

SPIN-H : A semi-parametric Spatially Informed Hawkes process.

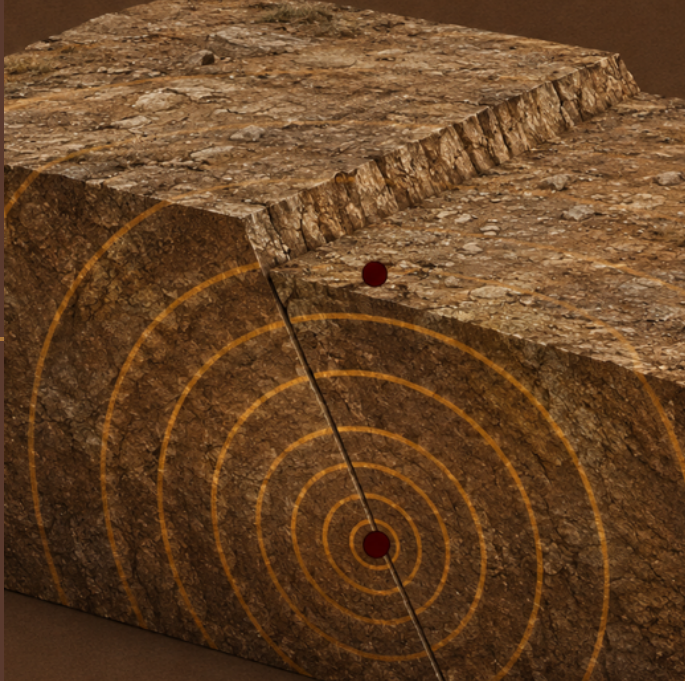
An application to french prob-
abilistic seismic hazard analysis

Ibrahim SEYDI

Sophie DONNET · Merlin KELLER

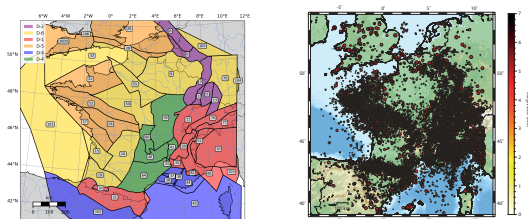
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02/10/2026



Industrial context, motivations and limitations

This work contributes to the development of learning methods for the **selection and validation of seismic source models** used in EDF's seismic risk assessment.



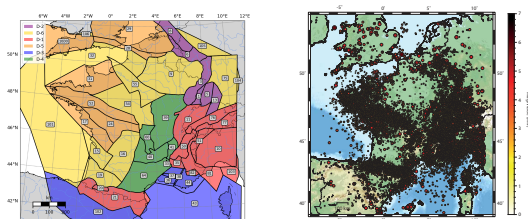
(Left) Seismotectonic model of France proposed by EDF. (Right) French earthquake catalog for PSHA.

Limitations of current models :

- **Zoning models** : too many zones $\Rightarrow$ highly uncertain parameters (data scarcity)
- **Zoneless models** : better data fit, but ignore physical/geological constraints

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- **Zoneless models** : better data fit, but ignore physical/geological constraints

$\Rightarrow$ **Need for a "hybrid" model integrating both data and geological expertise, while properly accounting for uncertainties**

Seismic data and marked Hawkes process

Full seismic catalog main/after-shocks

$$\mathcal{D} = \{(t_i, x_i, y_i, m_i)\}_{i=1}^N, \quad (x_i, y_i) \in \mathcal{X} \subset \mathbb{R}^2, \quad t_i \in (0, T], \quad m_i \in \mathbb{R}_+.$$

Conditional intensity

$$\begin{aligned} \lambda^*(t, x, y, m \mid \mathcal{H}_t) &= p_M(m) \times \lambda(t, x, y \mid \mathcal{H}_t), \\ \lambda(t, x, y \mid \mathcal{H}_t) &= \underbrace{\mu(x, y)}_{\text{background}} + \underbrace{\sum_{j: t_j < t} \phi(t - t_j, x - x_j, y - y_j, m_j)}_{\text{self-exciting triggering}}. \end{aligned}$$

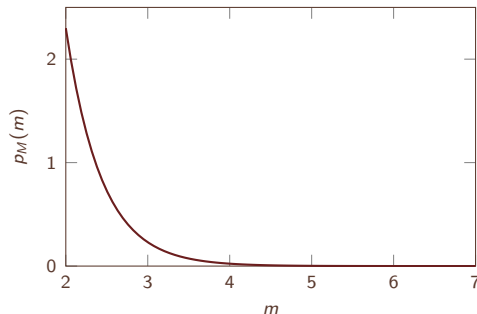
- $\mu(x, y)$: spatially varying rate of spontaneous background events
- ϕ : self-exciting contribution of each past event to future seismicity

Model — Magnitude distribution p_M

Gutenberg–Richter law (truncated exponential) :

$$p_M(m) = \frac{\beta e^{-\beta(m-m_c)}}{1 - e^{-\beta(m_{\max}-m_c)}} \mathbb{1}_{\{m_c \leq m \leq m_{\max}\}} , \quad \beta = \ln(10) b$$

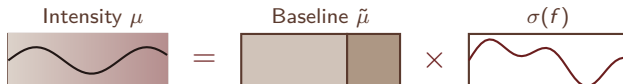
- $b \approx 1$ (seismology), so $\beta \approx 2.3$
- Independent of (t, x, y)
- Same for background and triggered events
- $\int_{m_c}^{m_{\max}} p_M(m) dm = 1 \Rightarrow$ separable from spatio-temporal part



Model — Background intensity μ

The background intensity is modeled as a **Spatially Structured sigmoidal Gaussian Cox process (SSGC)** :

$$\mu(x, y) = \tilde{\mu}(x, y) \sigma(f(x, y)) = \frac{\tilde{\mu}(x, y)}{1 + e^{-f(x, y)}}$$



Baseline $\tilde{\mu}$: macroscopic structure

Piecewise constant over J sub-regions $\mathcal{X}_j$:

$$\tilde{\mu}(x, y) = \exp\left(\sum_{j=1}^J \varepsilon_j u_j(x, y)\right) = e^{\varepsilon_j}$$

for $(x, y) \in \mathcal{X}_j$, where u_j is the indicator of $\mathcal{X}_j$.

Latent GP f : residual fluctuations

Zero-mean GP with squared-exponential kernel :

$$f \mid \nu, \ell \sim \mathcal{GP}(0, k(\cdot, \cdot \mid \nu, \ell))$$

$$k = \nu \exp\left(-\frac{\|s-s'\|^2}{2\ell^2}\right)$$

ν : marginal variance, ℓ : length-scale.

$\tilde{\mu}$ captures the prior macroscopic spatial structure ; f models local residual fluctuations around this baseline.

Model — Triggering kernel ϕ

The excitation kernel ϕ decomposes into three factors :

$$\phi(t - t_j, x - x_j, y - y_j, m_j) = \phi_m(m_j) \phi_t(t - t_j) \phi_s(x - x_j, y - y_j \mid m_j)$$

Productivity

$$\phi_m(m_j) = A e^{\alpha(m_j - m_c)}$$

Exponential scaling with parent
magnitude

Modified Omori–Utsu

$$\phi_t(\Delta t) = \frac{(p-1) c^{p-1}}{(\Delta t + c)^p}$$

Temporal aftershock decay ($c > 0$
and $p > 1$)

Spatial dispersion

$$\phi_s = \frac{q-1}{\pi R(m_j)} \left[1 + \frac{r^2}{R(m_j)} \right]^{-q}$$

$$R(m_j) = d e^{\gamma(m_j - m_c)}, \quad d > 0, \\ \gamma > 0, q > 1$$

- ϕ_t : normalized density on $\mathbb{R}_+$
- ϕ_s : normalized density on $\mathbb{R}^2$; radius scales with magnitude
- Total aftershock rate : $\iint \phi dx dy dt = A e^{\alpha(m_j - m_c)}$

Towards Bayesian inference — Structured Gaussian prior on zonal baselines

The zonal coefficients $\varepsilon = (\varepsilon_1, \dots, \varepsilon_J)^\top$ are chosen to be **not independent**. They are endowed with a centered Gaussian prior that encodes spatial proximity :

$$\varepsilon \sim \mathcal{N}(\mathbf{0}_J, \Sigma_\varepsilon), \quad (\Sigma_\varepsilon)_{jj'} = \delta_0 \exp\left(-\frac{\|c_j - c_{j'}\|^2}{2\delta_1^2}\right)$$

where $c_j = (c_x^j, c_y^j)$ are the **centroids** of region $\mathcal{X}_j$.

- $\delta_0 > 0$: marginal variance
- $\delta_1 > 0$: correlation length-scale

This regularizes the baseline by inducing correlations between adjacent zones, tempering the strength of the prior belief

Towards Bayesian inference — Priors on remaining parameters

Weakly informative Gamma priors enforce positivity :

Parameter	Constraint	Prior	Rationale
A, α, c, d, γ	> 0	$\text{Gamma}(1, 1) = \text{Exp}(1)$	diffuse, mean 1
p	> 1	$p - 1 \sim \text{Gamma}(1, 1)$	shifted support
q	> 1	$q - 1 \sim \text{Gamma}(1, 1)$	shifted support
β	> 0	$\text{Gamma}(2, 1)$	mode ≈ 1 , mean 2

GP kernel hyperparameters

(ν, ℓ) calibrated prior to sampling via a heuristic based on kriging the following predictions of the latent GP :

$$z_i = \frac{2 N |\mathcal{X}_j|}{N_j} \hat{p}(x_i, y_i) - 2, \quad (x_i, y_i) \in \mathcal{X}_j, \quad (1)$$

using a linearization of sigmoïd fonction, and then can be **held fixed** throughout inference.

Towards Bayesian inference — Posterior & Likelihood

The posterior is : $p(\theta \mid \mathcal{D}) \propto L_N(\theta) p(\varepsilon) p(f \mid v, \ell) p(v, \ell) p(\beta) p(\theta_\phi)$. The likelihood of $\mathcal{D}$ under the Hawkes process model factorizes as :

$$L_N(\theta) = \underbrace{\prod_{i=1}^N p_M(m_i)}_{\text{magnitudes}} \times \underbrace{\left[\prod_{i=1}^N \lambda(t_i, x_i, y_i \mid \mathcal{H}_{t_i}) \right] \exp\left(-\int_0^T \int_{\mathcal{X}} \lambda \, dx \, dy \, dt\right)}_{\text{spatio-temporal}}$$

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The integrated intensity splits into background and triggering (recall $\lambda = \mu + \sum \phi$) :

Triggering integral

The triggering term is tractable by Fubini :

$$\int_0^T \int_{\mathcal{X}} \sum_{j:t_j < t} \phi \, dx \, dy \, dt = \sum_{j=1}^N A e^{\alpha(m_j - m_c)} T_j S_j$$
$$T_j = 1 - \left(\frac{c}{T - t_j + c} \right)^{p-1}, \quad S_j \approx 1$$

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Background integral

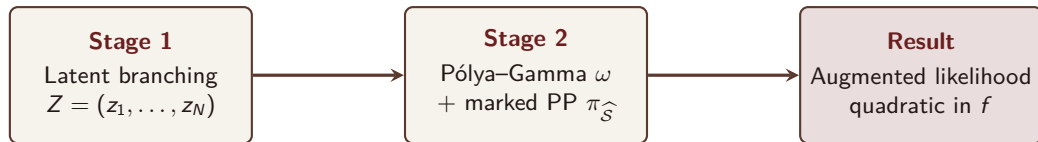
$$\int_0^T \int_{\mathcal{X}} \mu \, dx \, dy \, dt = T \sum_{j=1}^J e^{\varepsilon_j} \int_{\mathcal{X}_j} \sigma(f(x, y)) \, dx \, dy$$

*No closed form when f is a GP
 $\Rightarrow$ source of intractability*

Towards Bayesian inference — Data augmentation strategy

A **two-stage** augmentation strategy to :

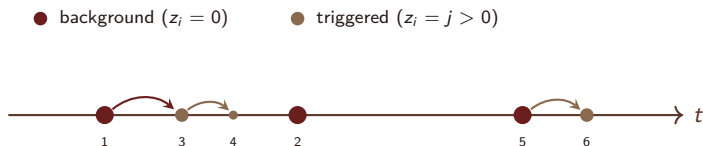
1. Decouple background (μ) from triggering (ϕ)
2. Render the background component conjugate to the GP prior



Data augmentation — Stage 1 : Latent branching structure

Introduce latent branching variables :

$$Z = (z_1, \dots, z_N), \quad z_i = \begin{cases} 0 & \text{if event } i \text{ is a background event,} \\ j > 0 & \text{if event } i \text{ was triggered by event } j. \end{cases}$$



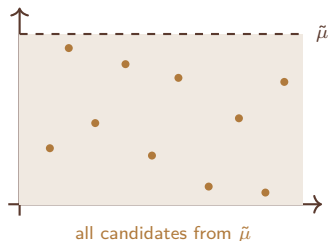
This **factorizes the augmented likelihood** into :

$$L_N = \underbrace{L_{\text{background}}(\epsilon, f)}_{\text{depends on } \mu} \times \underbrace{L_{\text{self-excited}}(\theta_\phi)}_{\text{depends on } \phi} \times p(Z)$$

Data augmentation — Stage 2 : Marked PP & PG augmentations

Classical thinning (simulation)

Start from $\tilde{\mu}$, reject to get $\tilde{\mu} \sigma(f)$



Step 1 : draw all candidates uniformly under $\tilde{\mu}$

Step 2 : the curve $\tilde{\mu} \sigma(f)$ sets the acceptance boundary

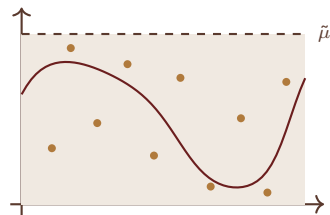
Step 3 : accept with prob. $\sigma(f)$, reject otherwise

Result : Inhomogeneous PP ($\tilde{\mu} \cdot \sigma(f)$)

Data augmentation — Stage 2 : Marked PP & PG augmentations

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curve $\tilde{\mu} \sigma(f)$ separates the two regions

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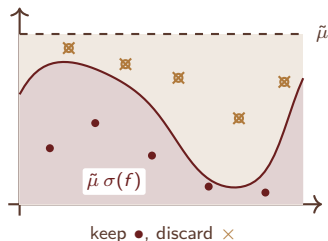
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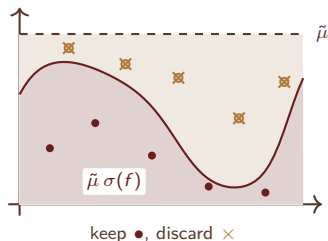
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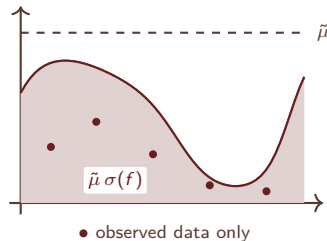
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Marked Poisson process augmentation

Start from data, add back latent events



Start : only observed background events ● are known

Identify : the thinned region $\tilde{\mu} \sigma(-f)$ above the curve

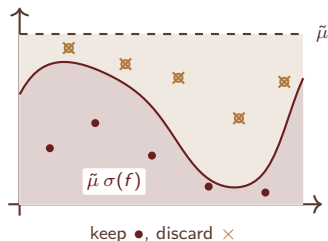
Augment : sample latent ● from $\tilde{\mu} \sigma(-f)$

Result : Union $\rightarrow$ homogeneous PP($\tilde{\mu}$) + mark P-G $\Rightarrow$ Tractable integral & conjugacy GP

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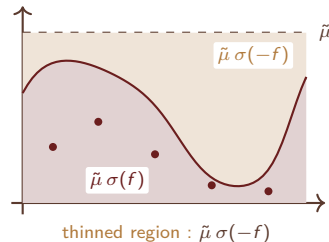
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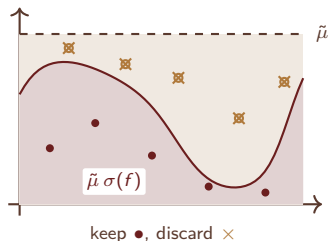
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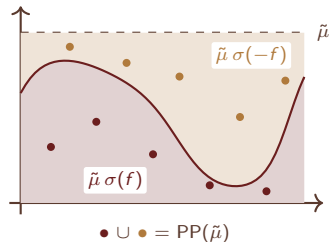
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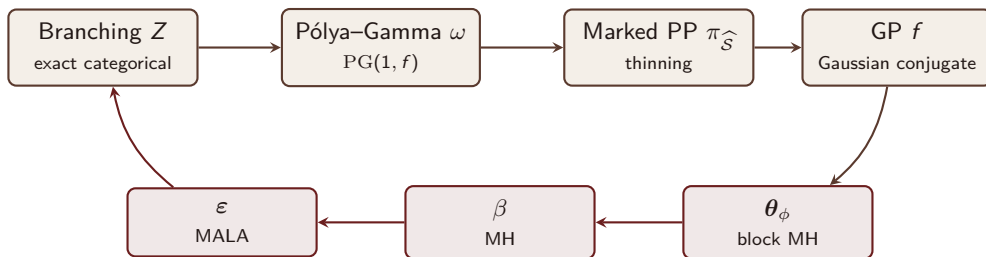
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Bayesian Inference — Gibbs sampling



- **Brown** : Conjugate updates (closed-form or exact sampling)
- **Red** : Non-conjugate (MALA or Metropolis–Hastings)

Bayesian inference — Variational Bayes

Mean-field variational family

$$q(\Theta, Z, \omega, \Pi_S, f, \varepsilon) = q(Z) q(\omega) q(\Pi_S) q(f) q(\varepsilon) q(\theta_\phi) q(\beta)$$

BRANCHING

$$q(Z)$$

Categorical
allocations

AUGMENTATION

$$q(\omega) q(\Pi_S)$$

Tractable ex-
pectations

LATENT FIELD

$$q(f) q(\varepsilon)$$

Gaussian ; La-
place/Newton

PARAMETERS

$$q(\theta_\phi) q(\beta)$$

Gamma factors

Fast inference with factorized posterior dependence

Scalable inference — HSGP and Temporal truncation

Two computational bottlenecks $\longrightarrow$ Two approximations

HSGP

Gaussian-process compression/approximation

$$f(\mathbf{s}) \simeq \Phi(\mathbf{s})^\top \mathbf{w}, \quad \mathbf{w} \sim \mathcal{N}(\mathbf{0}, I_M)$$

N_{aug} GP values $\longrightarrow M$ coefficients
($M \ll N_{\text{aug}}$)

Spectral sine basis on an expanded domain

TEMPORAL TRUNCATION

Sparse branching

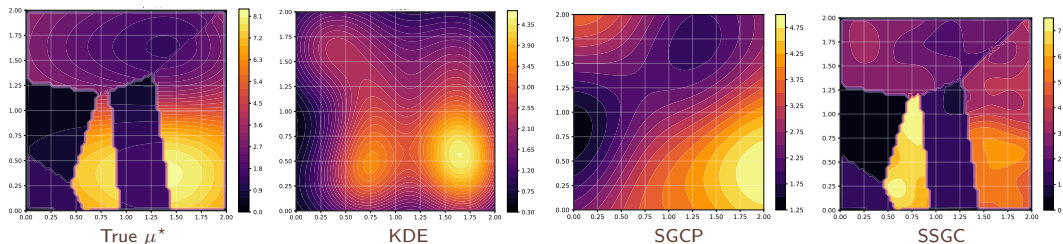
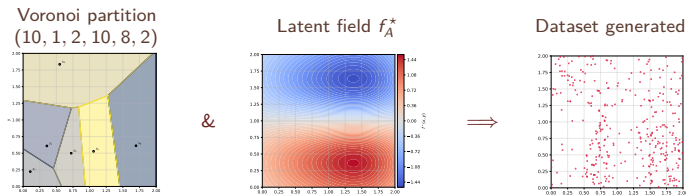
$$\mathcal{C}_i = \{j < i : t_i - t_j \leq \tau_{\max}\}$$

$$\tau_{\max} = c \left(\eta^{-1/p} - 1 \right), \quad \eta = 10^{-3}$$

$$\mathcal{O}(N^2) \longrightarrow \mathcal{O}(NK)$$

Experiment 1 — SSGC vs SGCP vs KDE

Three models compared : **SSGC** (our model, structured $\tilde{\mu}$ + prior $\epsilon \sim \mathcal{N}(\mathbf{0}, \Sigma_\epsilon)$) / **SGCP** ($\propto$ Donner and Opper (2018), $J=1$, constant $\tilde{\mu}$) / **KDE** (with Silverman bandwidth)



Experiment 2 — Numerical design

INFERENCE CONFIGURATIONS

Method	Inference	Gaussian process	Candidate parents
M1	Gibbs	exact	dense
M2	Gibbs	HSGP	dense
M3	Gibbs	HSGP	truncated
M4	MF-VI	HSGP	dense
M5	MF-VI	HSGP	truncated

COMMON SETTING

$$\mathcal{X} = [0, 2]^2, \quad J = 6$$

$$\tilde{\mu} = (8, 1, 2, 8, 7, 2)$$

$$m \in [2, 6], \quad \beta = 2.3$$

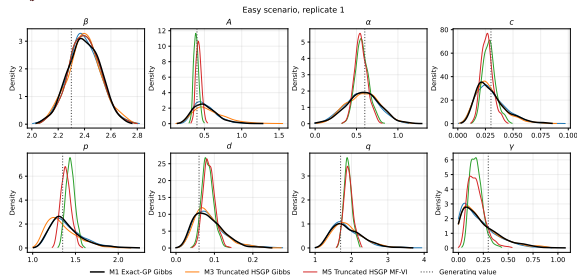
Same latent field f^*

TRIGGERING REGIMES

	T	A	c	p	d	q	α
Easy	51	0.40	0.03	1.35	0.06	1.70	0.60
Difficult	52	0.50	0.08	1.30	0.10	1.55	0.60

Easy concentrated ; **Difficult** broader
 ≈ 400 events per catalogue

Experiment 2 — Posterior parameter marginals



HSGP

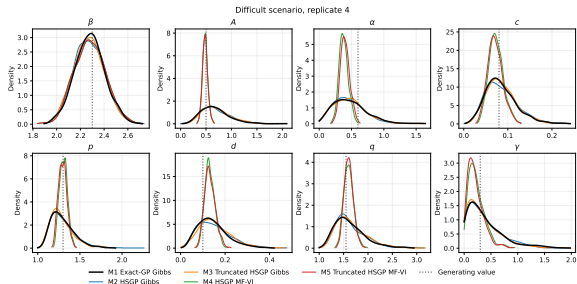
$$q_{\text{Gibbs}}^{\text{exact}} \simeq q_{\text{Gibbs}}^{\text{HSGP}}$$

Similar marginal shapes

MF-VI

$$\text{Var}_{\text{VI}}(\theta) < \text{Var}_{\text{Gibbs}}(\theta)$$

Marginals concentrated



Experiment 3 — Robustness under prior misspecification (SSGC)

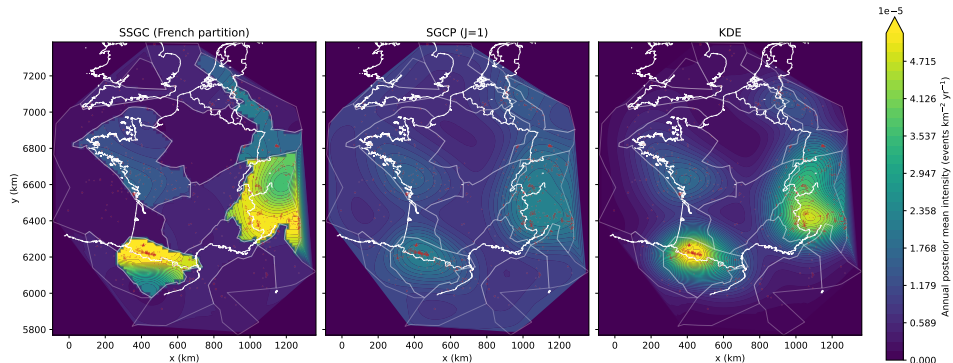
Scenario	True partition	Inference partition
M1 : Superfluous	$J^* = 1$ (homogeneous)	$J = 6$
M2 : Wrong partition	$J^* = 6$	$J = 5$
M3 : Missing zones	$J^* = 6$	$J = 1$ (homogeneous)

Reconstruction metrics — oracle vs misspecified (Settings A & B)

Scen.	Model	Setting A			Setting B		
		RMSE	MAE	CRPS	RMSE	MAE	CRPS
M1	Oracle (SGCP, $J=1$)	0.147	0.109	0.106	0.734	0.591	0.588
	SSGC superfluous ($J=6$)	0.882	0.597	0.541	1.243	0.983	0.897
M2	Oracle (SSGC, $J=6$)	0.943	0.616	0.604	0.917	0.721	0.697
	Wrong partition ($J=5$)	1.727	1.288	1.234	1.980	1.597	1.506
M3	Oracle (SSGC, $J=6$)	0.952	0.620	0.607	0.949	0.751	0.728
	Missing zones ($J=1$)	2.038	1.582	1.578	2.160	1.739	1.734

■ **Hierarchy** : oracle $\gg$ wrong partition $\gg$ missing zones $>$ superfluous zones

Experiment 4 — FCAT-17 real-data application



935
mainshocks

$M_w \geq 3$
complete period since 1965

11
seismotectonic subdomains
D-1a, D-1b, D-2, D-3, ...

Predictive out of sample score : SSGC : -11.314 / SGCP : -12.874 / KDE : -12.545

Take-Home messages & Perspectives

The SPIN-Hawkes / SSGC model

- Spatially informative background via structured baseline $\tilde{\mu}(x, y)$ with centroid-based spatial correlations
- Zero-mean residual GP isolating local fluctuations
- Parametric and interpretable self-excited component
- Two-stage augmentation (branching + marked PP/Pólya–Gamma) $\Rightarrow$ Gibbs sampler & VB
- Two scaling strategies for large catalogs : HSGP & Temporal truncature of parent candidate
- Promising results from tests on simulated data and on a part of french catalog

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Limitation & Perspectives

- Validation and comparison on full **real seismological catalogs**
- Development of methods for handling **missing or unobserved seismic events**

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