Incorporating multi-source uncertainties in fast building wall thermal resistance estimation through physics-based and multifidelity statistical learning models with functional outputs

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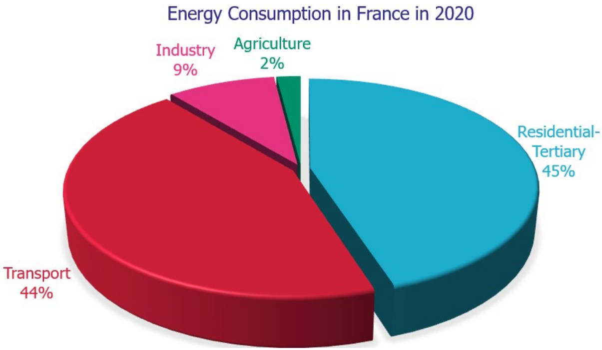
Context



Ref : SDES, Bilan énergétique de la France.

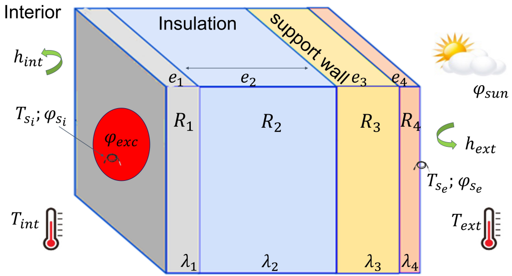
- Reducing energy consumption.
- Renovating buildings.
- Good understanding of energy performance.
- Identifying of thermal resistance of walls by associating confidence to the estimation.

Objectives

- Propose a thermal model with **functional inputs and outputs** and with reduced **external dependency**.
- Reduce the **calculation costs** associated with direct thermal model evaluation.
- Provide a **precise estimate** of the thermal proprieties ($\boldsymbol{z} \in R^8$) with a credible interval.
- Quantify the differents sources of **uncertainties**.
- Study of the influence of the **experimental time** and the **instrumentation** used.

Case studied

Cartesian geometry for an ITI wall



- Measured : $T_{\mathrm{ext}}, T_{\mathrm{int}}, T_{\mathrm{S_i}}, T_{\mathrm{S_e}}, \varphi_{\mathrm{S_i}}, \varphi_{\mathrm{S_e}}$.
- Can be measured : $\Phi_{\mathrm{sun}}, \Phi_{\mathrm{exc}}$.
- Unknown : e_i, $T_{\mathrm{W_0}}$, h_{int}, h_{ext}, $e\rho C_i$, R_i.
- $e_{Tot} = \sum_{i=1}^{4} e_i$.
- $R_{Tot} = \sum_{i=1}^{4} R_i$ / $e\rho C_{Tot} = \sum_{i=1}^{4} e\rho C_i$.

$$\boldsymbol{z} = (R_1, R_2, R_3, R_4, e\rho C_1, e\rho C_2, e\rho C_3, e\rho C_4)$$

1 Introduction

2 Bayesian Formalism

3 Multi-Fidelity approach

4 Applications

5 Conclusion

Bayesian Formalism

Estimation of $\boldsymbol{z}$ in Bayesian framework

$$\pi(\boldsymbol{z}|\boldsymbol{y}^{\mathrm{mes}}) = \frac{\pi(\boldsymbol{z}) \times \pi(\boldsymbol{y}^{\mathrm{mes}}|\boldsymbol{z})}{\int_{\boldsymbol{z}'} \pi(\boldsymbol{z}') \times \pi(\boldsymbol{y}^{\mathrm{mes}}|\boldsymbol{z}') d\boldsymbol{z}'}, \quad \boldsymbol{z} \in \Omega(\mathbb{Z}), \tag{1}$$

- $\boldsymbol{y}^{\mathrm{mes}}$ gathers the discretisation of the measured values of $\varphi_{\mathrm{S_i}}$ and $\varphi_{\mathrm{S_e}}$ in $N_t >> 1$.
- $\pi(\boldsymbol{z}|\boldsymbol{y}^{\mathrm{mes}})$: A posteriori distribution.
- $\pi(\boldsymbol{z})$: A priori probability density function (PDF) of $\boldsymbol{z}$, uniform in $\Omega(\mathbb{Z}) \subset R^8$.
- $\pi(\boldsymbol{y}^{\mathrm{mes}}|\boldsymbol{z})$: Likelihood function.

Bayesian Formalism

$$\boldsymbol{y}^{\mathrm{mes}} = \boldsymbol{y} + \varepsilon_{\varphi}^{\mathrm{mes}}, \tag{2}$$

$$\boldsymbol{y}^{\mathrm{mes}} = \boldsymbol{u}^{3D}(\boldsymbol{z}; T_{\mathrm{S_i}}, T_{\mathrm{S_e}}, T_{\mathrm{W_0}}) + \varepsilon_{\varphi}^{\mathrm{mes}}, \tag{3}$$

$$\boldsymbol{y}^{\mathrm{mes}} = \boldsymbol{u}^{\mathrm{Meta}}(\boldsymbol{z}; T_{\mathrm{S_i}}^{\mathrm{Smooth}}, T_{\mathrm{S_e}}^{\mathrm{Smooth}}, T_{\mathrm{w}}^{\mathrm{Poly}}) + \varepsilon_{\varphi}^{\mathrm{mes}} + \varepsilon_{\mathrm{T}}^{\mathrm{mes}} + \varepsilon^{\mathrm{Meta}}(\boldsymbol{z}) + \varepsilon^{\mathrm{Res}}, \tag{4}$$

$$\pi(\boldsymbol{y}^{\mathrm{mes}}|\boldsymbol{z}) = \frac{\exp\left(-\frac{1}{2}\left(\boldsymbol{y}^{\mathrm{mes}} - \boldsymbol{u}^{\mathrm{Meta}}(\boldsymbol{z})\right)^T \boldsymbol{W}(\boldsymbol{z})^{-1}\left(\boldsymbol{y}^{\mathrm{mes}} - \boldsymbol{u}^{\mathrm{Meta}}(\boldsymbol{z})\right)\right)}{\sqrt{(2\pi)^{N_t}\det(\boldsymbol{W}(\boldsymbol{z}))}}, \quad \boldsymbol{z} \in \Omega(\mathbb{Z}). \tag{5}$$

$\boldsymbol{W}(\boldsymbol{z})$ correspond to the sum of the covariance matrix related to each error.

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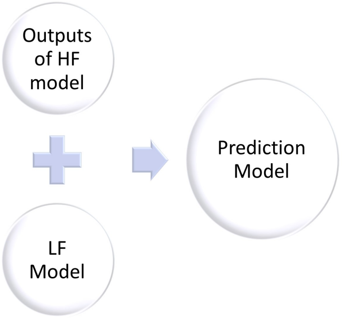
1. Introduction
2. Bayesian Formalism
3. **Multi-Fidelity approach**
4. Applications
5. Conclusion

Multi-Fidelity

Objective and Data prepartion

Statistical approach proposed to create a meta-model of the complexe model.

- Construct a **fast predictor** of the High Fidelity model.
- Enable the most accurate prediction of quantities of interest at the **lowest cost**.

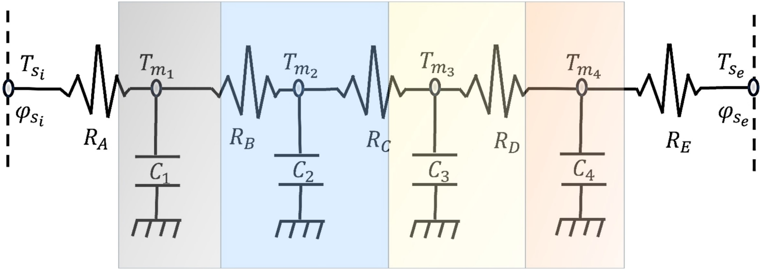


Models used

Level of fidelity

$$HF\ model\ :\ e_i \times \rho C_i \frac{\partial T_i}{\partial t} - \frac{1}{R_i} \frac{\partial^2 T_i}{\partial {x_i}^2} = 0 \quad x_i \in [0, 1]$$

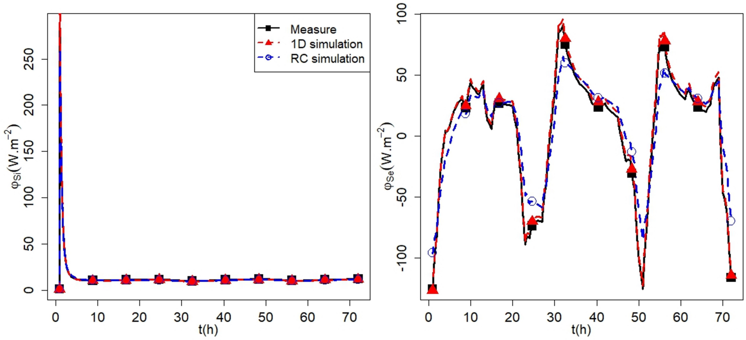
LF model :



- $R_A = R_1/2$
- $R_C = R_2/2 + R_3/2$
- $R_D = R_3/2 + R_4/2$
- $R_E = R_4/2$
- $C_i = e\rho C_i$ for $i \in [1, 4]$

Models used

Comparison for a given set of parameters



Multi-Fidelity

Creation of the meta-model using Gaussian Process (GP)[1]

$$\boldsymbol{Y} \sim \mathrm{GP}(\boldsymbol{\mu}, \boldsymbol{C}),$$

$$\boldsymbol{\mu}(\boldsymbol{z}) = \alpha \times \boldsymbol{u}^{\mathrm{RC}}(\boldsymbol{z}; T_{\mathrm{S_i}}^{\mathrm{Smooth}}, T_{\mathrm{S_e}}^{\mathrm{Smooth}}, T_{\mathrm{w}}^{\mathrm{Poly}}),$$

- α is a correlation coefficient estimated between the HF and LF model,
- $\boldsymbol{u}^{\mathrm{RC}}$ corresponds to the discretization of the output of the LF model.

$$\boldsymbol{C}(\boldsymbol{z}, \boldsymbol{z}') = \boldsymbol{R}_t \times C_z(\boldsymbol{z}, \boldsymbol{z}'),$$

- $\boldsymbol{R}_t$ is a $(N_t \times N_t)$-dimensional positive definite matrix,
- $C_z(\boldsymbol{z}, \boldsymbol{z}')$ choosen as Matérn-5/2 class of covariance functions.

[1] G. Perrin. Adaptive calibration of a computer code with time-series output. Reliability engineering and system safety, 196:106728, 2020

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Multi-Fidelity

Prediction Quality

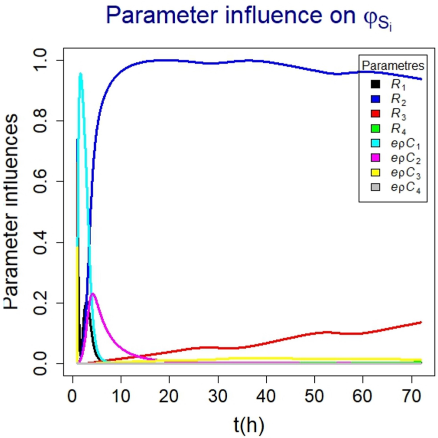
$$\Delta_i^2 = \frac{1}{\mathrm{N_t}} \sum_{j=1}^{\mathrm{N_t}} \left((\boldsymbol{u}_i^{\mathrm{1D}})_j - (\boldsymbol{u}_i^{\mathrm{Meta}})_j \right)^2, \; \delta_i = \frac{1}{\mathrm{N_t}} \sum_{j=1}^{\mathrm{N_t}} |(\boldsymbol{u}_i^{\mathrm{1D}})_j - (\boldsymbol{u}_i^{\mathrm{Meta}})_j|.$$

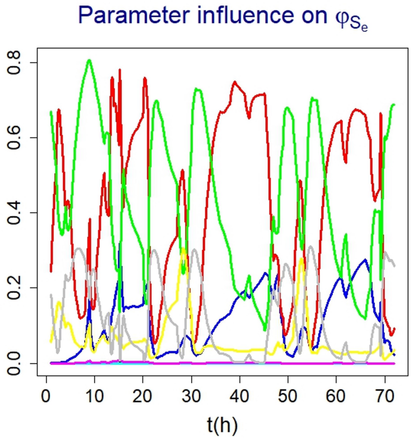
$$\mathrm{RMSE}^* = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \Delta_i^2}, \; \mathrm{MAE}^* = \frac{1}{n} \sum_{i=1}^{n} \delta_i$$

Indicator	$\varphi_{\mathrm{S_i}}$	$\varphi_{\mathrm{S_e}}$
RMSE^* ($\mathrm{W.m^{-2}}$)	0.10	0.13
MAE^* ($\mathrm{W.m^{-2}}$)	0.05	0.06
Measurement error	1.09	2.59

Sensibility Analysis

Total Sobol Index





Applications

- Numericals tests for two weather conditions (M_1 & M_2),
- Estimation of the thermal properties of four internal insulated walls (IIW)

Configuration	Stabilisation time (h)	Excitation time (h)	Measurements
$\boldsymbol{C_1}$	**72**	**72**	$\boldsymbol{\varphi_{S_i}, \varphi_{S_e}}$
C_2	72	72	φ_{S_i}
C_3	0	72	$\varphi_{S_i}, \varphi_{S_e}$
C_4	0	72	φ_{S_i}
C_5	0	10	$\varphi_{S_i}, \varphi_{S_e}$
C_6	0	10	φ_{S_i}

Numerical Application

$e_{Tot} = 0.36\ m$ & $R_{Tot} = 4.72\ m^2.K.W^{-1}$

Layer	R^{Ref}	R^{Est}	$e\rho C^{\mathrm{Ref}}$	$e\rho C^{\mathrm{Est}}$
#1 - Internal coating	0.05	0.06[0.05; 0.07]	9.7	9.22[8.82; 9.64]
#2 - Insulation	4.37	4.20[4.05; 4.36]	5.7	4.98[3.67; 7.18]
#3 - Wall support	0.28	0.24[0.16; 0.32]	160	120[100; 166]
#4 - External coating	0.008	0.04[0.01; 0.07]	15	39.1[15.5; 57.3]
Total Wall	**4.72**	**4.53[4.41; 4.70]**	**190**	**171[151; 211]**

By applying the proposed method, in the recommanded configuration we are able to estimate with a **high accuracy** the R_{Tot} (especially R_2) and $e\rho C_{\mathrm{Tot}}$.

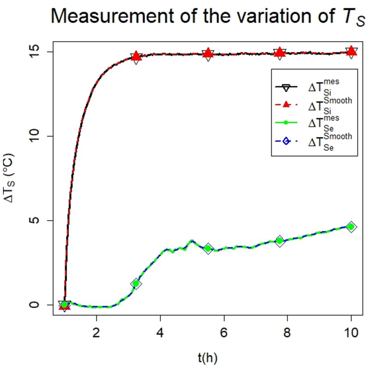
Numerical Application

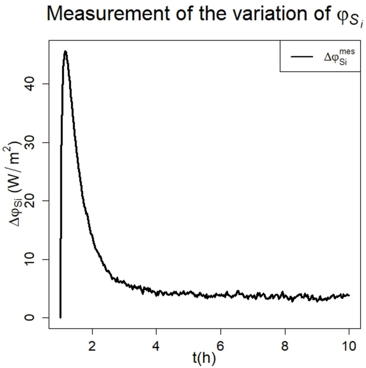
$e_{Tot} = 0.36\ m$ & $R_{Tot} = 4.72\ m^2.K.W^{-1}$

Estimated parameters	R^{Est} ($R^{\text{Ref}} = 4.72\,\text{m}^2.\text{K.W}^{-1}$)		$e\rho C^{\text{Est}}$ ($e\rho C^{\text{Ref}} = 190\,\text{kJ.K}^{-1}.\text{m}^{-2}$)	
Weather	M_1	M_2	M_1	M_2
C_1	**4.53[4.41; 4.70]**	**4.82[4.34; 5.08]**	**171[151; 211]**	**195[158; 796]**
C_2	4.50[4.42; 4.59]	4.42[4.18; 4.77]	181[143; 383]	210[159; 765]
C_3	4.42[4.30; 4.54]	4.88[4.75; 5.04]	173[147; 205]	721[468; 816]
C_4	4.41[4.19; 4.76]	4.49[4.30; 4.75]	188[137; 749]	286[149; 667]
C_5	5.23[4.73; 5.70]	5.53[4.63; 6.78]	189[163; 772]	673[229; 826]
C_6	5.10[4.92; 7.14]	6.37[4.46; 11.97]	199[138; 769]	288[199; 816]

Experimental Application[2].

$e_{Tot} = 0.29\ m$ & $R^{Mat}_{Tot} = 4.01\ m^2.K.W^{-1}$





[2] Ha et al. Measurement prototype for fast estimation of building wall thermal resistance under controlled and natural environmental conditions.Energy and Buildings, 268:112166, 2022

Experimental Application

Identification results

	R^{Mat}	R^{Est}
R_1	0.05	0.14[0.09; 0.14]
R_2	3.75	3.65[3.39; 4.02]
R_3	0.20	2.61[0.17; 2.93]
R_4	0.01	0.04[0.01; 0.29]
$\boldsymbol{R_{\text{Tot}}}$	**4.01**	**6.42[4.24; 6.86]**

- **Few measurements and instruments** used in this experimental application,
- An **estimate of** R_2 contributing most in R_{Tot} is provided with high accuracy,
- An accurate estimate of the **wall's minimum resistance** is provided, which is interesting information.

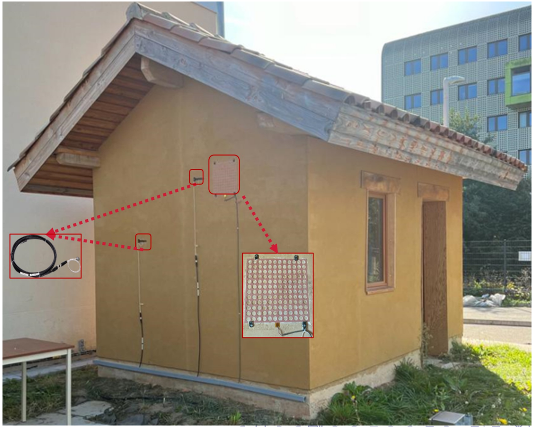
Conclusion

- A model with **reduced dependence** on the external environment is proposed.
- Create a meta-model based on the **multi-fidelity approach**.
- Quantify the various **sources of uncertainty**.
- Identify R_{tot} based on a Bayesian framework and using the **1D predictor**.
- Study of the influence of excitation time, instrumentation and wall initial condition.
- Numerical and experimental applications to real wall data.

Perspectives

In progress...

- Create a **3D meta-model** based on the multi-fidelity approach by adding a third level of fidelity.
- Switch to **hygrothermal models** to study humidity in addition to thermal resistance and capacitance.
- **Experimental applications** on the raw-earth house built and instrumented in SenseCity.



Thank you !

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H. Nasser et al., Incorporating Multi-Source Uncertainties in Fast Building Wall Thermal Resistance Estimation Through Physics-Based and Multi-Fidelity Statistical Learning Models with Functional Outputs, 2024. Available at SSRN 4862899.

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- Ha et al. Benchmark of identification methods for the estimation of building wall thermal resistance using active method: Numerical study for iwi and single-wall structures. Energy and Buildings, 224:110130, 2020.
- Ha et al. Measurement prototype for fast estimation of building wall thermal resistanceunder controlled and natural environmental conditions.Energy and Buildings, 268:112166, 2022.
- G. Perrin. Adaptive calibration of a computer code with time-series output. Reliability engineering and system safety, 196:106728, 2020

Appendix - Data Preparation

Layer - $\#i$	R_i	$e_i \times \rho C_i$
Internal layer - $\#1$	$0.0125 \leq R_1 \leq 0.15$	$6000 \leq e_1 \rho C_1 \leq 45000$
Insulation - $\#2$	$0.66 \leq R_2 \leq 10$	$1080 \leq e_2 \rho C_2 \leq 42000$
Support wall - $\#3$	$0.065 \leq R_3 \leq 3$	$97500 \leq e_3 \rho C_3 \leq 750000$
External layer - $\#4$	$0.0055 \leq R_4 \leq 0.3$	$5000 \leq e_4 \rho C_4 \leq 60000$

Table: Range of possible values for the parameters of the 1D model.

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Appendix - Estimation of α

$$\mathcal{V}_n(\alpha) \propto \frac{1}{(2\pi)^{\frac{N_t}{2}}} \frac{1}{det(cov)^{\frac{1}{2}}} exp(-\frac{1}{2}(\alpha y^{0D} - y^{1D})^T cov^{-1}(\alpha y^{0D} - y^{1D})) \quad (6)$$

$$\propto exp(-\frac{1}{2}(\alpha^2 y^{0D^T} cov^{-1} y^{0D} - 2\alpha y^{0D^T} cov^{-1} y^{1D})) \quad (7)$$

$$Log(\mathcal{V}_n(\alpha)) \propto -\frac{1}{2}\alpha^2 \underbrace{y^{0D^T} cov^{-1} y^{0D}}_{C_1} + \alpha \underbrace{y^{0D^T} cov^{-1} y^{1D}}_{C_2} \quad (8)$$

So the estimation of α :

$$\alpha^* = \frac{C_2}{C_1} \quad (9)$$

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Appendix Sobol - Concept of the method 1

S_i corresponds to the first order Sobol indices, which are interpreted as the part of the variance of Y explained by X_i only.

$$0 \leq S_i = \frac{Var(\mathbb{E}(Y|X_i))}{Var(Y)} \leq 1 \tag{10}$$

Evaluation of the total Sobol index

$$T^i = S^i + \sum_{j \neq i} S_{ij} + \sum_{1 \leq j \leq k \leq d, i \neq j, i \neq k} S_{ijk} \cdots + S_{12 \ldots d} \tag{11}$$

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Appendix Sobol- Concept of the method 2

Using the Hoeffding-Sobol decomposition :

$$g(X) = g_0 + \sum_{i=1}^{d} g_i(X_i) + \sum_{1<i<j<d} g_{i,j}(X, X_j) + \cdots + g_{1,2,\ldots,d}(X_a, X_2, \ldots, X_d) \tag{12}$$

$$var(g(X)) = \sum_{i=1}^{d} V_i + \sum_{1\leq i\leq j\leq d} V_{ij} + \cdots + V_{1,\ldots,d} \tag{13}$$

$$1 = \sum_{i=1}^{d} S_i + \sum_{1\leq i\leq j\leq d} S_{ij} + \cdots + S_{12\ldots d} \tag{14}$$

$$0 \leq S_i, S_{ij}, \cdots + S_{12\ldots d} \leq 1 \tag{15}$$

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Appendix : Influence of exchanges coeff 1

Concept

Berger et al. 2022 propose an estimation of exchange coefficients within a Bayesian framework and demonstrate the significance of these coefficients on the measurements.

In the previous work :

$$h_i = 7.7 W.(m^2.K)^{-1}.$$

$$h_e = 25 W.(m^2.K)^{-1}.$$

To study the influence of the exchanges coefficients, we consider :

$$h_i = 5 \ to \ 9.3 \ W.(m^2.K)^{-1}.$$

$$h_e = 4.3 \ to \ 37 \ W.(m^2.K)^{-1}.$$

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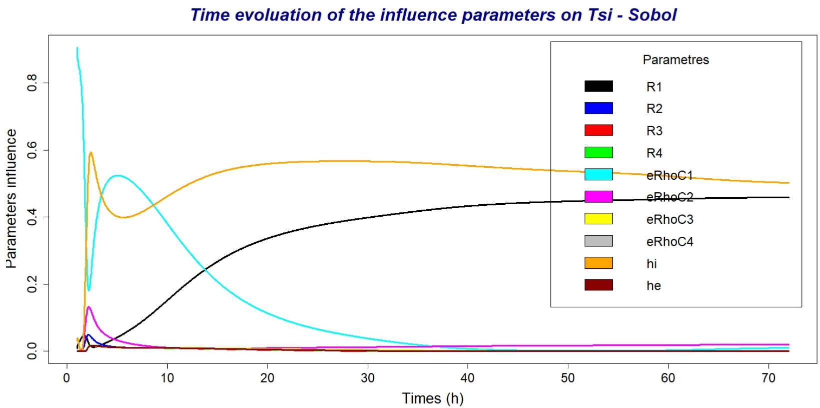
Appendix : Influence of exchanges coeff 1

Sobol Study



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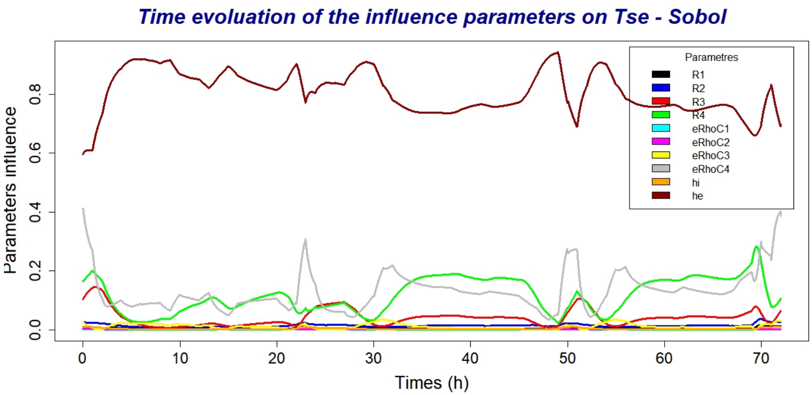
Appendix : Influence of exchanges coeff 2

Sobol Study



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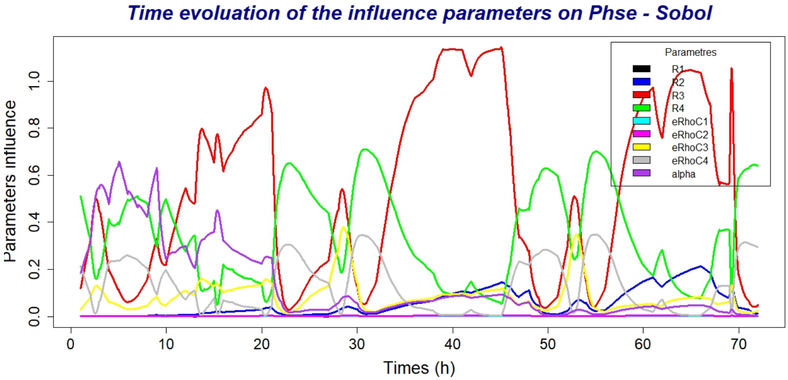
Appendix : Influence of IC 1

Influence of steady-state IC

$$IC = \alpha IC_{Ref} + (1 - \alpha) IC_{Stat} \text{ with } \alpha \in [0, 1]$$



Appendix : Influence of IC 2

Polynomial IC

Temperature equation :

$$T_{i_0}(x) = a_i x + b_i \ \ pour \ i = 1, 2$$

$$T_{i_0}(x) = a_i x^2 + b_i x + c_i \ \ pour \ i = 3, 4$$

According to Fourier's law, the flux equations :

$$\varphi_{i_0}(x) = -\frac{a_i}{R_i} \ \ pour \ i = 1, 2$$

$$\varphi_{i_0}(x) = -\frac{2a_i x + b_i}{R_i} \ \ pour \ i = 3, 4$$

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Appendix : Influence of IC 3

Polynomial IC

4 Measurements Conditions at t=0

$$T_1(x=0) = T_{S_i}$$

$$\varphi_1(x=0) = \varphi_{S_i}$$

$$T_4(x=4) = T_{S_e}$$

$$\varphi_4(x=4) = \varphi_{S_e}$$

6 Continuity Conditions at t=0

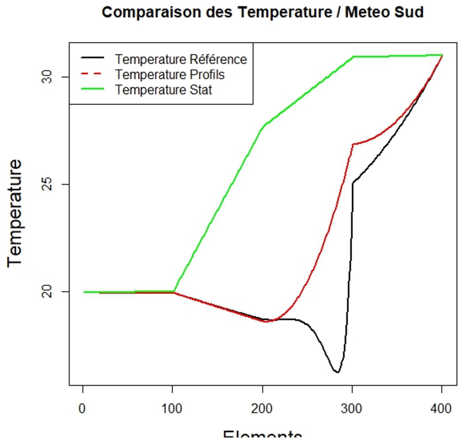
$$T_1(x=1) = T_2(x=1) \qquad \varphi_1(x=1) = \varphi_2(x=1)$$

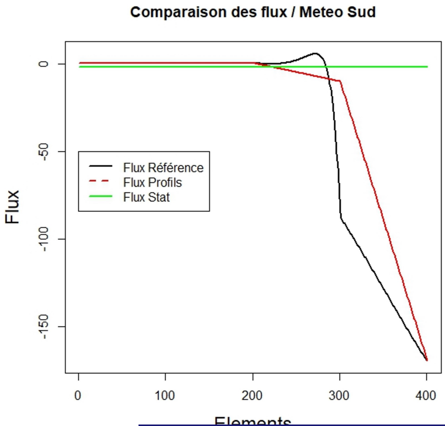
$$T_2(x=2) = T_3(x=2) \qquad \varphi_2(x=2) = \varphi_3(x=2)$$

$$T_3(x=3) = T_4(x=3) \qquad \varphi_3(x=3) = \varphi_4(x=3)$$

Appendix : Influence of IC 4

Comparison of polynomial and steady-state IC - Carpentras Weather





Appendix : Influence of IC 5

Impact on quantities of interest - Carpentras Weather