad \text{(unknown target)}\,.$$

Under technical conditions, $\exists$ constants c_1, c_2, c_3, a_1 and a_2 such that $\forall\, 0 < \tau < 1/2$, with probability at least $1 - 2\,\tau$

$$\|\widehat{\boldsymbol{\theta}}_{N,m} - \boldsymbol{\theta}^*\|^2 \leq c_1 \sqrt{\frac{\log(a_1 \tau^{-1})}{N}} + \frac{c_2 \sqrt{\log(a_2 \tau^{-1})} + c_3\, m^{1/10}}{\sqrt{m}}\,.$$

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Theorem (Central Limit Theorem)

In progress !

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Simulation of h is limited !

- **Calibration may be very greedy ...**
  Both calibration methods may need several computations of h involving new PDE system resolutions.
  - In most of our applications, one run of h (i.e numerical resolution + post-processing) $\sim$ 6 hours
  - So for 50 calibration algorithm iterations, we have to wait $\sim$ 13 days !

- **Strategy adopted:**

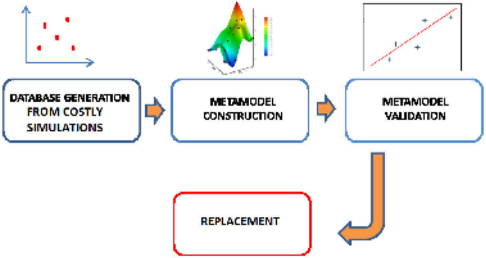
  Replace the CPU time expensive model $h(\mathbf{X}, \boldsymbol{\theta})$ by a mathematical approximation (analytical) $\widetilde{h}(\mathbf{X}, \boldsymbol{\theta})$, very cheap to evaluate.

Outline

Meta model strategy

A well adopted strategy (among others...) :

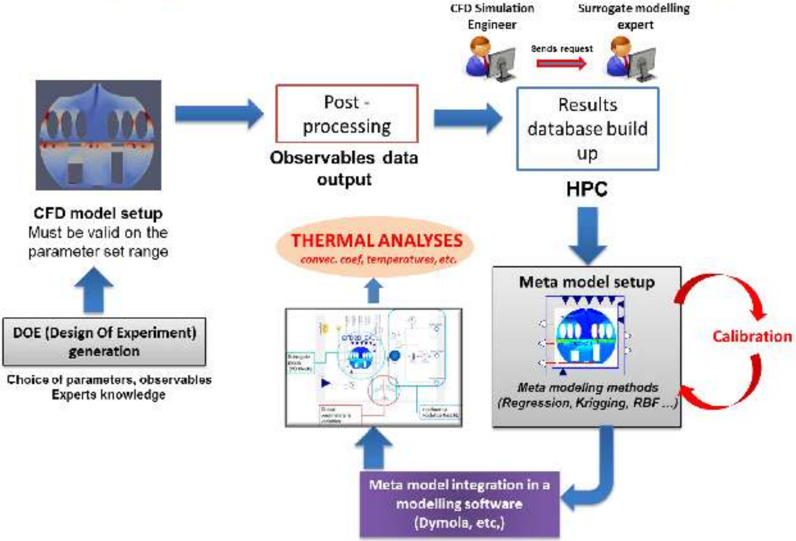
- **Sample, Build, Validate and Replace**



- **Different types of meta models**
    - **Regression-based:** (*Neural network, Polynomial Chaos, Least squares, etc.*)
    - **Interpolation-based:** (*Radial Basis Functions, Gaussian processes/Kriging, etc.*)
- Calibration methods only involve the metamodel, i.e one calibrates the metamodel ! (no more the PDE system...)

Outline

Summary : global process of thermal analysis



Conclusions & Issues

- Asymptotic study of the estimator $\widehat{\boldsymbol{\theta}}_{N,m}$
- Mathematical study of calibration procedures induced by the Stochastic Optimization of $\boldsymbol{\theta} \mapsto \mathcal{J}(\mathbf{X}, \boldsymbol{\theta})$
- Quantify the robustness of equipment specification when considering the uncertainties
- Improve existing metamodel-based algorithms (adaptive metamodelling, on-line refinement, etc.)
- HPC capabilities for metamodel constructions
- Facilitate metamodels exportation (distribution to suppliers, etc.)
- Extend the method for Multi-Fidelity learning data (varying mesh size, etc.)

Thank you for your attention !

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