Quantification of Uncertainty in Numerical simulation of Fluid Flows

Pietro Marco Congedo

BACCHUS Team - INRIA Bordeaux Sud-Ouest
Associated Team AQUARIUS - Joint Team from INRIA and Stanford University

March 28

MaiMoSine, Grenoble
Workshop on Validation, Verification and UQ

Outline

1. Introduction
2. UQ definitions
3. Mathematical setting
4. Some methods for solving UQ propagation problems
5. Problems in Fluid Mechanics
6. Some highligths from a new method

Virtual Prototyping, Robust Design and UQ

- Virtual Prototyping (VP) $\rightarrow$ key technology for industry to respond to the challenges of designing sustainable products, environmental challenges and cost effectiveness
- Important limitation of the extensive application of analytical tools connected to the level of uncertainty they introduce in the analysis and design process

Virtual Prototyping, Robust Design and UQ

- Virtual Prototyping (VP) $\rightarrow$ key technology for industry to respond to the challenges of designing sustainable products, environmental challenges and cost effectiveness
- Important limitation of the extensive application of analytical tools connected to the level of uncertainty they introduce in the analysis and design process
- Leads to a global uncertainty on the results of the analysis, on which design decisions have to be taken
- Managing the design process in light of these uncertainties is the key to robust design

- Virtual Prototyping (VP) $\rightarrow$ key technology for industry to respond to the challenges of designing sustainable products, environmental challenges and cost effectiveness
- Important limitation of the extensive application of analytical tools connected to the level of uncertainty they introduce in the analysis and design process
- Leads to a global uncertainty on the results of the analysis, on which design decisions have to be taken
- Managing the design process in light of these uncertainties is the key to robust design
- Introducing the probabilistic nature of the uncertainties in the simulation is highly challenging, as the whole process transforms the resolution of deterministic physical conservation laws, to non-deterministic methods, governed by stochastic partial differential equations (SPDE)
- As a consequence, predicted quantities, such as loads, lift, ..., are represented by a probability density function (pdf), providing a domain of confidence, associated to the considered uncertainties

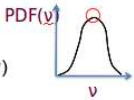
Meaning of quantifying uncertainties in the numerical simulation

EDP modèle

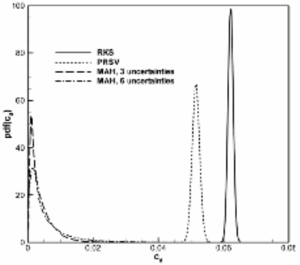
$$\frac{\partial u}{\partial t}+\frac{\partial f(u)}{\partial x}=\nu\frac{\partial^2 u}{\partial x^2}+S(x)$$

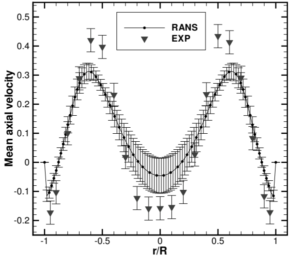
+ INCERTITUDE

Paramètres de l'équation
(Ex. propriétés physiques ν)

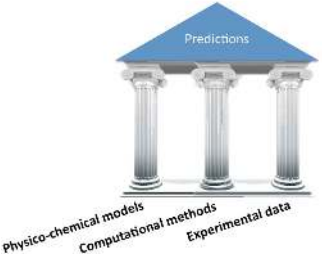


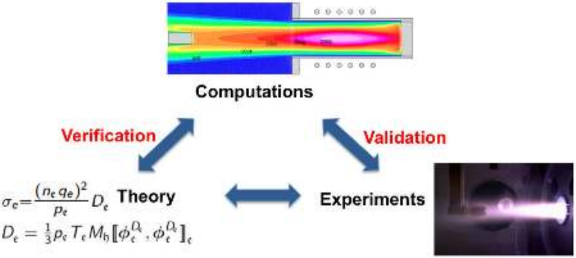
OBJECTIF :
SOLUTION
STATISTIQUE
DE L'EDP





Three pillars for predictive engineering simulations





- Computation cannot be truly predictive without the coupling to theory and experiments...
- This coupling is precisely the verification and validation process!

⇒
- **Verification**: Is the computational method implemented correctly?
- **Validation** : Are we solving the right equations?
- Uncertainty Quantification is the end-to-end study of the reliability of scientific predictions

Uncertainty Quantification

- Uncertainty identification: Data (e.g., operational uncertainties, geometrical variability); Model (e.g., physical model approximations, grid dependence, convergence)
- Uncertainty categorization (Epistemic) or Irreducible (Aleatory) Reducible

- Uncertainty identification: Data (e.g., operational uncertainties, geometrical variability); Model (e.g., physical model approximations, grid dependence, convergence)
- Uncertainty categorization (Epistemic) or Irreducible (Aleatory) Reducible
- Uncertainty quantification: Statistical description of input uncertainties (e.g., mean value and standard deviation); Distribution type (defined by a probability density function pdf-)
- Uncertainty propagation: Probabilistic definition of the output quantities; (Applying methods such as Monte Carlo Simulation; Methods of Moments; Polynomial Chaos,...
- Uncertainty analysis: Analysis of variance (ANOVA); Allocation of output uncertainty to specific sources; Identify the factors that contribute most to risk

Some definitions

Errors vs. uncertainties

- **Errors**: associated to the translation of a mathematical formulation into a numerical algorithm
  $\rightarrow$ Round-off errors and limited convergence of certain iterative algorithms
  $\rightarrow$ Implementation mistakes (bugs) or usage errors
- **Uncertainties**: associated to the choice of the physical models and to the specification of the input parameters

Uncertainty Classification

- **Aleatory**: not strictly due to a lack of knowledge $\rightarrow$ **can not be reduced**
  $\rightarrow$ characterized using probabilistic approaches
  $\rightarrow$ Ex: determination of material properties or operating conditions
- **Epistemic**: potential deficiency due to a lack of knowledge
  $\rightarrow$ It can arise from assumptions introduced in the derivation of the mathematical model or from simplifications
  $\rightarrow$ **can be reduced** (for example by improving the measures)

- Epistemic uncertainties are a property of the models applied in the analysis, including the choices made by the modeler
- The aleatory uncertainties are a strict property of the system being analyzed

Some definitions

- Epistemic uncertainties are a property of the models applied in the analysis, including the choices made by the modeler
- The aleatory uncertainties are a strict property of the system being analyzed
- Different ways of treatment and quantification
- Methods for handling epistemic uncertainties generally place some type of bounds on the resulting output uncertainty, largely based on (subjective) estimates of error and input uncertainty levels.

Predictions under uncertainty

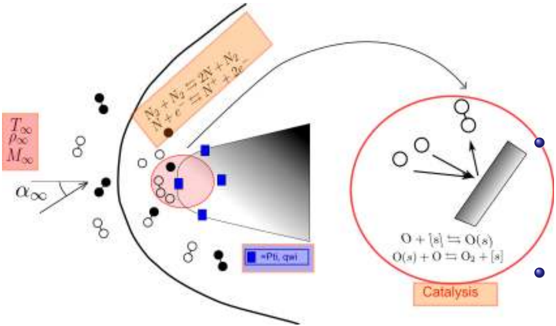
Introduction of UQ in numerical simulations: 3 Steps

- Data assimilation: (ex. conditions inferred from experiments) define a random vector $\boldsymbol{\xi} = (\xi_1, \ldots, \xi_N)$ that parametrizes uncertain input data with a specified PDF $p_{\boldsymbol{\xi}}$
- Uncertainty propagation: data uncertainty propagate into the forward model so that the output quantities of interest (QOIs) are random
  $\rightarrow$ compute statistics of the QOI
- Post-Processing analysis: reliability assessments, validation metrics

How taking into account experimental uncertainties and observations in the numerical simulation

- Forward problem
- Backward problem

Forward problem example



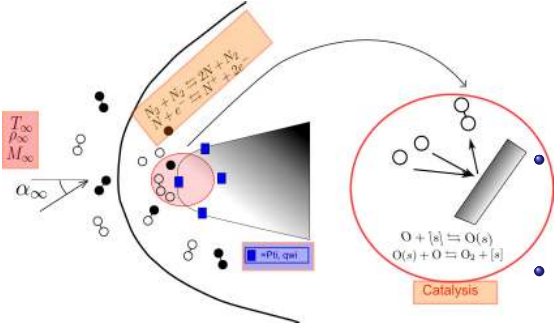
- Input data
  - Freestream pressure p_∞ and Mach number M_∞
  - Catalytic recombination coefficient γ and gas reaction rate coefficients $k_r, \quad r \in \{1, \ldots, n_R\}$
- Output data
  - Stagnation-point pressure p_{st} and heat flux q_{st}

- **CFD code**

$\Rightarrow$ **Sources of uncertainties**

- Unknown: $p_\infty \sim \mathcal{U}(16.3, 24.3)$ and $M_\infty \sim \mathcal{U}(13.7, 17.3)$
- Arrhenius gas reaction rate coefficients
- Epistemic uniform uncertainty on $\gamma \sim \mathcal{U}(0.001, 0.002)$

Inverse problem example



- FADS measurements
  - Stagnation-point pressure p_{st} and heat flux q_{st}
- Uncertainties
  - Experimental data + chemical model parameters

- Investigation of one point of the trajectory of the EXPERT vehicle
  $(p_\infty = 20.3, T_\infty = 245.5, M_\infty = 15.5)$

$\Rightarrow$ **Objective**: determine uncertainties on freestream conditions p_∞ and M_∞ from the FADS data

The mathematical setting of the problem

Let the output of interest $u(\boldsymbol{x}, t, \boldsymbol{\xi}(\boldsymbol{\omega}))$ be governed by the equation:

$$\mathcal{L}(\boldsymbol{x}, t, \boldsymbol{\xi}(\boldsymbol{\omega}); u(\boldsymbol{x}, t, \boldsymbol{\xi}(\boldsymbol{\omega}))) = \mathcal{S}(\boldsymbol{x}, t, \boldsymbol{\xi}(\boldsymbol{\omega})), \tag{1}$$

where $\mathcal{L}$ (algebraic or differential operator) and $\mathcal{S}$ are on $D \times T \times \Xi$, $\boldsymbol{x} \in D \subset \mathbb{R}^{n_d}$, with $n_d \in \{1, 2, 3\}$, $\boldsymbol{\xi}(\boldsymbol{\omega}) = \{\xi_1(\omega_1), \dots, \xi_d(\omega_d)\} \in \Xi$ with parameters space $\Xi \subset \mathbb{R}^d$

Probability framework (on the probability space $(\Omega, \mathcal{F}, P)$):
realizations $\boldsymbol{\omega} = \{\omega_1, \dots, \omega_d\} \in \Omega \subset \mathbb{R}$ with Ω set of outcomes, $\mathcal{F} \subset 2^{\Omega}$ is the σ-algebra of events, $P : \mathcal{F} \to [0, 1]$ is a probability measure.

The objective of uncertainty propagation is to find the **probability distribution** of $u(\mathbf{y}, \boldsymbol{\xi})$ and its **statistical moments** $\mu_{\mathrm{u}_i}(\mathbf{y})$ given by

$$\mu_{\mathrm{u}_i}(\mathbf{y}) = \int_{\Xi} u(\mathbf{y}, \boldsymbol{\xi})^i f_{\Xi}(\boldsymbol{\xi}) \mathrm{d}\boldsymbol{\xi}. \tag{2}$$

How compute this integral in an efficient way ?

Actually two kind of methodologies exist:

- *Intrusive*: the method requires intensive modifications of the numerical code NOTE: the number of equations is not preserved!
- *Non-intrusive*: No modifications of the deterministic scheme are demanded (the CFD code is a black-box)

- Deterministic Methods (moments, perturbation ...)
- Sampling techniques (Monte Carlo, Latin Hypercube)
- Stochastic collocation (Lagrangian interpolation)
- Probabilistic collocation (Chaos version of Lagrangian interpolation)
- (generalized-) Polynomial Chaos (gPC)
- Surrogate modelling

The gPC can be intrusive (Galerkin projection) or non-intrusive.

Let the output of interest $u(\boldsymbol{x}, t, \boldsymbol{\xi}(\boldsymbol{\omega}))$ be governed by the equation:

$$\mathcal{L}(\boldsymbol{x}, t, \boldsymbol{\xi}(\boldsymbol{\omega}); u(\boldsymbol{x}, t, \boldsymbol{\xi}(\boldsymbol{\omega}))) = \mathcal{S}(\boldsymbol{x}, t, \boldsymbol{\xi}(\boldsymbol{\omega})), \tag{3}$$

where $\mathcal{L}$ (algebraic or differential operator) and $\mathcal{S}$ are on $D \times T \times \Xi$, $\boldsymbol{x} \in D \subset \mathbb{R}^{n_d}$, with $n_d \in \{1, 2, 3\}$, $\boldsymbol{\xi}(\boldsymbol{\omega}) = \{\xi_1(\omega_1), \ldots, \xi_d(\omega_d)\} \in \Xi$ with parameters space $\Xi \subset \mathbb{R}^d$

Probability framework (on the probability space $(\Omega, \mathcal{F}, P)$):
realizations $\boldsymbol{\omega} = \{\omega_1, \ldots, \omega_d\} \in \Omega \subset \mathbb{R}$ with Ω set of outcomes, $\mathcal{F} \subset 2^{\Omega}$ is the σ-algebra of events, $P : \mathcal{F} \rightarrow [0, 1]$ is a probability measure.

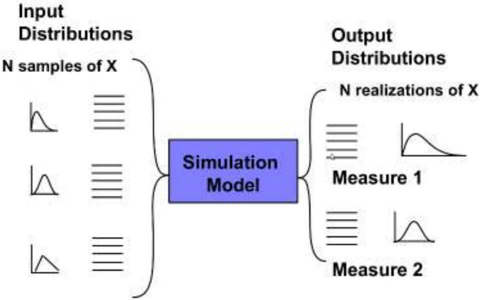
The objective of uncertainty propagation is to find the **probability distribution** of $u(\mathbf{y}, \boldsymbol{\xi})$ and its **statistical moments** $\mu_{\mathrm{u}_i}(\mathbf{y})$ given by

$$\mu_{\mathrm{u}_i}(\mathbf{y}) = \int_{\Xi} u(\mathbf{y}, \boldsymbol{\xi})^i f_{\xi}(\boldsymbol{\xi}) \mathrm{d}\boldsymbol{\xi}. \tag{4}$$

How compute this integral in an efficient way ?

First Possibility : Sampling Methods

- **Repeated** simulations with a proper selection of the input values
- Results **collected** to generate a statistical characterization of the outcome
  → Efficient Monte Carlo (MC), pseudo-MC (Latin hypercube) or quasi-MC
- Sampling **is not the most efficient** UQ method, but it is *easy* to implement, *robust*, and *transparent*.



Second Possibility : Quadrature Methods

Motivation

- Evaluation of **integrals** needed
  → *Natural* to employ conventional numerical integration techniques

How ?

- Quadratures based on **Newton-Cotes** formulas for equally spaced abscissas
- **Stochastic Collocation**: Gaussian quadrature, *i.e.* the Gauss-Legendre integration formula based on Legendre polynomials
  → *Natural extension* to multiple dimensions as tensor product of 1D interpolants
  → **Curse of Dimensionality** for high dimensions, Smolyak Algorithm

Polynomial Chaos (PC) expansions

[Wiener 38; Cameron & Martin 47; Ghanem & Spanos 91]
Every QOI u can be expanded in a convergent series of the form

$$u(\boldsymbol{\xi}) \approx u^{\mathrm{PC}}(\boldsymbol{\xi}) = \sum_{\alpha=0}^{\mathrm{P}} u_\alpha \Psi_\alpha(\boldsymbol{\xi}),$$

- $\mathrm{P} = (n_{\boldsymbol{\xi}} + \mathrm{No})!/n_{\boldsymbol{\xi}}!\mathrm{No}!$, No: expansion degree
- $\{\Psi_\alpha\}_{\alpha=0,\dots,\mathrm{P}}$ polynomial functions orthogonal w.r.t $p_{\boldsymbol{\xi}}$
- correspondence between $p_{\boldsymbol{\xi}}$ and $\{\Psi_\alpha\}$
- $\{u_\alpha\}_{\alpha=0,\dots,\mathrm{P}}$: deterministic spectral coefficients

Determination of $\{u_\alpha\}$ by a non-intrusive spectral method (NISP)

$$u_\alpha = \|\Psi_\alpha\|^{-2} \int u(\boldsymbol{\xi})\Psi_\alpha(\boldsymbol{\xi}) \approx \|\Psi_\alpha\|^{-2} \sum_{i=1}^{n} u(\mathbf{x}, t, \boldsymbol{\xi}_i)\Psi_\alpha(\xi_i)\omega_i$$

$(\boldsymbol{\xi}_i, \omega_i)$ quadrature formulae points and weigths $\rightarrow$ deterministic code used as a black box

From PC expansions of QOIs [Crestaux, Le Maitre & Martinez 09]

- derivation of means and variances

$$E(u^{\mathrm{PC}}) = u_0, \quad Var(u^{\mathrm{PC}}) = \sum_{\alpha=1}^{\mathrm{P}} u_\alpha^2(\mathbf{x}) \left\langle \Psi_i^2 \right\rangle$$

- estimation of sensitivity information using the analysis of variance (ANOVA) decomposition
  $\rightarrow$ Sobol first order indices $\{S_i\}_{i=1,\dots,n_{\boldsymbol{\xi}}}$ determining the contribution to the variance of the i^{th} random parameter to the QOI u
  $\rightarrow$ Sobol total order indices $\{S_{T,i}\}_{i=1,\dots,n_{\boldsymbol{\xi}}}$ determining the contribution of the i^{th} random parameter to the QOI u including interactions with other parameter $j \in \{1,\dots,n_{\boldsymbol{\xi}}\}, j \neq i$

Properties of classical methods in UQ

Sampling

- **Strengths**: Simple and reliable, convergence rate is dimension-independent
- **Weaknesses**: sqrt(N) convergence $\rightarrow$ expensive for accurate tail statistics

Stochastic expansions

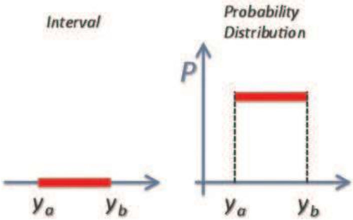
- **Strengths**: functional representation, exponential convergence rates
- **Problems**
  $\Rightarrow$ Discontinuity $\rightarrow$ Gibbs phenomena
  $\Rightarrow$ Singularity $\rightarrow$ divergence in moments
  $\Rightarrow$ Scaling to large n $\rightarrow$ exponential growth in number of simulation

- Second order probability
- **Interval valued probability**
- Evidence Theory
- ...

Interval valued probability

Minimal statement of knowledge: the input y is bounded $y \epsilon [y_a, y_b]$.

- **Nothing is known** expect that input uncertainties vary in some intervals
- Different than a random variable with a uniform probability: one true value vs. every outcome has the **same probability** of occurring



Interval valued probability

- Treating intervals in a probabilistic framework
  1. **Substitute** intervals with uniform r.v.s
  2. **Propagate** uncertainty using any of the probabilistic UQ methods
  3. Discard the probabilistic information and **compute the support** of the output

  → Very costly and requires precise evaluation of the tail distribution of the output quantity

Interval valued probability

- Treating intervals in a probabilistic framework
  1. **Substitute** intervals with uniform r.v.s
  2. **Propagate** uncertainty using any of the probabilistic UQ methods
  3. Discard the probabilistic information and **compute the support** of the output

  → Very costly and requires precise evaluation of the tail distribution of the output quantity
- Formulate the problem as a two-optimization problems, focused on the **bounds** of the output quantity in the space of the possible input

  → Worst case scenario analysis

Problem definition

Consider $n_{\xi 1}$ **epistemic** random variables and $n_{\xi 2}$ **aleatory** random variables. Using interval analysis in a mixed aleatory/epistemic uncertainty framework reduces to solve the following problems

$$\min_{\boldsymbol{\xi_1}(\boldsymbol{\omega})\subset\mathbb{R}^{n_{\xi 1}}} \mu'_{\mathrm{u}_i}(\mathbf{x},t) \qquad \textit{and} \qquad \max_{\boldsymbol{\xi_1}(\boldsymbol{\omega})\subset\mathbb{R}^{n_{\xi 1}}} \mu'_{\mathrm{u}_i}(\mathbf{x},t) \tag{5}$$

with

$$\mu'_{\mathrm{u}_i}(\mathbf{x},t) = \int_{\Xi_2} u(\mathbf{x},t,\boldsymbol{\xi_2})^i f_{\xi}(\boldsymbol{\xi_2})\mathrm{d}\boldsymbol{\xi_2}, \tag{6}$$

We are interested in a method that directly address the computation of 5 and 6 by minimizing the computational cost.

High number of uncertainties

- Use of different types of (sparse) numerical quadratures, sparse tensor products, adaptive methods
- **Problem reduction via ANOVA (or others) analysis and predominant uncertainties**

Problems in Fluid Mechanics: Some highlights from new methods

High number of uncertainties

- Use of different types of (sparse) numerical quadratures, sparse tensor products, adaptive methods
- **Problem reduction via ANOVA (or others) analysis and predominant uncertainties**

Mixed aleatory/epistemic uncertainty

- **Interaction between modelization/epistemic/ aleatory uncertainty**
- Global Method for treating at the same time the optimization and the stochastic space → Simplex^2 method

High number of uncertainties

- Use of different types of (sparse) numerical quadratures, sparse tensor products, adaptive methods
- **Problem reduction via ANOVA (or others) analysis and predominant uncertainties**

Mixed aleatory/epistemic uncertainty

- **Interaction between modelization/epistemic/ aleatory uncertainty**
- Global Method for treating at the same time the optimization and the stochastic space $\rightarrow$ Simplex^2 method

Discontinuous Flows (Compressible with shocks)

- **Develop adapted bases: multi-resolution or multi-elements gPC**
- Semi-intrusive multi-resolution stochastic method

Problems in Fluid Mechanics: Some highlights from new methods

High number of uncertainties

- Use of different types of (sparse) numerical quadratures, sparse tensor products, adaptive methods
- **Problem reduction via ANOVA (or others) analysis and predominant uncertainties**

Mixed aleatory/epistemic uncertainty

- **Interaction between modelization/epistemic/ aleatory uncertainty**
- Global Method for treating at the same time the optimization and the stochastic space → Simplex^2 method

Discontinuous Flows (Compressible with shocks)

- **Develop adapted bases: multi-resolution or multi-elements gPC**
- Semi-intrusive multi-resolution stochastic method

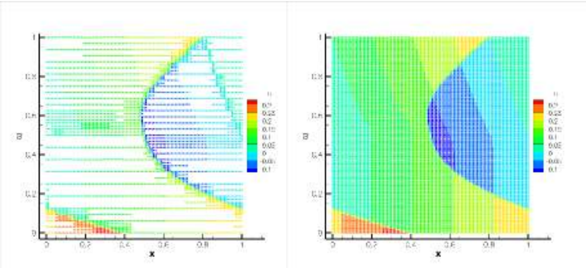
Coupling numerics/physics

- Use of observations in the model calibrations
- **Use of UQ tools in order to perform predictive numerical experiments**

Main Idea

Adaptive refinement/derefinement of the whole surface

Major issue in UQ+CFD → Number of CFD runs



Need for an intrusive approach
→ Coupled strategy in physical/stochastic spaces
→ Semi-Intrusive Method (No additional equations)

Weak Coupling (Fixed CFD and physical mesh)

Classical methods:
$N = N_x \times N_t \times N_\xi$
Remark: N_ξ is constant

Evaluations ratio:
The number of effective evaluations in the physical/stochastic space can be reduced
$\eta = \eta(t) = \frac{N_{\text{Adaptive}}}{N_{\text{Full}}}$

Aim: Increase/Decrease points for selected locations of the spatial-time space

$N_\xi = N_\xi(x, t)$

AIM

Building a generalized UQ framework for systems with discontinuous responses affected by a moderate number of uncertainties

Desired properties

- Generalized non-intrusive/intrusive framework
- Capability to manage whatever form of pdf (discontinuous, time varying)
- Easy implementation even in the intrusive form: semi-intrusive approach (*i.e.* for complex code/problems the deterministic scheme is fixed!)
- Efficient memory usage (the grids are $T \times \Omega \times \Xi$, $\Omega \subset \mathcal{R}^n$, $\Xi \subset \mathcal{R}^d$)
- Lower computational cost (with respect equivalent approaches)

The main idea

From Harten to the spatial-TE algorithm

Harten framework (essential elements)

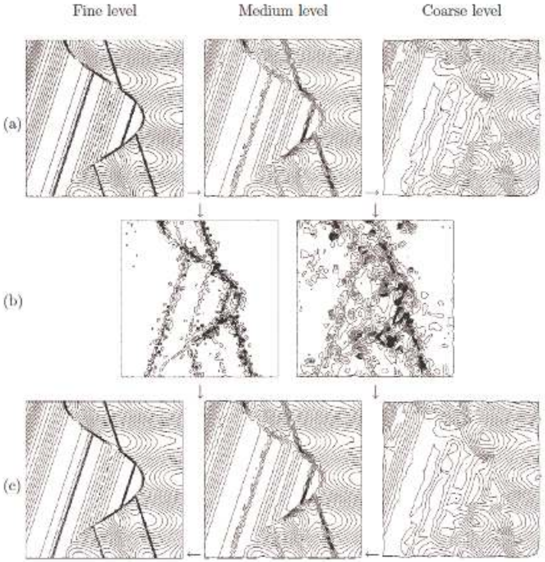
1. Hierarchical representation of the information between different resolution levels
2. Truncation of the non-essential information
3. Advancing between different time steps employing a CFL-like condition (in the physical space), *i.e.* you can predict a mesh from the previous time step

The spatial-TE algorithm

- From the coarsest level to the finest
- Same truncation criterion of Harten
- Generalized time advancing strategy efficient in the combined physical/stochastic space. Discontinuities in the responses could arise from the physical space, bifurcation in stochastic space or discontinuous (variation) of the pdf

Classical Multiresolution framework

Abgrall and Harten, Multiresolution Representation in Unstructured Meshes, SIAM Journal on Numerical Analysis, 1998



What we need:

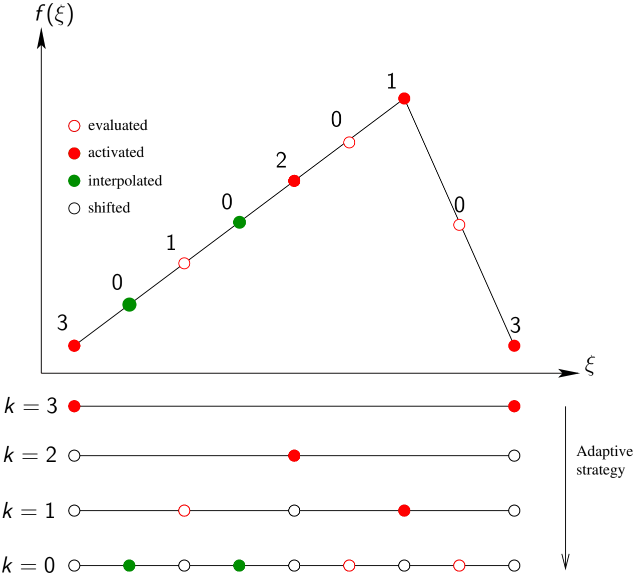
- A set of nested grid
- An interpolation operator
- A well resolved solution on th mesh
- A threshold ε

$$\Downarrow$$

$$||u^0 - \hat{u}^0|| \leq C\varepsilon,$$

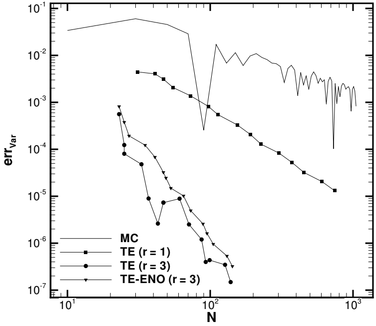
TE strategy

A constructive example

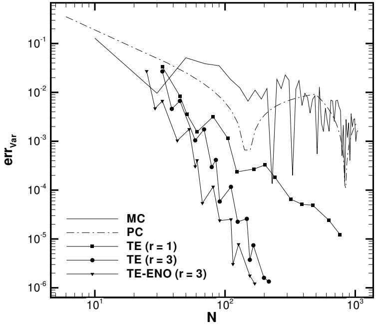


$$f_1 = f_1(\xi) = sin(2\xi^2\pi)$$

$$f_2 = \begin{cases} sin(2\xi^2\pi) & \text{if} \quad \xi \leq 11/20 \\ sin(2\xi^2\pi) + 1 & \text{otherwise.} \end{cases}$$



(c)

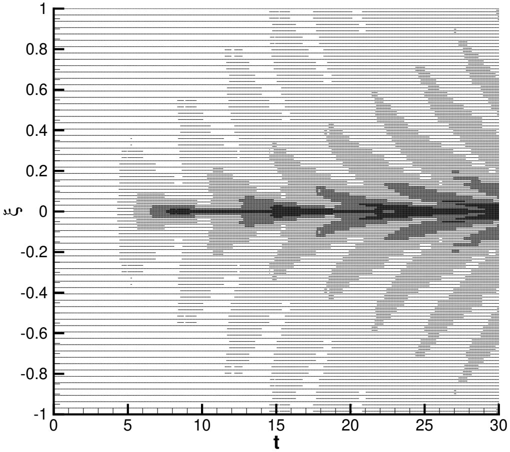


(d)

Figure: Relative error on the variance: f_1 (c) and f_2 (d)

$$\begin{cases} \dfrac{\mathrm{d}y_1}{\mathrm{d}t} = y_1 y_3 \\[2ex] \dfrac{\mathrm{d}y_2}{\mathrm{d}t} = -y_2 y_3 \\[2ex] \dfrac{\mathrm{d}y_3}{\mathrm{d}t} = -y_1^2 + y_2^2 \\[2ex] \mathbf{y}(t=0) = (1, 0.1\xi, 0) \\ \xi = 2\omega - 1 \quad \text{with} \quad \mathcal{U}(0,1) \end{cases}$$

Numerical solution: RK4 with $\Delta t = 0.05$ (600 time steps)

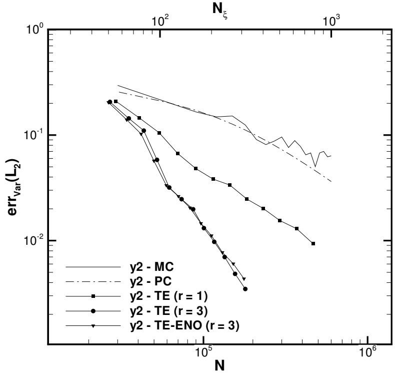


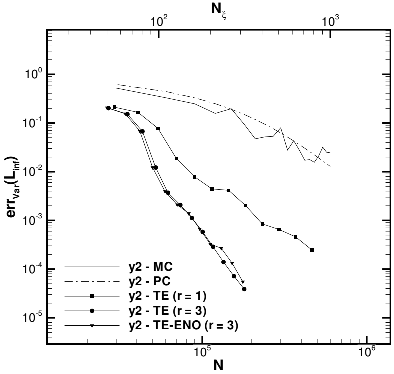
The classical intrusive PC fails to converge after $t = 8$

Numerical results

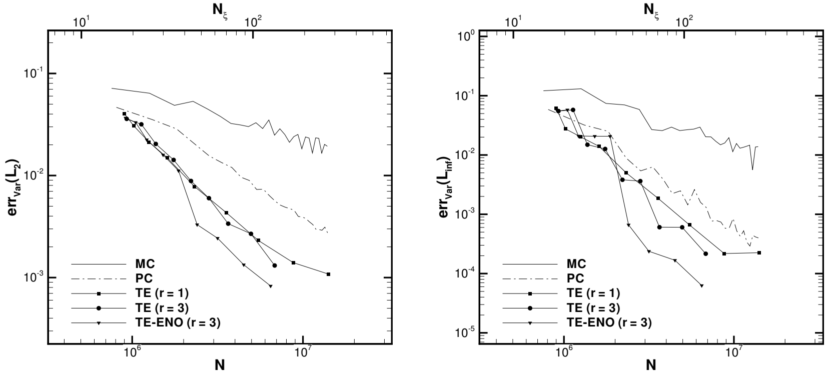
A non-linear vectorial ODE system: Kraichnan-Orszag

$$
\begin{cases}
\text{err}_{\mu^m}|_{L_p} = ||\mu^m(t) - \mu^m_{\text{ref}}(t)||_{L_p} = \left(\dfrac{1}{N_t} \displaystyle\sum_{i=1}^{N_t} \left| \dfrac{\mu^m_i(t) - \mu^m_{\text{ref},i}(t)}{\mu^m_{\text{ref},i}(t)} \right|^p \right)^{1/p} \\
\text{err}_{\mu^m}|_{L_\infty} = ||\mu^m(t) - \mu^m_{\text{ref}}(t)||_{L_\infty} = \max\limits_i \left| \dfrac{\mu^m_i(t) - \mu^m_{\text{ref},i}(t)}{\mu^m_{\text{ref},i}(t)} \right|,
\end{cases}
$$





$$\begin{cases} \dfrac{\partial u(x,t,\xi)}{\partial t} + \dfrac{\partial f(u(x,t,\xi))}{\partial x} = 0 \\ u(x,0,\xi) = u_0(x,\xi) = sin(x\pi\xi) \end{cases}$$



(c) (d)

Conclusions and perspectives

Conclusions

- Generalized framework for UQ
- Easy implementation starting from existing deterministic codes
- Good accuracy compared to classical techniques (MC and PC)

Work in progress

- Extension to two and three dimensional stochastic spaces
- Application to multidimensional physical codes
- Point-value and Cell-averages comparison (in a real battle)

Staff

- BACCHUS Team, INRIA Bordeaux Sud-Ouest, Leader: **R. Abgrall**
- AQUARIUS Team, Joint Team with UQ Lab (Stanford University)
  Leaders: **P.M. Congedo**, **G. Iaccarino**

Staff

- BACCHUS Team, INRIA Bordeaux Sud-Ouest, Leader: **R. Abgrall**
- AQUARIUS Team, Joint Team with UQ Lab (Stanford University)
  Leaders: **P.M. Congedo**, **G. Iaccarino**

Main activities

- Efficient and Flexible numerical methods for UQ
  - Semi-intrusive approach for unsteady discontinuous functions
  - Computation and decomposition of High-Order statistics
  - Bayesian methods (inverse problem, model calibration, ...)

Team Presentation

Staff

- BACCHUS Team, INRIA Bordeaux Sud-Ouest, Leader: **R. Abgrall**
- AQUARIUS Team, Joint Team with UQ Lab (Stanford University)
  Leaders: **P.M. Congedo**, **G. Iaccarino**

Main activities

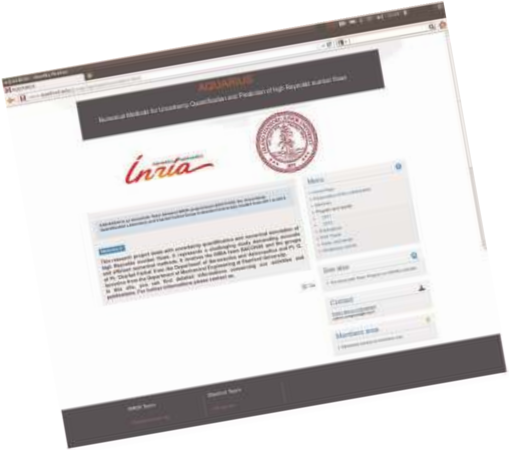
- Efficient and Flexible numerical methods for UQ
  - Semi-intrusive approach for unsteady discontinuous functions
  - Computation and decomposition of High-Order statistics
  - Bayesian methods (inverse problem, model calibration, ...)
- Methods for robust design optimization
  - Efficient strategy using response surface and stochastic dimension reduction
  - Simplex^2 Method
  - HR-Simplex

Staff

- BACCHUS Team, INRIA Bordeaux Sud-Ouest, Leader: **R. Abgrall**
- AQUARIUS Team, Joint Team with UQ Lab (Stanford University)
  Leaders: **P.M. Congedo**, **G. Iaccarino**

Main activities

- Efficient and Flexible numerical methods for UQ
  - Semi-intrusive approach for unsteady discontinuous functions
  - Computation and decomposition of High-Order statistics
  - Bayesian methods (inverse problem, model calibration, ...)
- Methods for robust design optimization
  - Efficient strategy using response surface and stochastic dimension reduction
  - Simplex^2 Method
  - HR-Simplex
- Application in Fluid Mechanics (Real-gas flows, Tsunami, Multiphase flows)



http://www.stanford.edu/group/uq/aquarius/index3.html