



# Electrical Machine design via Bayesian optimization under uncertainties

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- Context and Problematic
- Bayesian Optimization under uncertainty: EFISUR algorithm
- Modified EFISUR algorithm
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- Conclusions and perspectives

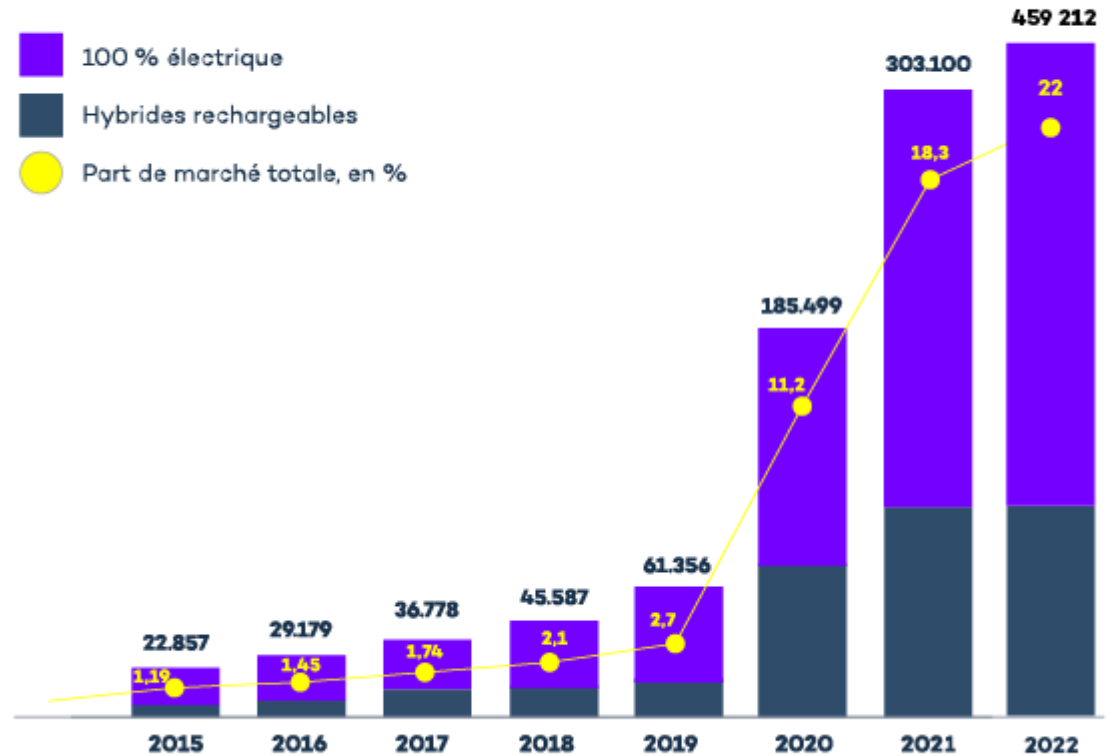
## ➤ Context

- The number of electric and hybrid vehicles (EV, HEV) is growing.
- Electric motor manufacturing requirements :
  - Minimizing cost
  - Maximizing efficiency
  - Ensure performances and specifications.

### CONSTRAINED MULTI-OBJECTIVE optimization

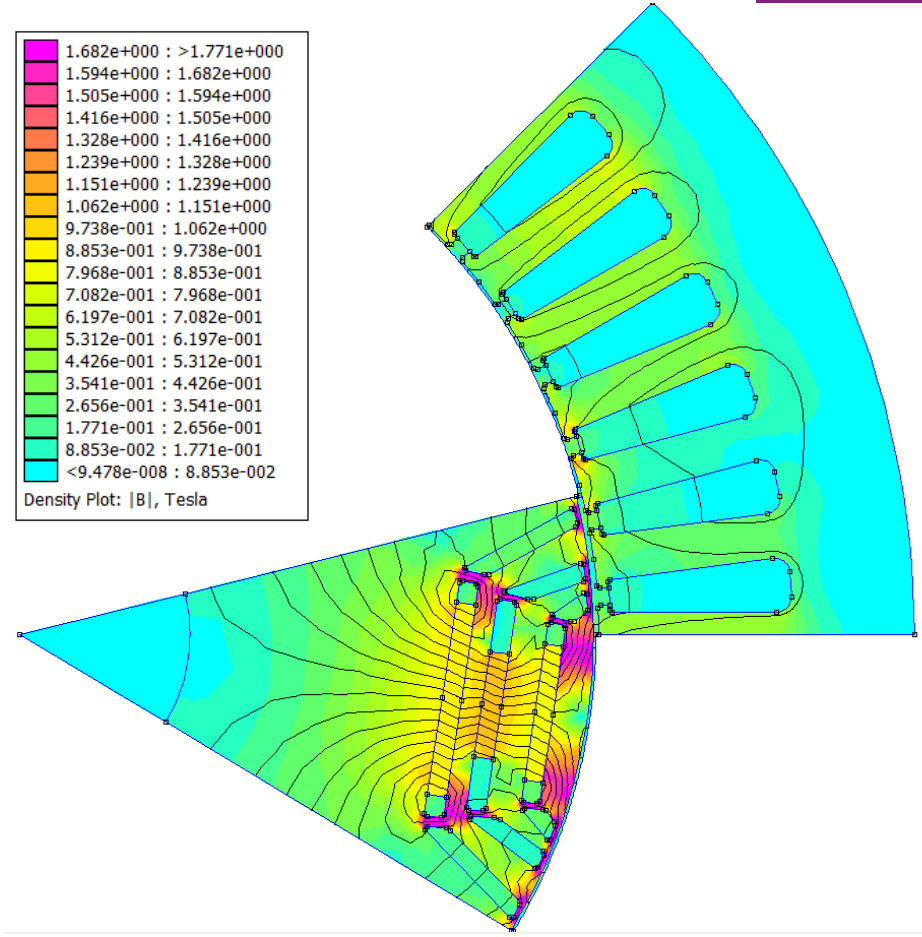
## L'évolution du nombre de véhicules vendus en France

En nombre de véhicules neufs



Source: PFA  
<https://www.virta.global/fr/marche-francais-vehicules-electriques-statistiques-predictions>

- Electrical machine optimization :
- Non-linear, generic model -> Finite element simulations (Time consuming).
- Complex multi-physical system: electromagnetic, mechanical, thermal.

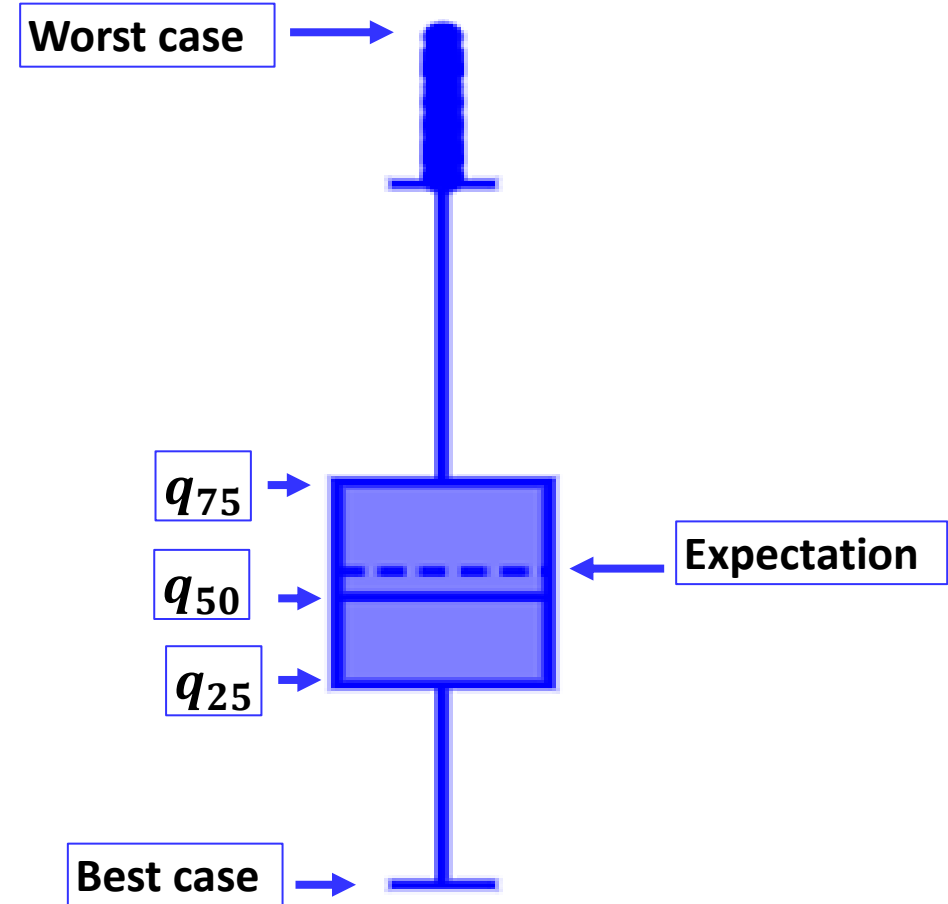
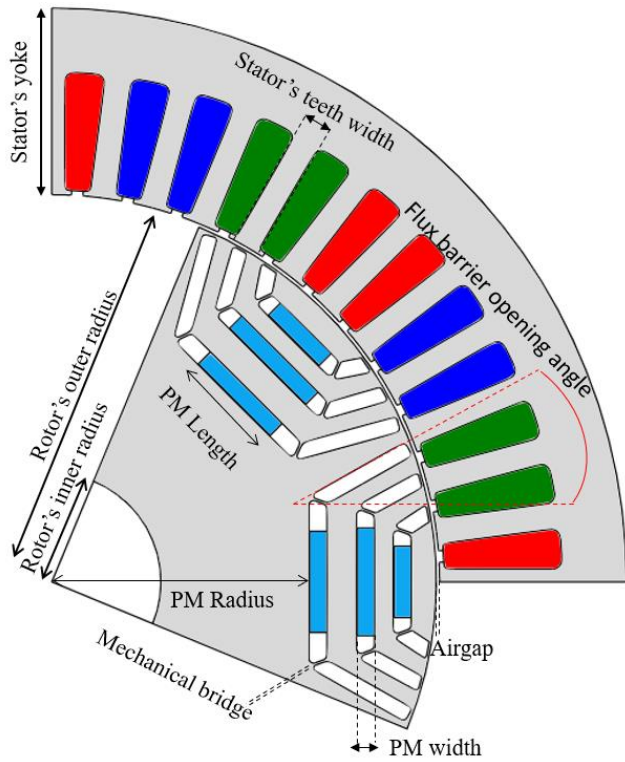


## BLACK-BOX constrained multi-objective optimization

# Context and Problematic: Uncertainties in an electrical machine and their impact on optimal values

Uncertainties:

- Magnetic properties of materials [1].
- Manufacturing tolerances on certain geometric parameters [2].
- Assembly tolerances on certain geometric parameters [3].



## Black-box constrained multi-objective optimization UNDER UNCERTAINTY

- Complex problem: Black-box, multiple objectives, constraints, input uncertainties
- Parallel computing facilities: several machines can be simulated at once
- Several uncertainties  $U$  can be defined as dispersion around controllable variables  $x$ .
  - For example, we might optimize permanent magnet dimensions, but there exists uncertainties around its desired dimensions.
- Limited budget of simulations.
- Bayesian Optimization has been successfully employed in a previous study of electrical machine optimization without input uncertainties [4].

# Bayesian optimization under uncertainty: EFISUR algorithm (Expected Feasible Improvement with Stepwise Uncertainty Reduction sampling)

Problem Definition :

$$\min_{\mathbf{x} \in D_x} \mathbb{E}_{\mathbf{U}} [f(\mathbf{x}, \mathbf{U})] \text{ s.t. } \mathbb{P}_{\mathbf{U}} (g_i(\mathbf{x}, \mathbf{U}), i = 1, \dots, l) \leq 0 \geq 1 - \alpha$$

**Assumption:** objective functions  $f$  and  $g$  as realizations of Gaussian processes :

$$F(\mathbf{x}, \mathbf{u}) \sim \mathcal{GP} (m_F(\mathbf{x}, \mathbf{u}), k_F(\mathbf{x}, \mathbf{u}, \mathbf{x}', \mathbf{u}'))$$
$$\forall i = \{1, \dots, l\}, G_i(\mathbf{x}, \mathbf{u}) \sim \mathcal{GP} (m_{G_i}(\mathbf{x}, \mathbf{u}), k_{G_i}(\mathbf{x}, \mathbf{u}, \mathbf{x}', \mathbf{u}'))$$

For the objective, the expectation of a Gaussian process (conditioned at any time  $t$ ) **is still a Gaussian process** :

$$Z^{(t)}(\mathbf{x}) = \mathbb{E}_{\mathbf{U}} \left[ F^{(t)}(\mathbf{x}, \mathbf{U}) \right]$$

For constraints, the probability of a Gaussian process is not a Gaussian process, but we can always define the following process whose realizations can be simulated :

$$C^{(t)}(\mathbf{x}) = 1 - \alpha - \mathbb{E}_{\mathbf{U}} \left[ \mathbf{1}_{\bigcap_{i=1}^l \left\{ G_i^{(t)}(\mathbf{x}, \mathbf{U}) \leq 0 \right\}} \right]$$

# Bayesian optimization under uncertainty: EFISUR algorithm

As in every Bayesian Optimization Algorithm, **an infill criterion is needed**. In the case of EFISUR, this criterion is divided in two, one to obtain  $x^{t+1}$ , and other one to obtain  $u^{t+1}$  [5].

The first one is the **Expected Feasible Improvement (EFI)**:

$$\mathbf{x}^{t+1} = \arg \max_{\mathbf{x} \in D_x} \text{EFI}^{(t)}(\mathbf{x})$$

$$\text{EFI}^{(t)}(\mathbf{x}) = \mathbb{E} \left[ \text{FI}^{(t)}(\mathbf{x}) \right] = \mathbb{E} \left[ \mathbf{1}_{\{C^{(t)}(\mathbf{x}) \leq 0\}} \left( z_{\min}^{\text{feas}} - Z^{(t)}(\mathbf{x}) \right)^+ \right] = \mathbb{P}(C^{(t)}(\mathbf{x}) \leq 0) \mathbb{E} \left[ \left( z_{\min}^{\text{feas}} - Z^{(t)}(\mathbf{x}) \right)^+ \right]$$

To obtain  $u^{t+1}$ , a criterion based on **Stepwise Uncertainty Reduction (SUR)** is proposed and has the following expression:

$$\mathbf{u}^{t+1} = \arg \min_{\tilde{\mathbf{u}} \in D_U} S(\mathbf{x}^{t+1}, \tilde{\mathbf{u}})$$

$$S(\mathbf{x}^{t+1}, \tilde{\mathbf{u}}) = \underbrace{\text{VAR} \left( \left( z_{\min}^{\text{feas}} - Z^{(t+1)}(\mathbf{x}^{t+1}) \right)^+ \right)}_{\text{Variance of future improvement}} \int \underbrace{\text{VAR} \left( \mathbf{1}_{\{G_i^{(t+1)}(\mathbf{x}^{t+1}, \mathbf{u}) \leq 0\}} \right)}_{\text{Variance of the variable quantifying constraints satisfaction}} \rho_U(\mathbf{u}) d\mathbf{u}$$

Variance of future  
improvement

Variance of the variable quantifying  
constraints satisfaction



# Our modified EFISUR algorithm

$$\begin{aligned} & \min_{\mathbf{x} \in D_x} \mathbb{E}_U[f(\mathbf{x}, \mathbf{U})] \\ & \text{s. t. } \mathbb{P}_U(g_i(\mathbf{x}, \mathbf{U}) \leq 0, i = 1, \dots, l) \geq 1 - \alpha \end{aligned}$$



Transition to the **(x+U) formulation** and **individual probability constraints** :

$$\begin{aligned} & \min_{\mathbf{x} \in D_x} \mathbb{E}_U[f(\mathbf{x} + \mathbf{U})] \\ & \text{s. t. } \mathbb{P}_U(g_i(\mathbf{x} + \mathbf{U}) \leq 0) \geq 1 - \alpha_i \\ & \quad i = 1, \dots, l \end{aligned}$$

- Many controllable variables in an electrical machine also carry uncertainties -> **Reduction of input dimension**

Switching to a **multi-objective formulation**:

$$\begin{aligned} & \min_{\mathbf{x} \in D_x} \mathbb{E}_U \left[ \sum_{j=1}^k w_j f_j(\mathbf{x} + \mathbf{U}) \right] \\ & \text{s. t. } \mathbb{P}_U(g_i(\mathbf{x} + \mathbf{U}) \leq 0) \geq 1 - \alpha_i, i = 1, \dots, l \end{aligned}$$

- By doing this the insides of the EFISUR algorithm rest **unchanged**

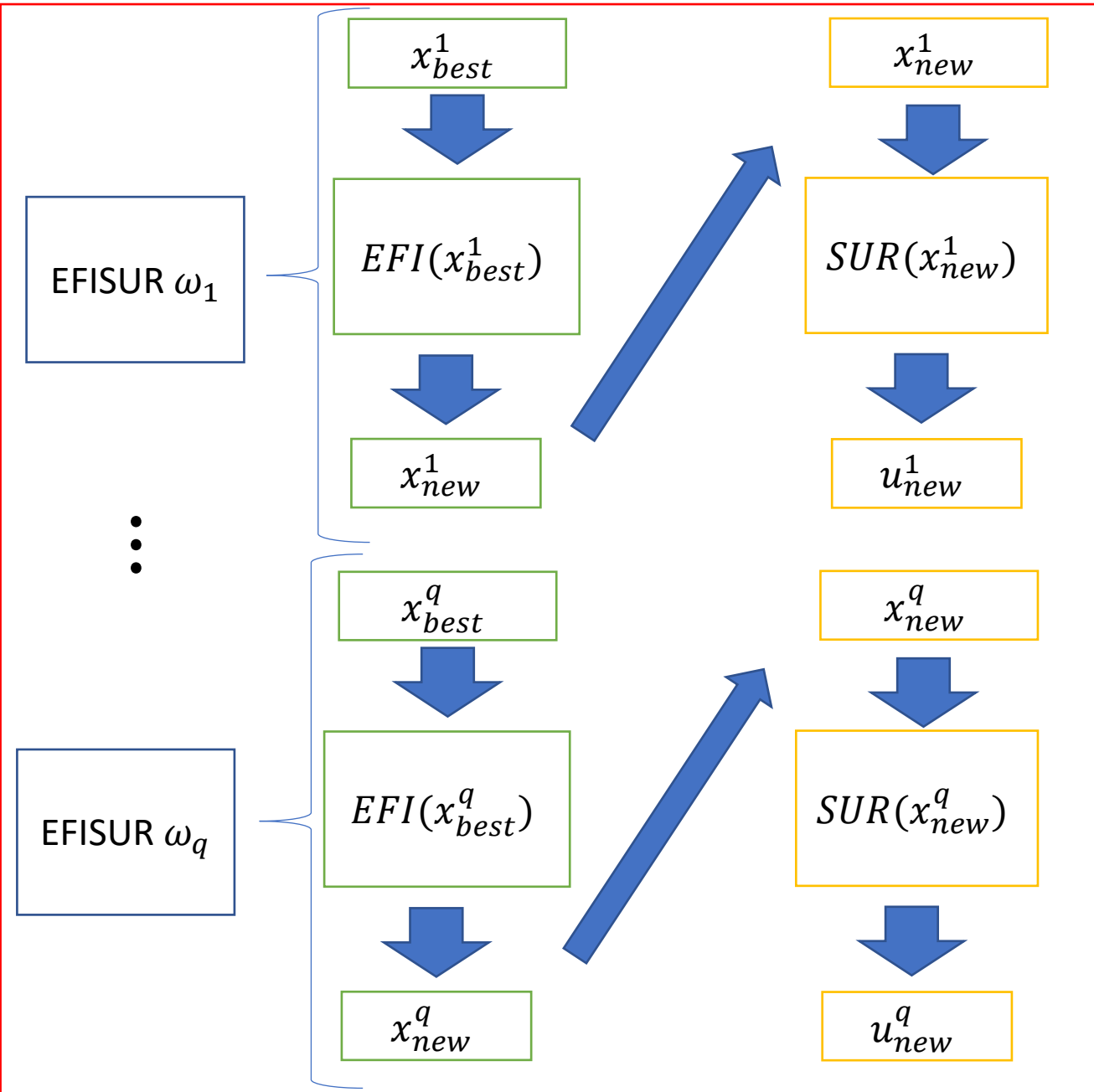
Switching to a **q-batch algorithm** :

Thanks to the weighted sum approach, we can solve several problems (each one with a different vector of weights) in **parallel** -> q-batch algorithm

- Each of the new q points enriches all the GPs.

Parallel computation

DOE and initial GP models

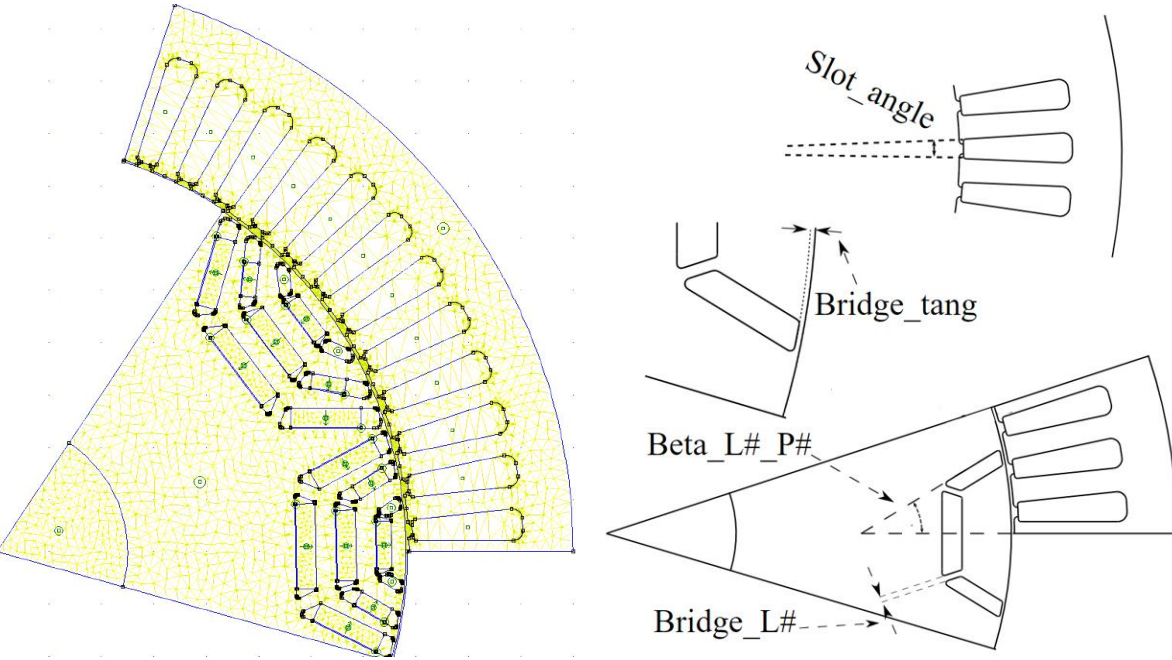


New points

- $x_{new}^1$
- $x_{new}^2$
- $\vdots$
- $x_{new}^q$
- $u_{new}^1$
- $u_{new}^2$
- $\vdots$
- $u_{new}^q$

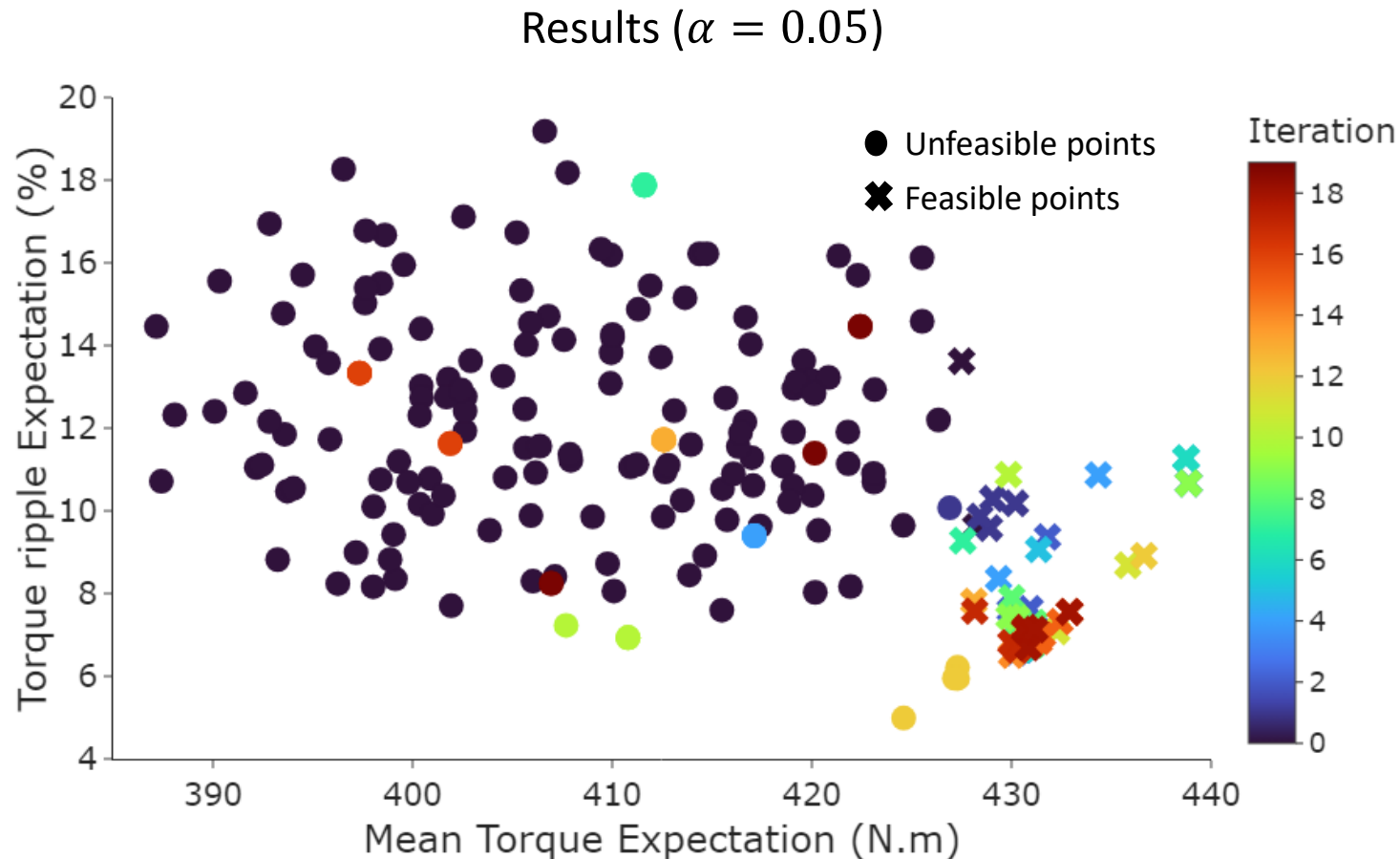


Update GP models



Objective function $\mathbb{E}_U[f_1(x + U)]$	Maximize the mean torque expectation
Objective function $\mathbb{E}_U[f_2(x + U)]$	Minimize the torque ripple expectation
Constraint $\mathbb{P}_U[f_1(x + U) \geq 420] \geq 1 - \alpha$ $\alpha \in \{0.01, 0.03, 0.05, 0.07, 0.10\}$	Achieve a mean torque superior to 420 N.m with a probability greater or equal to {99%, 97%, 95%, 93%, 90%}
Initial DOE size	140 points LHS maximin
Number of iterations and points added per iteration	19 iterations and 5 points per iteration: A total 95 points

- Number of controllable variables  $x$  : 11
- Number of uncertain variables  $U$  : 7
  - 5 (dispersions for 5  $x$ , called  $U_g$ )  $\rightarrow x + U_g$ ,
  - 2 magnetic properties of materials  $\rightarrow U_m$



- This figure shows the expectations (predicted by GPs) of all the simulated points through algorithm iterations

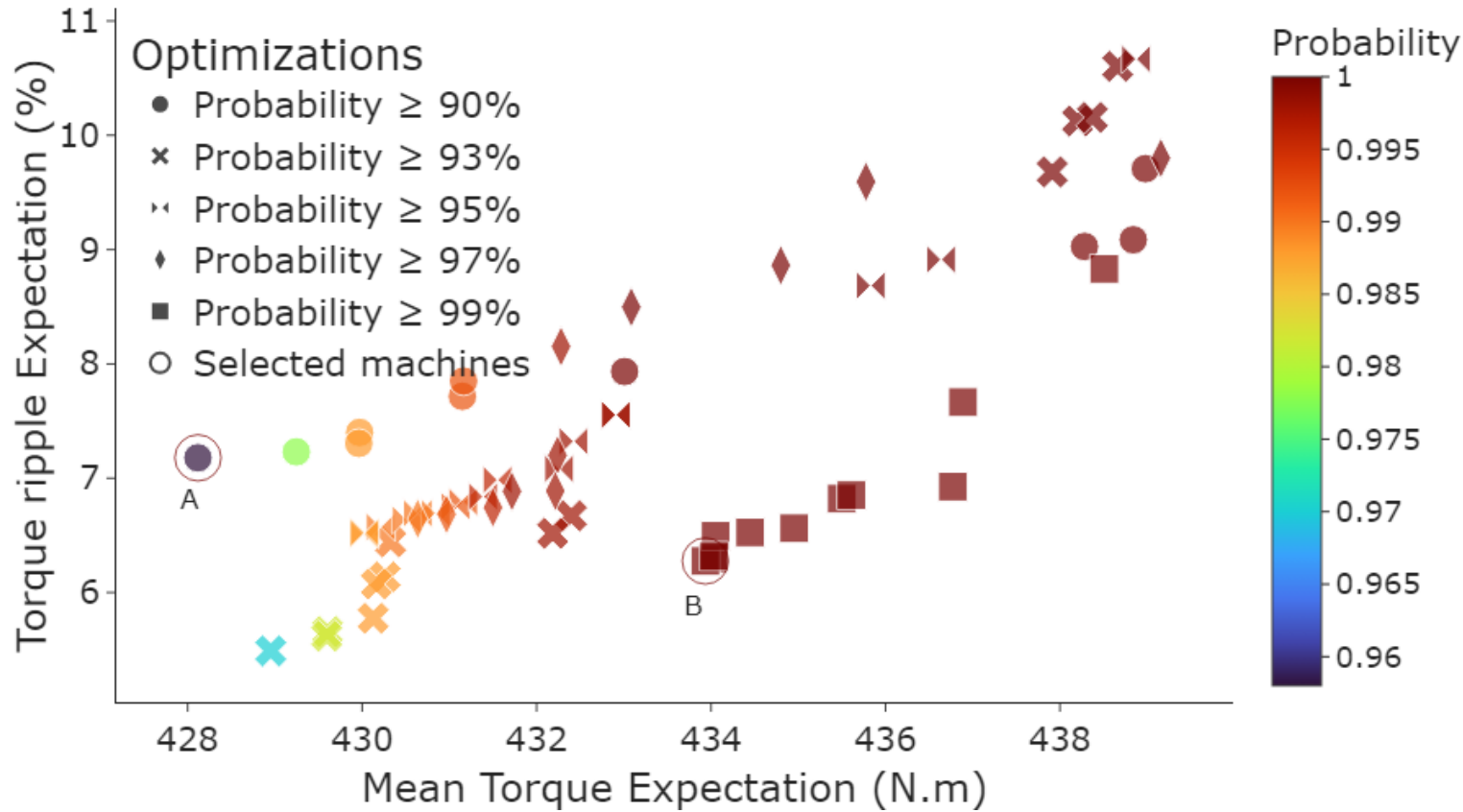
- None of the points in the initial design of experiments satisfies the constraint of having a mean torque of at least 420 N.m with a probability of at least 95%.

- On the other hand, the modified EFISUR algorithm proposes several feasible promising points.

- The same behavior was observed for the other probability thresholds

# Electrical Machines Optimization : Results

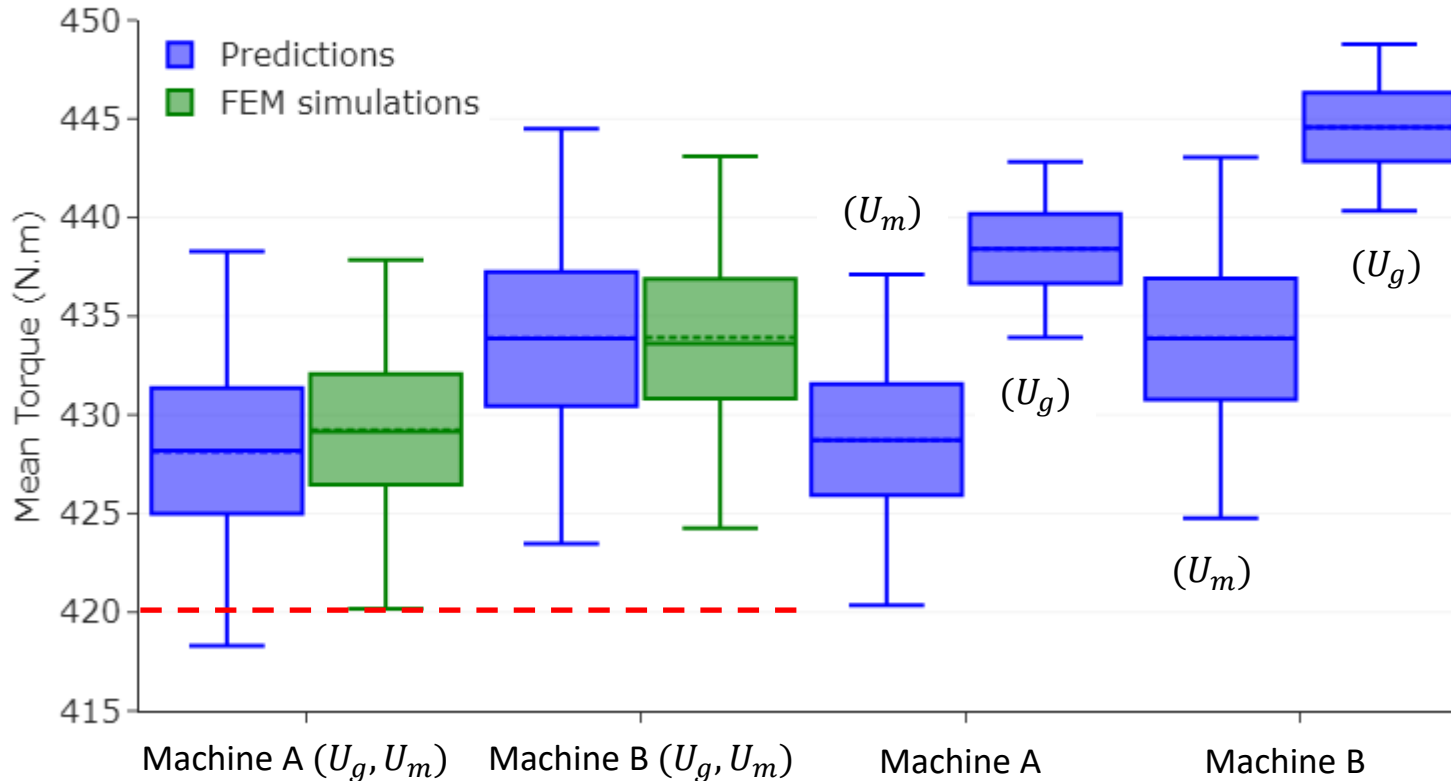
Results from robust optimizations



- This figure shows Pareto fronts for different probability thresholds
- Higher probability levels (e.g. 99%) results in a shorter Pareto front. This is somehow expected, higher the probability of passing 420 N.m, higher the expectation value.
- Machines A and B were selected for further analysis (see Table below for their designs).

Input	Slot_angle	Beta_L1P1	Beta_L1P2	Beta_L2P1	Beta_L2P2	Beta_L3P1	Beta_L3P2	Bridge_L1	Bridge_L2	Bridge_L3	Bridge_Tang
Machine A	2,47°	27,03°	38,65°	31,04°	47,04°	<b>36,99°</b>	<b>59,7°</b>	2,6 mm	<b>1,18 mm</b>	0,5 mm	<b>0,6 mm</b>
Machine B	2,47°	27,03°	38°	31,16°	47,07°	<b>33,7°</b>	<b>63°</b>	2,6 mm	<b>0,9 mm</b>	0,5 mm	<b>0,4 mm</b>

## Results verification



- Finite element simulations were performed to build the boxplots of two machines (green). These boxplots were compared with the values predicted by the metamodels (blue).

Prediction errors (Mean Torque GPs):

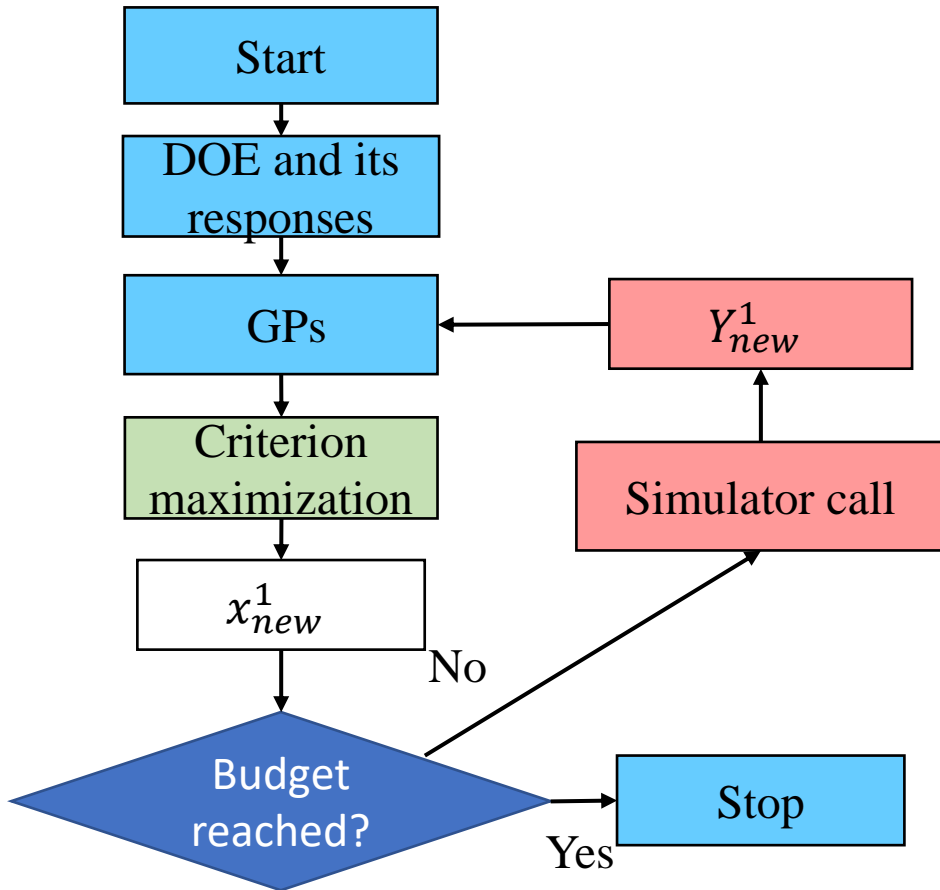
- Number of  $U$  (at  $x$  fixed  $x_A^*$ ,  $x_B^*$ ) points: 200
- RMSE machine A : 1.9 N.m
- RMSE machine B : 1.24 N.m
- Good  $\hat{f}$  accuracy around  $(x_A^*, x_B^*)$

- As we can see, the mean torque values are more sensitive to materials' properties ( $U_m$ ).

## EFISUR

$$\min_{\mathbf{x} \in D_x} \mathbb{E}_U[f(\mathbf{x}, \mathbf{U})]$$

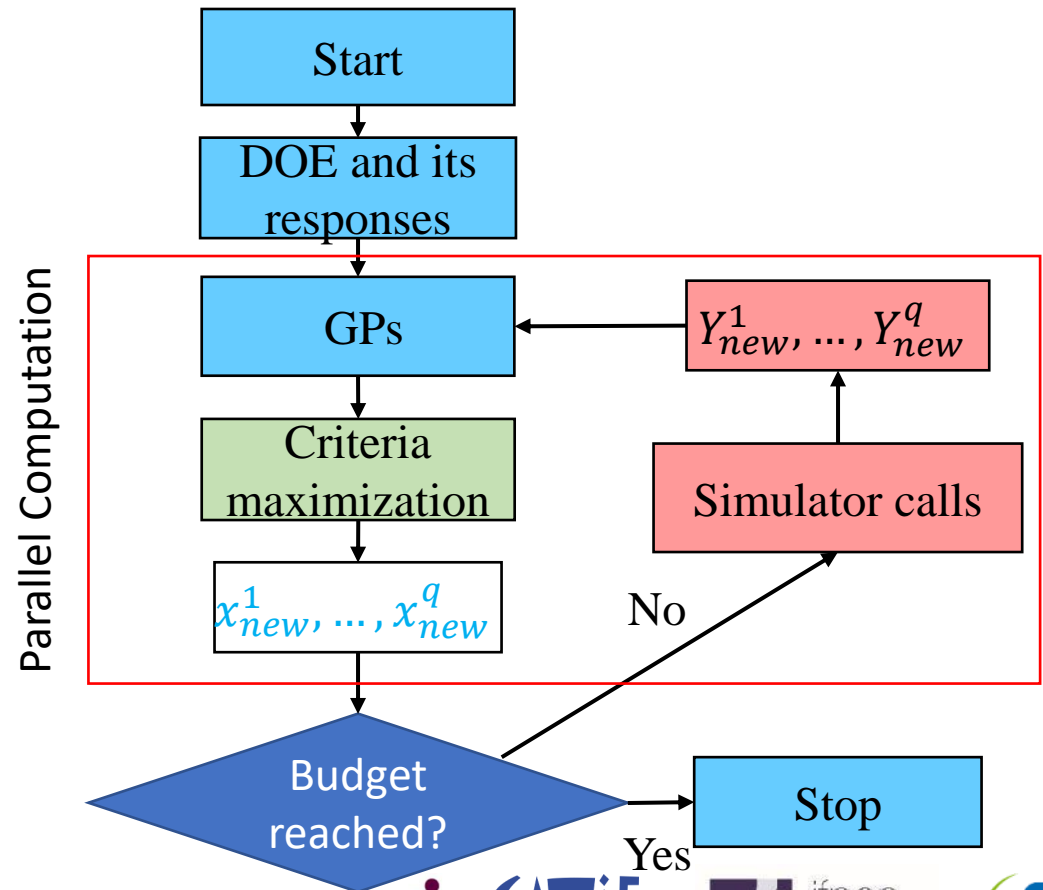
$$s.t. \mathbb{P}_U(g_i(\mathbf{x}, \mathbf{U}) \leq 0, i = 1, \dots, l) \geq 1 - \alpha$$



## Modified EFISUR

$$\min_{\mathbf{x} \in D_x} \mathbb{E}_U \left[ \sum_{j=1}^k w_j f_j(\mathbf{x} + \mathbf{U}) \right]$$

$$s.t. \mathbb{P}_U(g_i(\mathbf{x} + \mathbf{U}) \leq 0) \geq 1 - \alpha_i, i = 1, \dots, l$$





- The modified EFISUR algorithm has successfully solved a constrained multi-objective electrical machine optimization problem taking into account uncertainties
- After validating this algorithm on a simplified application, ongoing work on a more realistic problem with all the constraints related to electrical vehicle applications:
  - 2 objectives : efficiency and permanent magnets overall weight expectations
  - 7 probability constraints (mean torque, power, torque ripple,.... )
- Future work on Multifidelity Optimization since several simulators (with different accuracy) are available



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- [5] El Amri, R., Le Riche, R., Helbert C., Blanchet-Scalliet, C., Da Veiga, S. "A sampling criterion for constrained Bayesian optimization with uncertainties" In SMAI Journal of Computational Mathematics, 2023, 9, pp.285-309. {10.5802/smai-jcm.102}. {emse-03167452v2}