Optimal Uncertainty Quantification of a Risk Measurement

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Abstract:

Uncertainty quantification in the context of industrial operations can be conducted by considering the uncertain inputs of a computer code as a vector of random variables, named $\mathbf{X} = (X_1, ..., X_d) \sim \mu$. The computer code represented by a function $G(\cdot)$ usually replaces a physical experiment. The most widespread approach consists in running $G(\cdot)$ with different combinations of inputs in accordance with their probability distribution, in order to study the related uncertainty on the output $Y = G(X_1, ..., X_d)$ or to estimate a specific quantity of interest (QoI). A QoI is is a statistical quantity derived from Y, e.g. a performance as the mean of Y, a probability of failure, or a risk criterion as a high-level quantile of Y. Because the QoI only depends on the choice of the probability measure μ, it is formally represented as a scalar function defined on a measure space.

We propose to gain robustness on the quantification of this QoI. Indeed, the probability distributions characterizing the uncertain input may themselves be uncertain. For instance, contradictory expert opinion may yield difficulty to select a probability model, and the lack of information in the input variables affects inevitably the choice of the distribution. Therefore, there exists a difference between the probabilistic model and an unknown hypothetical perfect modeling of the physical input. As the uncertainty on the input distributions propagates to the QoI, an important consequence is that different choices of input distributions will lead to different values of the QoI.

To consider this uncertainty, we propose to evaluate the maximum of the QoI over a space of probability measures. The solution of this optimization problem is numerically computed thanks to a reduction theorem [7]. In this PhD we generalize the reduction theorem from [6] which introduces the Optimal Uncertainty Quantification (OUQ) framework. We have shown that under the assumption that the QoI is a quasi-convex lower semicontinuous function of the input probability measure μ, the maximum of the QoI is reached on the extreme points of the measure space whenever this domain is convex compact (this last assumption can be relaxed).

The choice of the domain is therefore important, we focus in this work on the space of (unimodal) probability measures specified by moment constraints, called moment class (respectively unimodal moment class) [8]. This means that we investigate bounds on the QoI over the set of all (unimodal) probability distributions with given moments. This is justified by our industrial context, mainly related to nuclear safety issues. Indeed, in practice the estimation of the input probability distributions, built with the help of the expert, often relies only on the knowledge of the mean or the variance of the input variables. The unimodality constraint is convenient to deal with input parameter with one most likely value.

In the context of the moment class, the extreme points are simply finite convex combinations of Dirac masses. To be more specific it holds that when N pieces of information are available on the moments of a measure μ, it is enough to assume that the measure is supported on at most $N+1$ points. Those properties have been studied by Winkler [9] based on the well known Choquet

theory [1]. For the unimodal moment class, we proved that the extreme points are finite convex combinations of uniform distributions with one parameter set to the mode [7].

One of the main issues is the computational complexity of the QoI optimization over the extreme points of the measure space. Semi-Definite-Programming [3] has already been explored by Lasserre [4], but the deterministic solver considered rapidly reaches its limits as the dimension of the problem increases. Other approaches, in particular stochastic algorithms, have also been implemented, as available for instance in the Python toolbox developed by McKerns [5], called Mystic framework, that fully supports the OUQ framework. However, this toolbox was built as a generic tool for generalized moment problems. Hence, using Mystic for enforcing classical moment constraints cannot be optimal. By restricting the work to the moment class and the unimodal moment class, we developed an original and practical approach based on the theory of canonical moments [2]. Canonical moments of a measure can be seen as the relative position of its moment sequence in the moment space. They are inherent to the measure and therefore present many interesting properties. A parameterization based on canonical moments allows an efficient exploration of the extreme points of the (unimodal) moment class where the maximum QoI is to be found [8]. Hence, we propose a original parameterization of the optimization problem leading to a simplified and constraints free optimization program. We have shown that in both toy examples and industrial applications, our algorithm greatly improves the efficiency of the maximum QoI search compared to the previous existing methods.

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Short biography – Robustness analysis is an emerging field in the domain of Uncertainty Quantification. It is especially required when evaluating industrial risks, as in nuclear safety. This PhD originates from the close colaboration of EDF R&D and the University of Toulouse III - Paul Sabatier. Key words: *robustness, uncertainty quantification, optimization, canonical moments.*