

Data assimilation based on physics-informed Gaussian processes for the reconstruction of incompressible flows

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Table of Contents

- 1 Introduction
- 2 Lagrangian Data Assimilation based on GPs
- 3 Reconstruction of Unsteady Flow past a Cylinder
- 4 Conclusions & Future Works

Introduction

Introduction

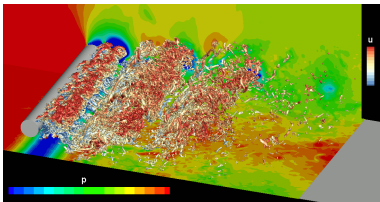
- Increasing interest in **physics-informed machine learning**: merging data-driven estimates with physical models [Brunton *et al.* 2020][1].

Some limitations of current methods:

- mesh-based, difficult to adapt to new scenarios
- need of multiple instances for **uncertainty quantification (UQ)**
- neural approaches are computationally expensive and do not necessarily provide theoretical guarantees

On the other hand, Gaussian process regression (GPR) provides :

- a robust mathematical background with **built-in UQ**
- lower computational costs than deterministic methods
- **flexible adaptation to data**, including **physics-informed** priors



High-fidelity benchmarking Lagrangian dataset of 3D flow past a cylinder (*particle tracking velocimetry* or PTV) [Leclaire *et al.*, 2022][2]

Physical framework

Consider an incompressible fluid flow past an aerodynamic profile (e.g. cylinder, airfoil), with velocity field $\mathbf{u} = (u_1, u_2)^T$. Let $\Omega \subset \mathbb{R}^2$ be a 2D spatial domain.

→ Ground truth with viscosity $\nu = 3.75 \cdot 10^{-4} \text{ m}^2/\text{s}$ (Re = 800) computed at 33280 cells.

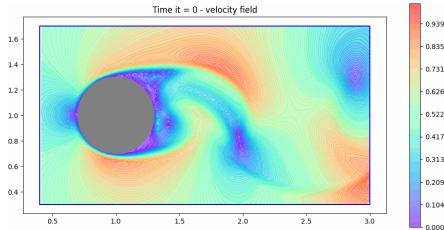
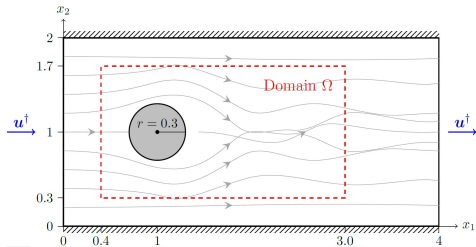
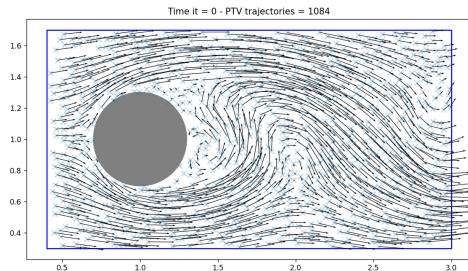


Diagram of inlet flow configuration in a tunnel setting

Ground truth velocity field (norm in m/s) based on the “incompressible wake flow” benchmarking dataset, generated using PhiFlow [\[Kohl et al. 2026\]\[3\]](#).

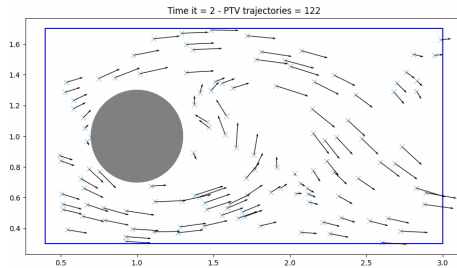
Lagrangian PTV dataset

- Initial condition available: $\mathbf{u}_0(\mathbf{x}, t)$
- Time-evolving dataset of PTV trajectories $(\mathbf{R}, \mathbf{V})$ with positions $t \mapsto \mathbf{r}_k(t)$ and velocities $t \mapsto \mathbf{v}_k(t)$



Initial condition at high resolution

Arrows ($\rightarrow$) represent Lagrangian PTV observations.



PTV trajectories generated from the ground truth at a sparse spatial scale

Lagrangian Data Assimilation based on GPs

Lagrangian dynamics of incompressible viscous flows

Consider the incompressible 2D Navier–Stokes equations on vorticity $\omega = \mathbf{curl} \cdot \mathbf{u}$, for $\mathbf{curl} = (-\partial_{x_2}, \partial_{x_1})^T$, with viscosity $\nu > 0$:

$$\begin{cases} \partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = \nu \Delta \mathbf{u} - \nabla p + \mathbf{f} \\ \mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0(\mathbf{x}), \quad \nabla \cdot \mathbf{u} = 0, \quad \text{on } \mathbf{x} \in \Omega \end{cases} \implies \begin{cases} \partial_t \omega + \mathbf{u} \cdot \nabla \omega = \nu \Delta \omega + \mathbf{curl} \cdot \mathbf{f} \\ \omega(\mathbf{x}, 0) = \omega_0(\mathbf{x}), \quad \text{on } \mathbf{x} \in \Omega \end{cases}$$

over the time interval $[0, T]$, given $\omega_0 = \mathbf{curl} \cdot \mathbf{u}_0$.

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over the time interval $[0, T]$, given $\omega_0 = \mathbf{curl} \cdot \mathbf{u}_0$. A Lagrangian system on trajectories $t \mapsto \mathbf{q}_m(t)$ reads:

$$\text{(Lagrangian N-S)} \quad \begin{cases} \dot{\mathbf{Q}}(t) = \mathbf{u}(\mathbf{Q}(t), t) \\ \dot{\mathbf{W}}(t) = \nu \Delta \mathbf{curl} \cdot \mathbf{u}(\mathbf{Q}(t), t) + \mathbf{curl} \cdot \mathbf{f} \\ \mathbf{Q}(0) = \mathbf{Q}_0, \\ \mathbf{W}(0) = \omega_0(\mathbf{Q}_0), \end{cases}$$

with $\mathbf{Q}(t) = \{\mathbf{q}_m(t)\}_{m=1}^M$, $\mathbf{W}(t) := \omega(\mathbf{Q}(t), t)$. $\rightarrow$ The dynamic law is determined by the unknown $\mathbf{u}$.

Lagrangian dynamics of incompressible viscous flows

→ How to simulate $t \mapsto (Q(t), W(t))$, without solving the full field $u(x, t)$?

- **Standard vortex/particle methods:** e.g. smoothed-particle hydrodynamics (SPH), reconstruction of velocity from vorticity using Biot-Savart law, representation of fields through smooth vortex *blobs*, including (or not) stochastic versions [Majda et al. 2001][4].

Lagrangian dynamics of incompressible viscous flows

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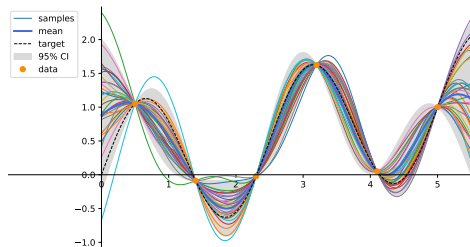
- **Standard vortex/particle methods:** e.g. smoothed-particle hydrodynamics (SPH), reconstruction of velocity from vorticity using Biot-Savart law, representation of fields through smooth vortex *blobs*, including (or not) stochastic versions [Majda et al. 2001][4].
- What about a **GP reconstruction** of the velocity field?

Closure using GP regression

We model u as a Gaussian process $\mathbf{U} \sim \mathcal{GP}(\mathbf{m}, \mathbf{K})$ indexed in space Ω with mean and covariance functions determining the prior distribution:

- $\mathbf{m} : \Omega \longrightarrow \mathbb{R}^2$ is a trend/reference: $\mathbb{E}(\mathbf{U}(\mathbf{x})) = \mathbf{m}(\mathbf{x})$;
- $\mathbf{K} : \Omega \times \Omega \longrightarrow \mathbb{R}^{2 \times 2}$ is a symmetric and positive semi-definite function or **covariance kernel**:

$$\text{Cov}(\mathbf{U}(\mathbf{x}), \mathbf{U}(\mathbf{x}')) = \mathbf{K}(\mathbf{x}, \mathbf{x}');$$



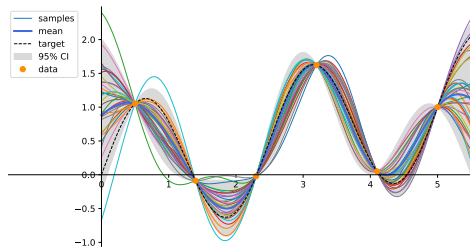
Visualization of scalar GP regression

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Visualization of scalar GP regression

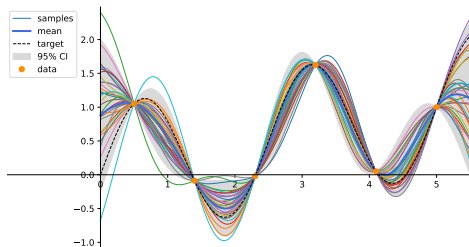
Consider the GP optimal recovery from virtual trajectories $t \mapsto (\mathbf{Q}^*(t), \mathbf{W}^*(t))$, given that $\mathbf{K}$ is smooth enough:

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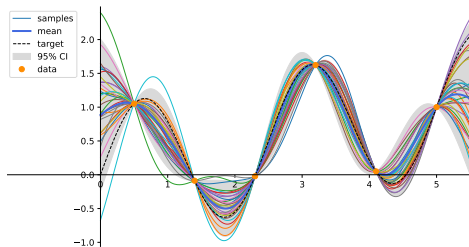
$$\mathbf{u}(\mathbf{x}, t) \simeq \hat{\mathbf{u}}(\mathbf{x}; \mathbf{Q}^*(t), \mathbf{W}^*(t)) := \mathbb{E}(\mathbf{U}(\mathbf{x}) \mid \mathbf{curl} \cdot \mathbf{U}(\mathbf{Q}^*(t)) = \mathbf{W}^*(t))$$

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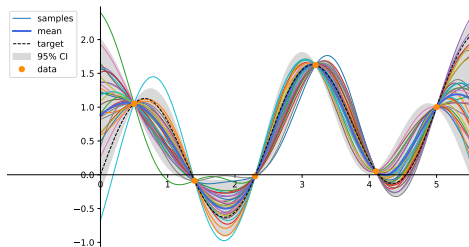
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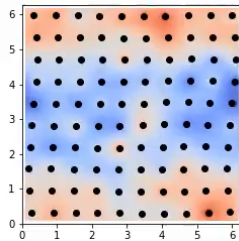
→ Posterior UQ estimates follow directly.

Gaussian Process Hydrodynamics (GPH)

This closure leads to an approximate Lagrangian system on $[0, T]$
[Owhadi 2023][5]:

$$\text{(GPH)} \quad \begin{cases} \dot{\mathbf{Q}}^*(t) = \hat{\mathbf{u}}(\mathbf{Q}^*(t); \mathbf{Q}^*(t), \mathbf{W}^*(t)), \\ \dot{\mathbf{W}}^*(t) = \nu \Delta \mathbf{curl} \cdot \hat{\mathbf{u}}(\mathbf{Q}^*(t); \mathbf{Q}^*(t), \mathbf{W}^*(t)) + \mathbf{curl} \cdot \mathbf{f}, \\ \mathbf{Q}^*(0) = \mathbf{Q}_0, \\ \mathbf{W}^*(0) = \omega_0(\mathbf{Q}_0). \end{cases}$$

→ Numerical collocation trajectories starting at $(\mathbf{Q}_0, \omega_0(\mathbf{Q}_0))$ can be simulated since (GPH) is a **closed** system.

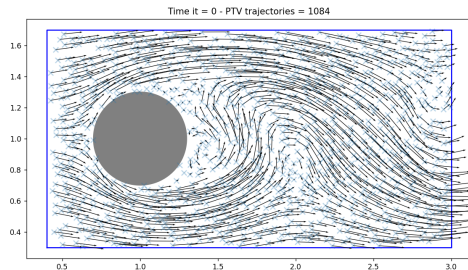


Virtual flow simulation using GPH on a periodic domain. Colormap is vorticity field.

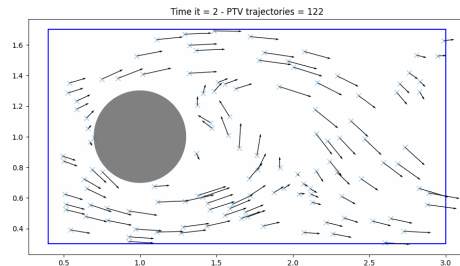
(from [Owhadi 2023][5]*)

*See youtube.com/watch?v=sLf-6ngp_G4

GP-based data assimilation



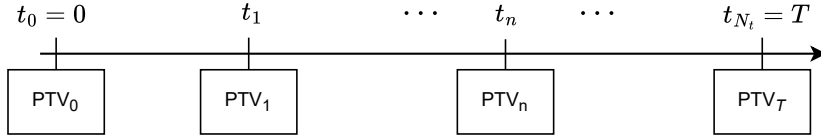
Initial condition at high resolution $\bar{\mathbf{V}}_0$



PTV trajectories $(\mathbf{R}_n, \mathbf{V}_n)_{n=0}^T$ generated from the ground truth at a sparse spatial scale

→ How to take into account both sources of information: evolution dynamics and measurements?

GPR reconstruction from PTV



Considering $\mathbf{U} \sim \mathcal{GP}(\mathbf{0}, \mathbf{K})$, we can reconstruct the velocity field for $n = 1, \dots, N_t$:

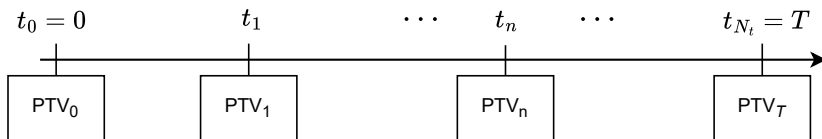
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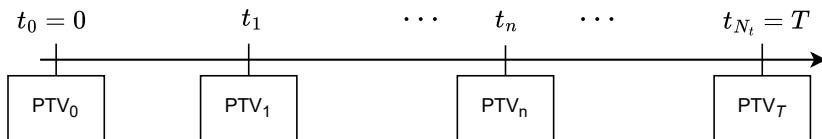
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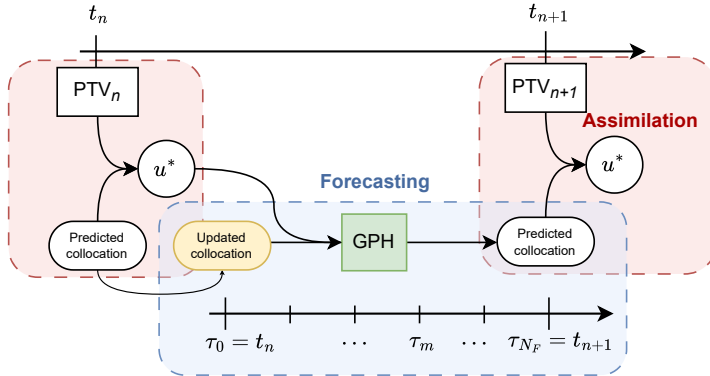
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and for the initial node:

$$\mathbf{u}^*(\mathbf{x}, t_0) = \mathbb{E}(\mathbf{U}(\mathbf{x}) \mid \mathbf{U}(\mathbf{R}_0) + \boldsymbol{\varepsilon}_0 = \mathbf{V}_0, \mathbf{U}(\mathbf{X}_0) + \bar{\boldsymbol{\varepsilon}}_0 = \bar{\mathbf{V}}_0)$$

GP-based data assimilation



→ Forecasting using GPH dynamical model:

Propagate vorticity state between two data time nodes

→ Data assimilation/analysis using physics-informed GPs:

Merge information from forwarded state with available data, accounting for **numerical model uncertainty** on (Q_n^*, W_n^*)

GP-based data assimilation

We consider a multi-source recovery, with ϵ_n and η_n Gaussian noises independent of $\mathbf{U}$:

$$\mathbf{u}^*(\mathbf{x}, t_n) := \mathbb{E}(\mathbf{U}(\mathbf{x}) \mid \mathbf{U}(\mathbf{R}_n) + \epsilon_n = \mathbf{V}_n, \text{curl} \cdot \mathbf{U}(\mathbf{Q}_n^*) + \eta_n = \mathbf{W}_n^*)$$

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On the time interval $[t_n, t_{n+1}]$, we consider a GPH simulation:

$$\text{(Forecast)} \quad \begin{cases} \dot{\mathbf{Q}}^*(t) = \hat{\mathbf{u}}(\mathbf{Q}^*(t); \mathbf{Q}^*(t), \mathbf{W}^*(t)), \\ \dot{\mathbf{W}}^*(t) = \nu \Delta \mathbf{curl} \cdot \hat{\mathbf{u}}(\mathbf{Q}^*(t); \mathbf{Q}^*(t), \mathbf{W}^*(t)) + \mathbf{curl} \cdot \mathbf{f}, \\ \mathbf{Q}^*(t_n) = \mathbf{Q}_n^*, \\ \mathbf{W}^*(t_n) = \mathbf{curl} \cdot \mathbf{u}^*(\mathbf{Q}_n^*, t_n). \end{cases}$$

Using a variant* of the structure: $\hat{\mathbf{u}}(\mathbf{x}; \mathbf{Q}^*(t), \mathbf{W}^*(t)) := \mathbb{E}(\mathbf{U}(\mathbf{x}) \mid \mathbf{curl} \cdot \mathbf{U}(\mathbf{Q}^*(t)) = \mathbf{W}^*(t))$

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→ We simulate then $(\mathbf{Q}_{n+1}^*, \mathbf{W}_{n+1}^*)$ and then **assimilate** the PTV data at node t_{n+1} .

Physics-informed GP priors: which kernel?

Pre-defined GP: Consider a scalar stream function GP $Z \sim \mathcal{GP}(0, G)$, over Ω , with smooth G of class $C^{4,4}$.

Velocity GP is thus: $\mathbf{curl} Z = (-\partial_{x_2} Z, \partial_{x_1} Z)^T \sim \mathcal{GP}(\mathbf{0}, K)$, with $K = \mathbf{curl}_x \otimes \mathbf{curl}_{x'}$ G .

- **Divergence-free condition is enforced everywhere:** $\nabla \cdot \mathbf{curl} Z = 0$, on Ω .

- **Multi-scale power law (energy decay):** $\mathbb{E} \left(\|\mathbf{curl} Z(\mathbf{x} + \mathbf{h}) - \mathbf{curl} Z(\mathbf{x})\|^2 \right) = O(\|\mathbf{h}\|^2)$ as $\|\mathbf{h}\| \rightarrow 0$.

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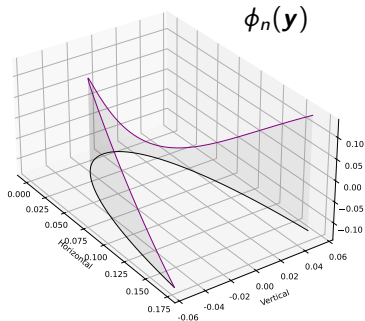
- **Boundary-constrained GP:** [Padilla-Segarra *et al.*, IJUQ, 2026][6]

We modify Z to obtain its BCGP version Z_0 so that: $Z_0 = 0$, on the compact set Γ (cylinder boundary).

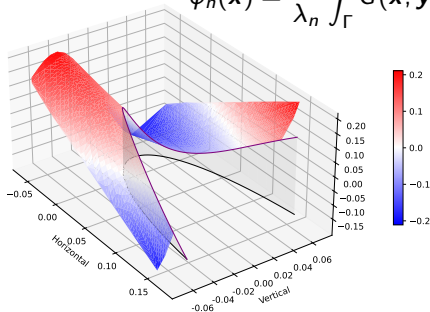
This is equivalent to: $\mathbf{curl} Z_0 \cdot \mathbf{n} = 0$ on Γ , a **slip boundary** condition on the obstacle.

→ We choose then $\mathbf{U} := \mathbf{curl} Z_0 \sim \mathcal{GP}(\mathbf{0}, K_0)$, with $K_0 = \mathbf{curl}_x \otimes \mathbf{curl}_{x'} G_0$.

Boundary-constrained GPs: localized eigenfunctions with canonical extensions



$$\tilde{\phi}_n(x) = \frac{1}{\lambda_n} \int_{\Gamma} G(x, y) \phi_n(y) d\mu(y)$$

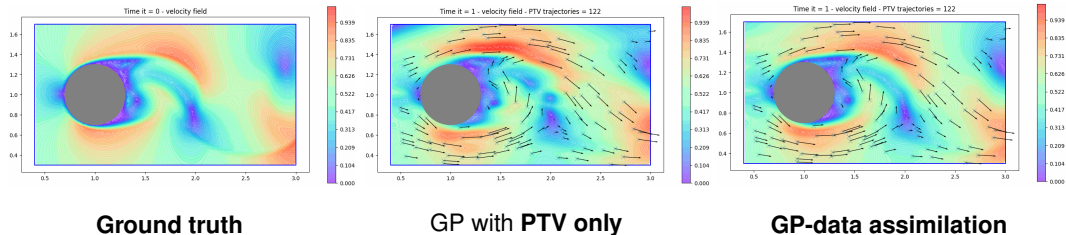


Localized eigenfunction (left) wrt measure μ supported on compact set Γ , and extension to 2D domain Ω (right)

$$G_0(x, x') = G(x, x') - \sum_{n=0}^{+\infty} \lambda_n \tilde{\phi}_n(x) \tilde{\phi}_n(x')$$

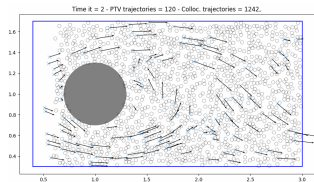
Reconstruction of Unsteady Flow past a Cylinder

Reconstruction of unsteady flow past a cylinder profile

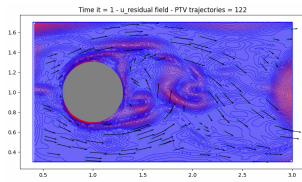


Velocity field (norm in m/s) of incompressible flow around a cylinder profile. Arrows ($\rightarrow$) represent Lagrangian PTV observations.

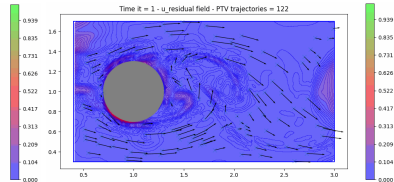
Reconstruction of unsteady flow past a cylinder profile



**Observation
configuration for DA**



GP with PTV only



GP-data assimilation

Observation configuration and residual field (norm in m/s) of velocity of incompressible flow around a cylinder profile. Arrows ($\rightarrow$) represent Lagrangian PTV observations. Circles ($\circ$) represent virtual collocation trajectories.

Conclusions & Future Works

Conclusions

- We presented a **GP-based data assimilation** scheme for 2D incompressible flows
- Combining **physics-informed** GP priors: **continuous BC**, incompressibility, multi-scale power law
- **Flexibility**: mesh-less, integrated UQ, adaptable to aerodynamic obstacles with different geometries

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Based on:

- A.P.-S., H. Owhadi, É. Savin. Data assimilation based on physics-informed Gaussian processes for the reconstruction of incompressible flows. In preparation, 2026.
- [6] A.P.-S., P. Noble, O. Roustant, É. Savin. Physics-informed, boundary-constrained Gaussian process regression for the reconstruction of fluid flow fields. *International Journal for Uncertainty Quantification*, 2026.
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Guidelines on future works

Physics-informed GP priors of flow fields

- 1 **Kernel design:** improve hyperparameter estimations (cross-validation with Kernel Flows), set POD-type kernels from data (see [Akian *et al.* 2022][7]).
- 2 **Boundary conditions:** other types of profile boundary conditions (adherence $\mathbf{u} = \mathbf{0}$), vector-valued spectral Karhunen-Loève expansions (extensions to [Padilla-Segarra *et al.* IJUQ 2026][6]).

Towards “digital twins” for the reconstruction of incompressible flows

- 1 **Adaptive design:** improved allocation of virtual trajectories (exploit built-in UQ and consistency error fields).
- 2 **Integrated and robust system:** couple data assimilation and sparse GPs, aiming at a sensor placement problem based on UQ.
- 3 **The 3D case:** framework is adaptable, but vorticity is now a vector-valued field.

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Thank you for your attention!

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