Management of uncertainties in engineering practice

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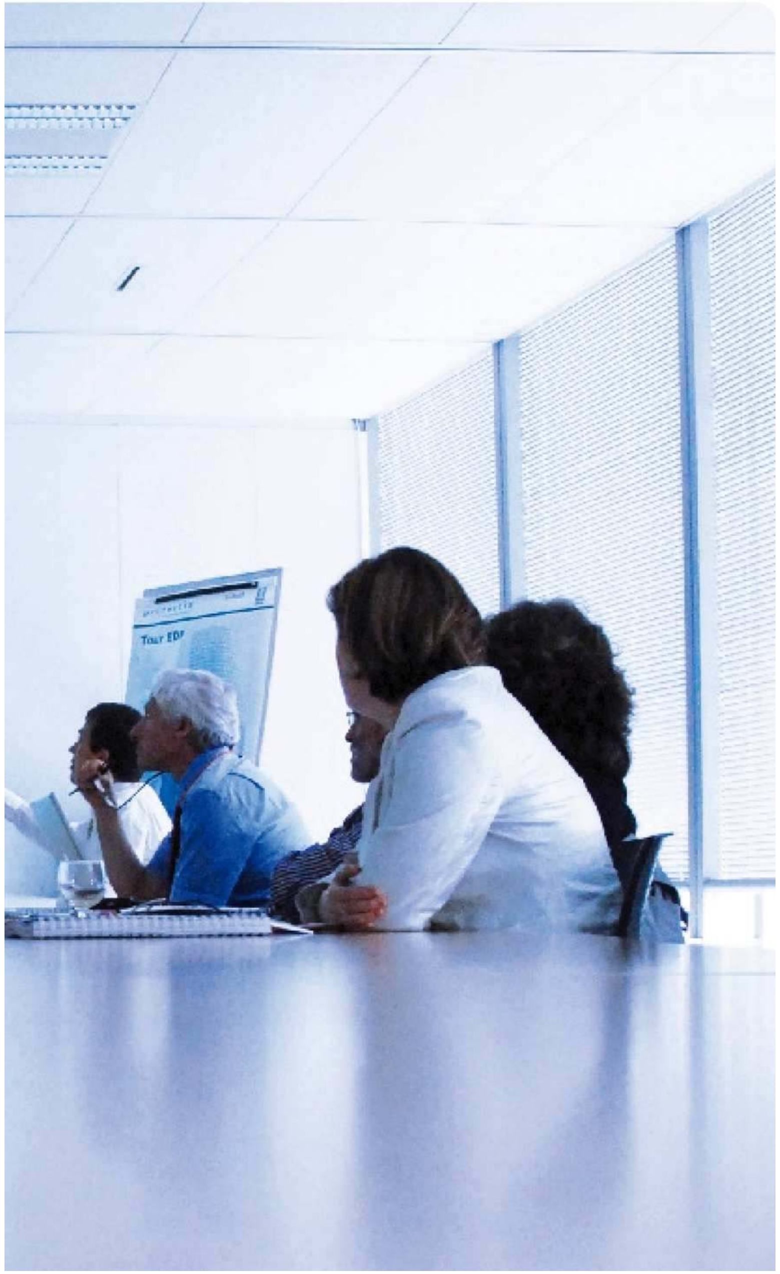
LEADING THE ENERGY CHANGE

Summary

- Common uncertainty management framework
- Examples of applied studies in different domains relevant for EDF :
  - Nuclear Power Generation
  - Thermal Power Generation
  - Hydraulics
  - Mechanics



Common uncertainty management framework

Which uncertainty sources?

- The modeling process of a phenomenon contains many sources of uncertainty:
  - model uncertainty: the translation of the phenomenon into a set of equations. The understanding of the physicist is always incomplete and simplified,
  - numerical uncertainty: the resolution of this set of equations often requires some additional numerical simplifications,
  - parametric uncertainty: the User fulfills the model with a set of deterministic values ... but could also have entered another set of values !

- Different kinds of uncertainties taint engineering studies, we focus here on parametric uncertainties (as it is common in practice)

Which (parametric) uncertainty sources?

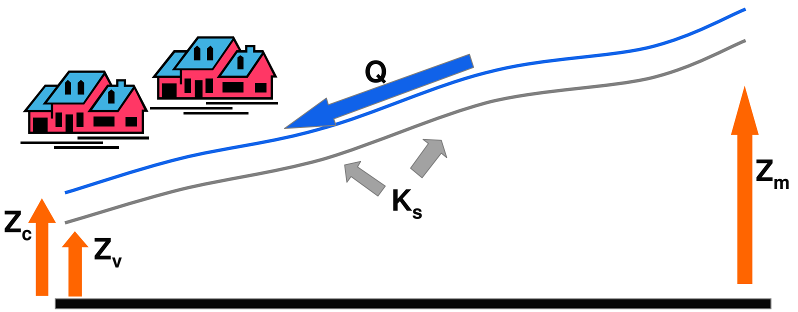
- Epistemic uncertainty
  - It is related to the lack of knowledge or precision about a parameter which is deterministic in itself (or can be considered deterministic under some accepted hypotheses). E.g. a characteristic of a material.

- Stochastic (or aleatory) uncertainty
  - It is related to the real variability of a parameter, which cannot be reduced (e.g. the discharge of a river in flood risk assessment of a riverside area). The parameter is stochastic in itself.

- Reducible vs non-reducible uncertainties
  - Epistemic uncertainties are (at least theoretically) reducible
  - Instead, stochastic uncertainties are (in general) irreducible (the discharge of a river will never be predicted with certainty)
    - A counter-example: stochastic uncertainty tainting the geometry of a mechanical piece → Can be reduced by improving the manufacturing line … The reducible aspect is quite relative since it depends on whether the cost of the reduction actions is affordable in practice

A (very) simplified example

Flood water level calculation



Uncertainty

$$Z_c = Z_v + \left[\frac{Q}{K_s \cdot \sqrt{(Z_m - Z_v)/L} \cdot B} \right]^{3/5}$$

Strickler's Formula

- Z_c : Flood level (variable of interest)
- Z_m et Z_v : level of the riverbed, upstream and downstream (random)
- Q : river discharge (random)
- K_s : Strickler's roughness coefficient (random)
- B, L : Width and length of the river cross section (deterministic)

General framework

Input Variables	***Model***	***Output variables of interest***
Uncertain : X Fixed : d	$G(X,d)$	$Z = G(X, d)$

Which output variable of interest?

- Formally, we can link the output variable of interest *Z* to a number of continuous or discrete uncertain inputs *X* through the function G:

$$Z = G(X, d)$$

  - *d* denotes the "fixed" variables of the study, representing, for instance a given scenario. In the following we will simply note: $Z = G(X)$

- The output variable of interest can be of dimension 1 or >1
- The function G can present itself as:
  - an analytical formula or a complex finite element code,
  - with high / low computational costs (measured by its CPU time),

- The uncertain inputs are modeled thanks to a random vector *X*, composed of *n* univariate random variables (X_1, X_2, …, X_n) linked by a dependence structure.

Which goal?

- Four categories of industrial objectives:
  - Industrial practice shows that the goals of any quantitative uncertainty assessment usually fall into the following four categories:
    - Understand: to understand the influence or rank importance of uncertainties, thereby to guide any additional measurement, modeling or R&D efforts.
    - Accredit: to give credit to a model or a method of measurement, i.e. to reach an acceptable quality level for its use.
    - Select: to compare relative performance and optimize the choice of maintenance policy, operation or design of the system.
    - Comply: to demonstrate compliance of the system with an explicit criteria or regulatory threshold (e.g. nuclear or environmental licensing, aeronautical certification, ...)
  - There may be several goals in any given study or along the time: importance ranking may serve for model calibration or model simplification at an earlier stage, which becomes, after some years of research, the basis for the selection of the best design and the final demonstration of compliance with a decision criteria.

Which criteria?

- Different quantities of interest
  - These different objectives are embodied by different criteria upon the output variable of interest.

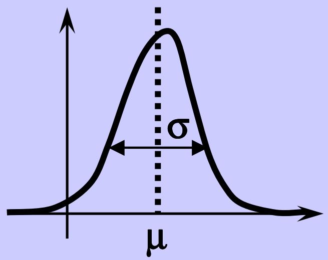
- These criteria can focus on:
  - its range : we only want to evaluate its min and maximum possible values. For example, in the prior stage of the design of a new concept.
  - its central dispersion : we want to evaluate its expected values and its dispersion around it. For example, in the design stage of a product.
  - its probability of exceeding a threshold : usually, the threshold is extreme. For example, in the certification stage of a product.

- Formally, the quantity of interest is a particular feature of the pdf of the variable of interest Z

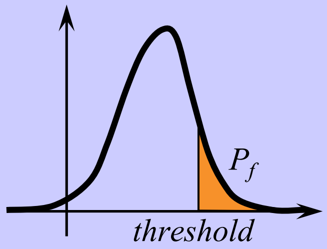
Why these questions are so important?

- The proper identification of:
  - the uncertain input parameters and the nature of their uncertainty sources,
  - the output variable of interest and the goals of a given uncertainty assessment,
- is the key step in the uncertainty study, as it guides the choice of the most relevant mathematical methods to be applied

What is **really** relevant in the uncertainty study?



Mean, median, variance, (moments) of *Z*



(Extreme) quantiles, probability of exceeding a given threshold

A particular quantity of interest: the "probability of failure"

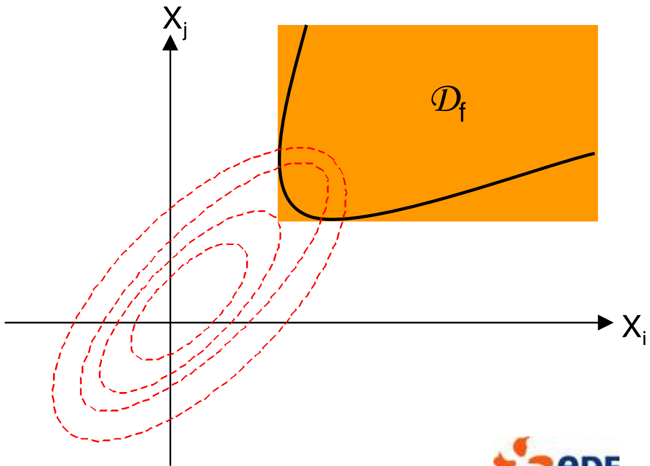
- G models a system (or a part of it) in operating conditions
  - Variable of interest *Z* → a given state variable of the system (e.g. a temperature, a deformation, a water level etc.)
- Following an « operator » point of view
  - The system is in safe operating condition if *Z* is above (or below) a given "safety" threshold

- System "failure" event: $Z \leq 0$
  - Classical formulation (no loss of generality) in which the threshold is 0 and the system fails when Z is negative
  - Structural Reliability Analysis (SRA) "vision": Failure if *C*-*L* $\leq$ 0 (Capacity – Load)

- Failure domain: $\mathcal{D}_f = \left\{ x \in \mathcal{X} : G(x) \leq 0 \right\}$
- Problem: estimating the mean of the random variable "failure indicator":

$$I_{\mathcal{D}_f}(x) = \mathbb{1}_{\{G(x) \leq 0\}}$$

$$p_f = \int_{\mathcal{D}_f} f(x) dx = \int_{\mathcal{X}} I_{\mathcal{D}_f}(x)\ f(x) dx = \mathbb{E}\left[I_{\mathcal{D}_f}(X) \right]$$



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Need of a generic and shared methodology

- There has been a considerable rise of interest in many industries in the recent decade

- Facing the questioning of their institutional control or certification authorities in an increasing number of different domains or businesses, large industrial companies have felt that domain-specific approaches are no more appropriate.

- In spite of the diversity of terminologies, most of these methods do share in fact many common algorithms.

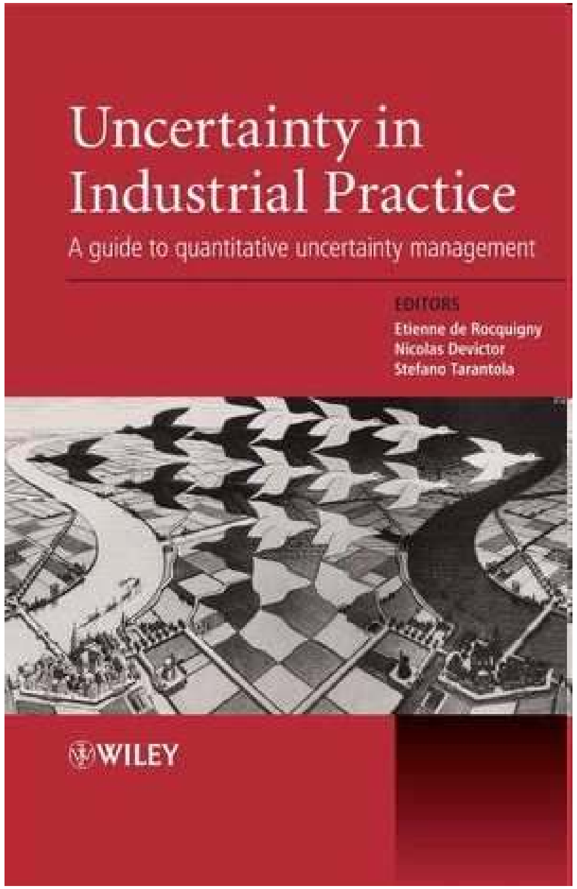
- That is why many industrial companies and public establishments (with a major role of EDF and CEA) have set up a common methodological framework which is generic to all industrial branches. This methodology has been drafted from industrial practice, which enhances its adoption by industries.

Shared global methodology

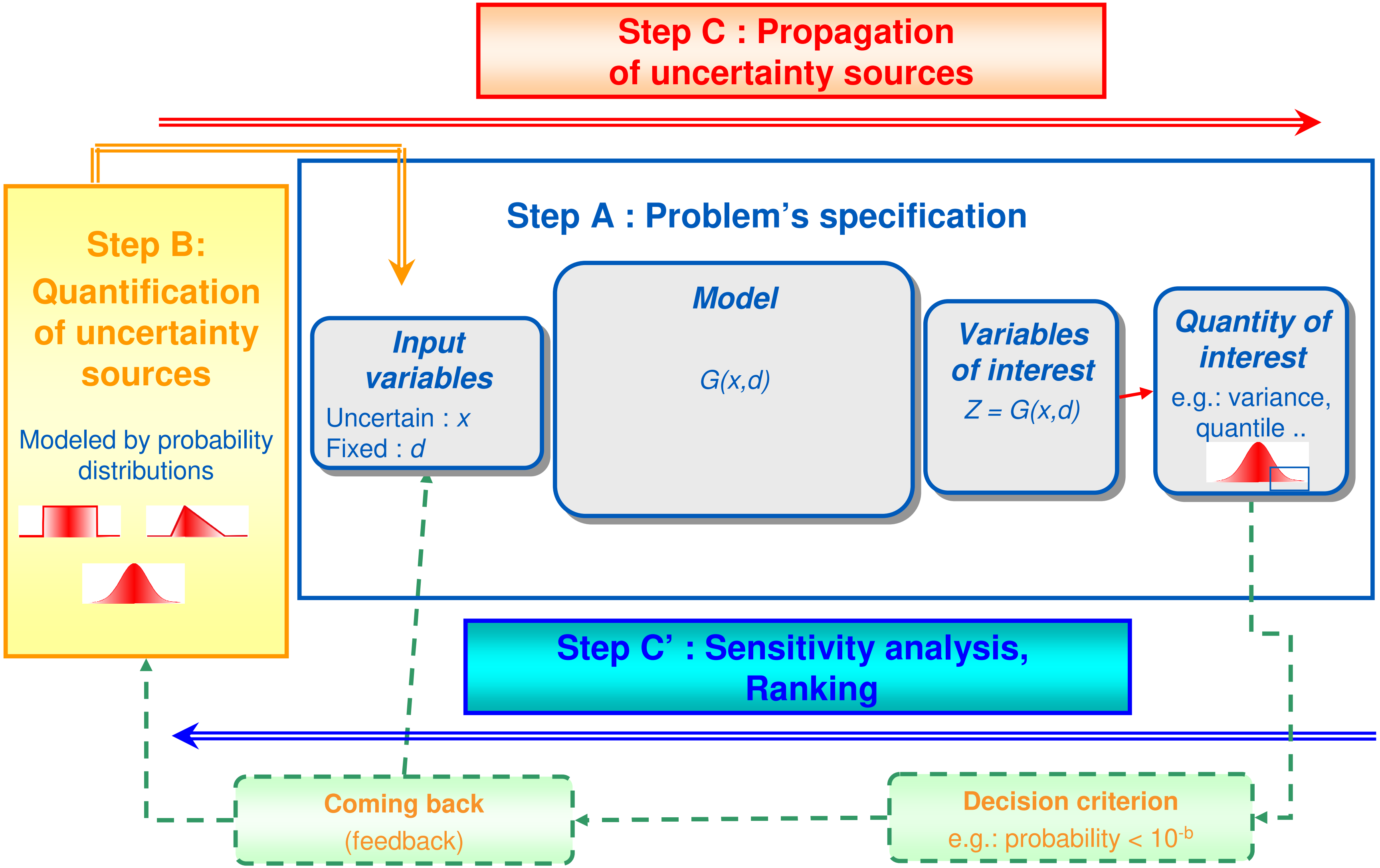
The global "uncertainty" framework is shared between EDF, CEA and several French and European partners (EADS, Dassault-Aviation, CEA, JRC, TU Delft …)

Uncertainty handbook
(ESReDA framework, 2005-2008)

ESReDA
European Safety, Reliability & Data Association



Uncertainty management - the global methodology



Some comments (Step B). Available information

- Different context depending on the available information
  - Scarce data (or not at all) → Formalizing the expert judgment
    - A popular method: the maximum entropy principle → Between all pdf complying with expert information, choosing the one that maximizes the statistical entropy :

$$H(X) = -\int_X f(x) \log(f(x))\, dx$$

← Measure of the "vagueness" of the information on X provided by f(x)

Information	Maximum Entropy pdf
$X \in [a, b]$	Uniform $X \sim \mathcal{U}(a, b)$
$\mathbb{E}(X) = \mu$ $X \in [0, \infty[$	Exponential $X \sim \mathcal{E}(1/\mu)$
$\mathbb{E}(X) = \mu$ $\mathbb{V}(X) = \sigma^2$	Normal $X \sim \mathcal{N}(\mu, \sigma)$

    - Other common choice: Triangular distribution (range + mode)

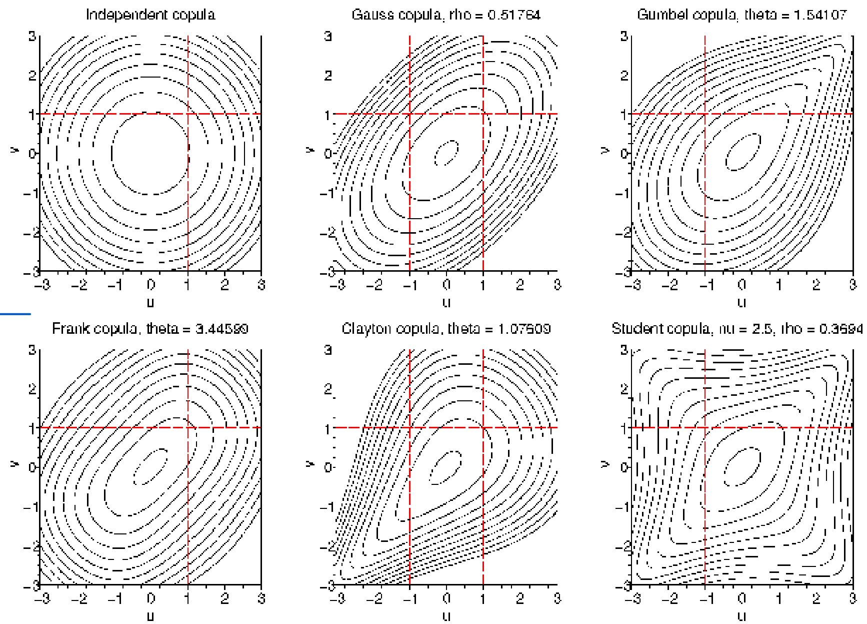
  - Feedback data available → Statistical fitting (parametric, non-parametric) in a frequentist or Bayesian framework

Some comments (Step B). Dependency

- Taking into account the dependency between inputs is a crucial issue in uncertainty analysis
  - Using copulas structure → CDF of the vector X as a function of the marginal CDF of $X_1 \dots X_n$:

$$F(x_1, x_2, \cdots, x_n) = C\left(F(x_1), F(x_2), \cdots, F(x_n)\right)$$

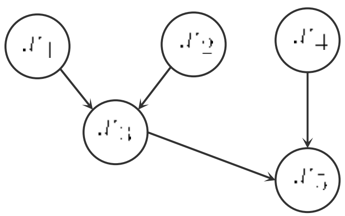
Example: All bivariate densities here have the same marginal pdf's (standard Normal) and the same Spearman rank coeff. (0.5)



  - Using conditional distributions
    - often based on "causality" considerations

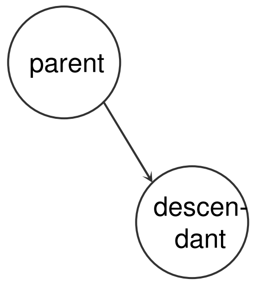
$$f(x_1, x_2) = f(x_1) \cdot f(x_2|x_1)$$

    - Directed Acyclic Graphs (Bayesian Networks) are helpful for representing the dependency structure



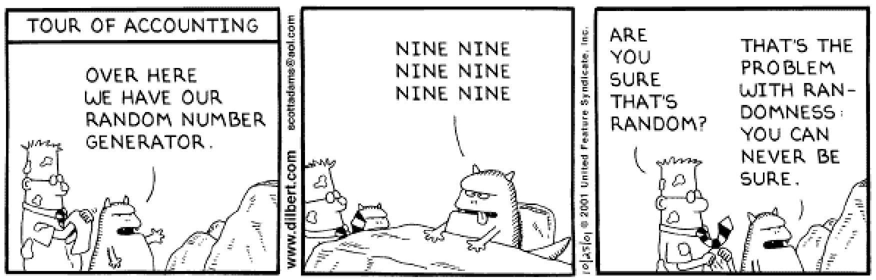
$$f(x_1, x_2, \cdots, x_n) = \prod_{i=1}^{n} \left(f(x_i|\mathrm{pa}(x_i))\right)$$

Set of the "parents" of x_i



Some comments (Step C and C’). CPU time

- These topics will be covered by the main lectures of this school
  - Main issue in the industrial practice: the computational burden !
  - In most problems, the “cost” depends on the number of runs of the deterministic “function” G
- If the code G is CPU time consuming
  - Be careful with Monte-Carlo simulations!
  - Rule of thumb: for estimating a rare probabilities of 10^{-r}, you need 10^{r+2} runs of G !
  - Appropriate methods (advanced Monte Carlo, meta-modeling)
  - Appropriate software tools for:
    - Effectively linking the deterministic model G(X) and the probabilistic model F(X)
    - Perform distributing computations (High Performance Computing)
  - Avoid DIY solutions !



Examples. Nuclear Power generation

Nuclear production at EDF

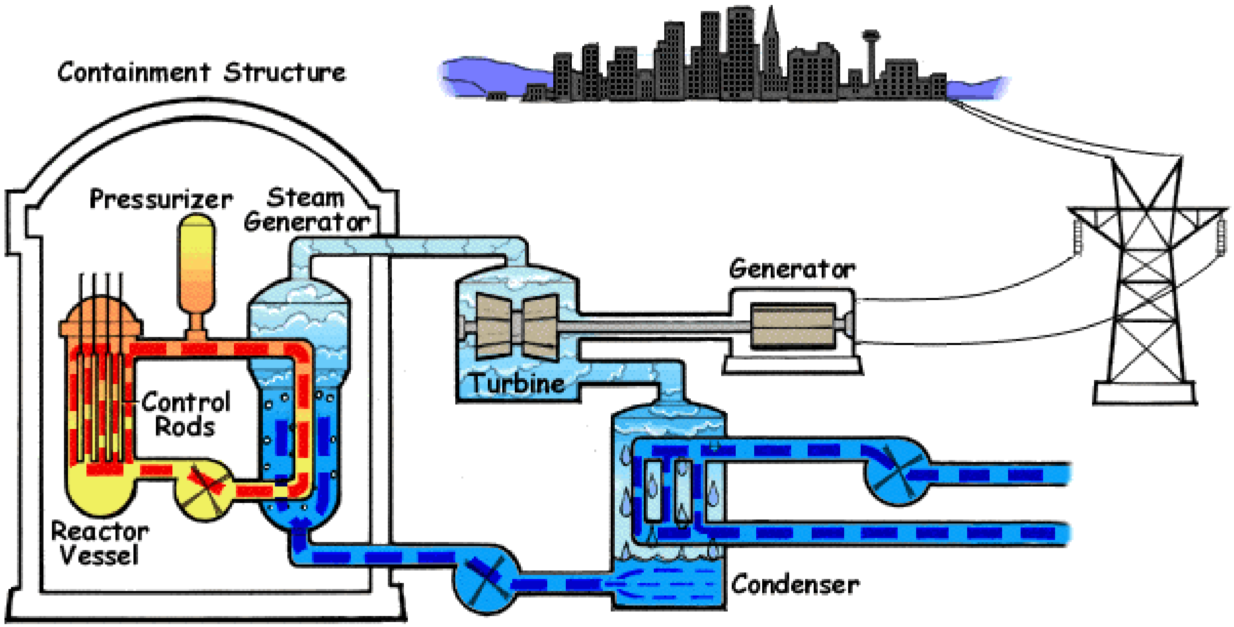
- 58 operating nuclear units in France, located in 19 power stations

- PWR (Pressurized water reactor) technology
  - 3 power levels

- Installed power: 63.1 GW

- Thanks to standard technologies and exploiting conditions, a feedback of more than 1000 operating years

PWR Power unit principles



- **Two separate loops:**
  - **Primary (pressurized water**
  - **Secondary**
- **Three safety barriers (fuel beams, vessel, containment structure)**
- **Highly important stakes**
  - **in terms of safety**
  - **In terms of availability: 1 day off = about 1 M€**

The nuclear reactor pressure vessel (NRPV)

- A key component
- Height: 13 m, Internal diameter: 4 m, thickness: 0,2 m, weight: 270 t
- Contains the fuel bars
- Where the thermal exchange between fuel bars and primary fluid takes place
- It is the second “safety barrier”
- It cannot be replaced !
  - Nuclear Unit Lifetime $\leq$ Vessel Lifetime

- Extremely harsh operating conditions
  - Pressure: 155 bar
  - Temperature: 300 °C
  - Irradiation effects: the steel of the vessel becomes progressively brittle, increasing the failure risk during an accidental situation

NRPV Safety assessment: a particular UQ problem

- The problem formulation is typical in most nuclear safety problems:
  - **Given some hard (and indeed very rare) accidental conditions**, what is the "failure probability" of the component?
  - It is the case of "structural reliability analysis" (SRA)
  - The physical phenomenon is described by a computer code

The system is safe if Z is lower (or greater) than a fixed value (equal to zero, without loss of generality)

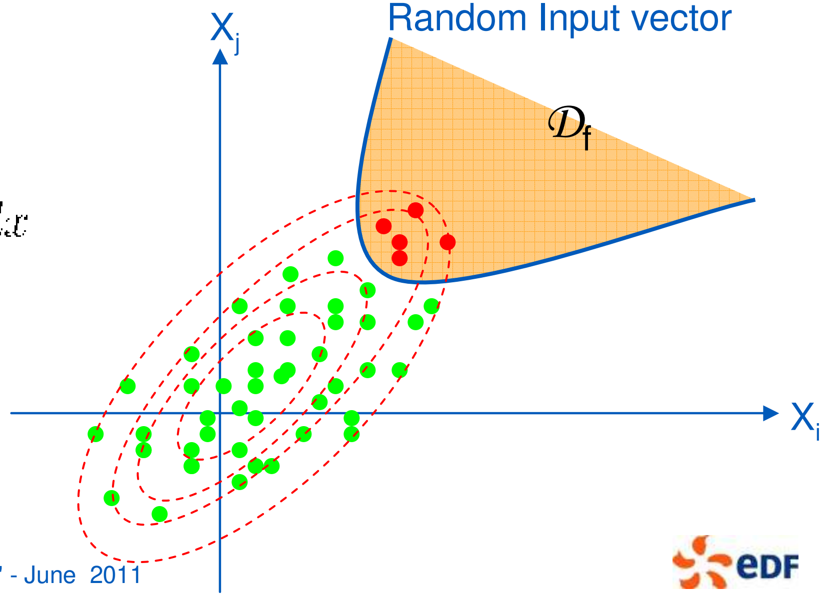
State variable of the system → $Z = G(X)$ ← Random Input vector

  - Failure condition: $Z \leq 0$
  - Failure probability

$$P_f = \mathbb{P}(Z \leq 0) = \int_{\mathcal{X}} \mathbb{1}_{\{G(x)\leq 0\}} f(x)dx = \int_{\mathcal{D}_f} f(x)dx$$

$$\mathcal{D}_f = \{x \in \mathcal{X}, G(x) \leq 0\}$$

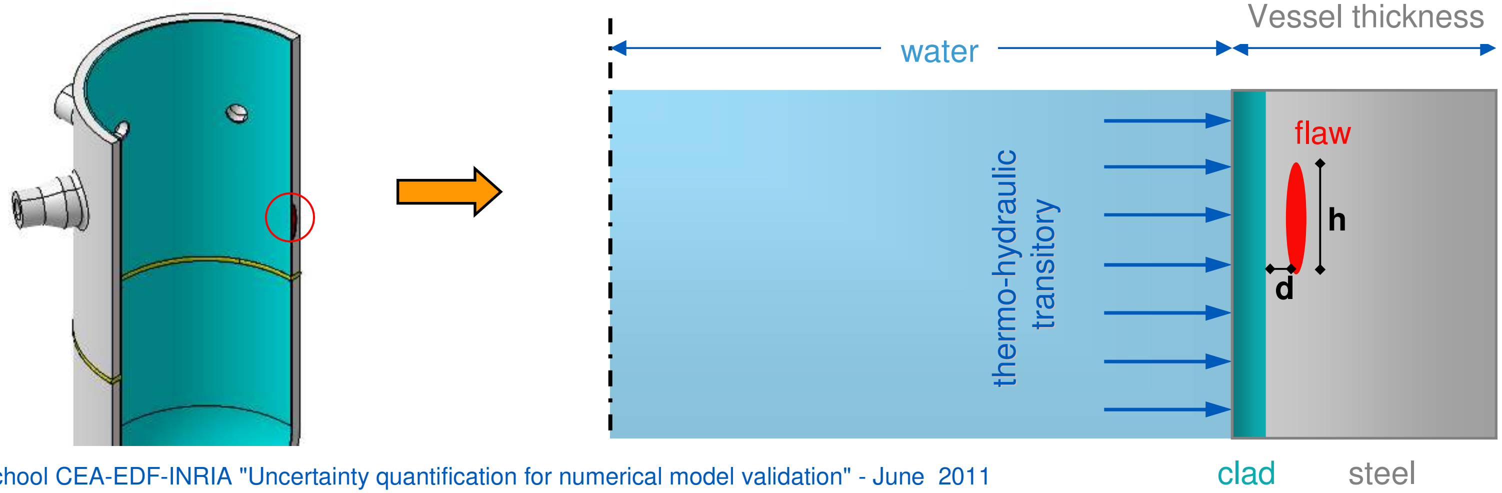
↑ Domain of failure



NRPV Safety assessment example [Munoz-Zuniga et al., 2009] (1/3) Step A

- Accidental conditions scenario: cooling water (about 20 °C) is injected in the vessel, to prevent over-warming
  - → Thermal cold choc → Risk of fast fracture around a manufacturing flaw

- Thermo-mechanical fast fracture model:
  - thermo-hydraulic representation of the accidental event (cooling water injection, primary fluid temperature, pressure, heat transfer coefficient)
  - thermo-mechanical model of the vessel cladding thickness, incorporating the vessel material properties depending on the temperature t
  - a fracture mechanics model around a manufacturing flaw
  - Outputs: Stress Intensity $K_{CP}(t)$ in the most stressed point
    Steel toughness, $K_{IC}(t)$ in the most stressed point
  - Goal: Evaluate the probability that for at least one t, the function $G = K_{IC} - K_{CP}$ is negative



NRPV Safety assessment example [Munoz-Zuniga et al., 2009] (2/3) Step B

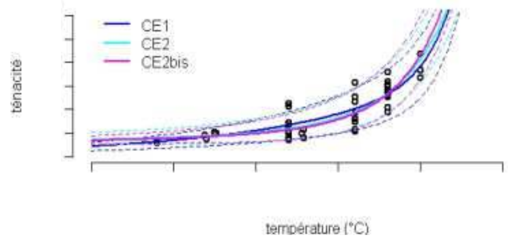
- A huge number of physical variables …
- In this example, three are considered as random. Penalized values are given to the remaining variables

Variable	Distribution	Distribution parameters	Comments
$u_{K_{Ic}}$	Normal	K_{Ic}^{RCC-M} and variation coefficient : $c_{K_{Ic}} = 15\%$	Support truncated at $[-4\sigma; +4\sigma]$
h	Weibull	Scale parameter $\alpha = 3.09$ mm and shape $\beta = 1.08$ mm	Distribution estimated by fitting exercise over inspection data
d	Uniform	$[0.1; 100]$ (mm)	The flaw is supposed to be in the inner half-thickness

1) Toughness low limit, playing in the steel toughness law $K_{IC}(t)$
Normal dispersion around a reference value K_{IC}^{RCC}

2) Dimension of the flaw h,

3) Distance between the flaw and the interface steel-clad d,



- A more complex example with 7 randomized inputs is given in [Munoz-Zuniga et al., 2010]

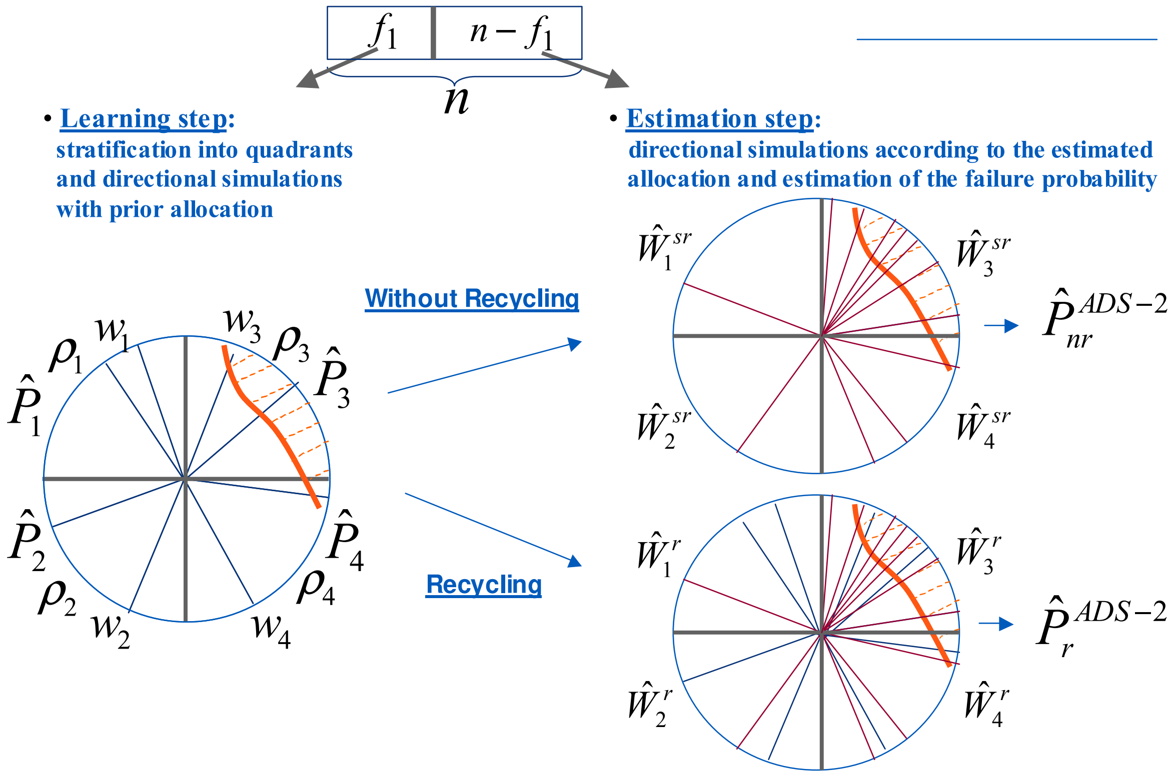
NRPV Safety assessment example [Munoz-Zuniga et al., 2009] (3/3) Step C

- A numerical challenge:
  - High CPU time consuming model
  - Standard Monte Carlo Methods are inappropriate to give an accurate estimate of P_f
  - An innovative Monte Carlo sampling strategy has been developed: “ADS-2” (Adaptive Directional Stratification)

- A numerical challenge:
  - Standard transformation
  - Directional sampling
  - Adaptive strategy to sample more “useful” direction

	Method	p	n	$f_1(n)$	$\hat{P}_f$	95% CI length	Nb. of calls to G_{Min}
1	DS	3	50	/	4.3×10^{-6}	1.3×10^{-5}	208
2	2-ADS	3	50	10	6.3×10^{-6}	7.0×10^{-6}	820
3	DS	3	200	/	3.7×10^{-6}	1.6×10^{-5}	822
4	2-ADS	3	200	40	8.0×10^{-6}	5.0×10^{-6}	3290
5	DS	3	1000	/	7.66×10^{-6}	7.0×10^{-6}	4028

Example of results. **NB P_f is here conditional to the occurrence of very rare accidental conditions**

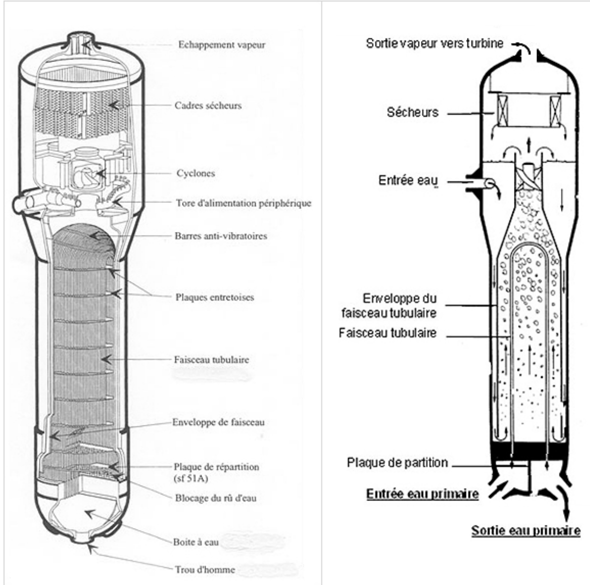


Another key component: the Steam Generator

- 3 or 4 steam generators in each unit ; 3000 to 5000 tubes / SG
- Many kinds of degradation factors

- Longstanding works, since 90's
  - e.g. assessment the probability of a Steam Generator (SG) tube rupture caused by multicircumferential cracking [Ardillon et al. 1996]
  - Ongoing works about other failure situations

- Safety issues
- Financial / Availability issues → It could be replaced but highly expensive



Bugey, 2010

Examples.
Thermal Power Production

A metrology example: assessing uncertainty in CO_2 emissions [de Rocquigny et al., 2008] (1/4)

- Context: Greenhouse Gases emission reduction
  - Extensive Regulatory coverage
  - Directive 2003/87/EC of October 13^{th} 2003
  - Decision of January 29^{th} 2004 (“EU Guidelines”)
  - Every producer receives an emission allowance/permit (quota)
- The methodology for a producer to provide CO_2 emissions is sketched by the EU Guidelines
  - An important point: uncertainty assessment: “*the operator shall have an understanding of the impact of uncertainty on the overall accuracy of his reported emission data” … “the operator shall manage and reduce the remaining uncertainties of the emissions data in his emissions report*”
- Important issue for EDF thermal production
  - The example shown hereby concerns a 2·600 MWe pulverized coal-fired plant
  - Only CO_2 resulting from coal combustion is considered

Thermal production at EDF

More than 30 units

Installed power: 11.5 GWh

Coal-fired, Oil-fired, Gas-fired plants and combustion turbines

Provide electricity for semi-base load and peak load (periods of high demand)

Assessing uncertainty in CO_2 emissions [de Rocquigny et al., 2008] (2/4)

Step A

- Variable of interest Z: CO_2 emissions of the plant in one year
- Quantity of interest: "relative uncertainty" u_Z .According to guidelines:

$$u_Z = 2 \cdot \frac{\sigma_Z}{\mu_Z}$$

Twice the coefficient of variation of Z

or

$$u_Z = \frac{q_{0.975} - q_{0.025}}{2}$$

Half-length of 95% conf. Interval of Z

Evaluation of Z

- Direct evaluation of CO_2 emission is not reliable and not recommended
- CO_2 emission is evaluated from fuel consumption Q

$$Z = \sum Q \cdot LHV \cdot F_{oxi} \cdot FE_{CO_2}$$

fuel raw Low Heat Value (TJ/t) — Oxidation Factor () — CO_2 emission factor (t/TJ)

- Three different methods of indirect evaluation (measurements chains + analytical combination of the results)
  - 1) Coal weighing. The coal is directly metered by a beltweigher
  - 2) Mass balance approach (NB Coal stocks are determined by a volumetric survey of stockpiles)

$$Q = Q_{purchased} + (Stock_{T_0} - Stock_{T_{end}}) - Q_{other}$$

  - 3) Evaluating the coal consumption from the plant's gross heat rate (PHR) and electric gross power output W

$$Q = \frac{W \cdot PHR}{HHV}$$

W: Gross Electric power output (MWh) — PHR Plant's Gross Heat Rate (kJ/kWh) →

HHV mean high heat value (kJ/kg)

$$PHR = OPHR + \sum \text{Ext. dev.} + \sum \text{Int. dev.}$$

Optimal Plant Heat Rate + External & Internal Perturbation Factors

Assessing uncertainty in CO_2 emissions [de Rocquigny et al., 2008] (3/4)

- Final goals of the study:
  - Demonstrate the compliance with uncertainty criteria (must be lower than limiting values given by Guidelines)
  - Select between the three available methods the one providing the lowest uncertainty
  - Understand the importance of various sources of uncertainties with respect to the uncertainty of Z

- **Step B**
  - Numerous uncertain input variables, depending on the measurement model (1, 2, 3)
  - pdf's have been assessed using GUM* standard recommendations (data and/or experts judgment) for measurement errors (in our case: normal, uniform, triangular pdf)

*Joint Committee for Guides in MetrologyEvaluation of measurement data — Guide to the expression of uncertainty in measurement (GUM)

- **Step C and C'**
  - Popular method traditionally used in metrology community: First order Taylor's approximation
  - Let $\mu = \mathbb{E}[X]$. By developing at first order $Z = G(X)$ around $\mathbb{E}[X]$:

$$\mathbb{E}[Z] = G(\mu)$$

$$\mathbb{V}[Z] = \mathbb{E}\left[(Z - \mathbb{E}[Z])^2\right] = \mathbb{E}\left[\left(\sum_{i=1}^{N} \frac{\partial G}{\partial X_i}\bigg|_{X=\mu} (X_i - \mu_i)\right)^2\right] =$$

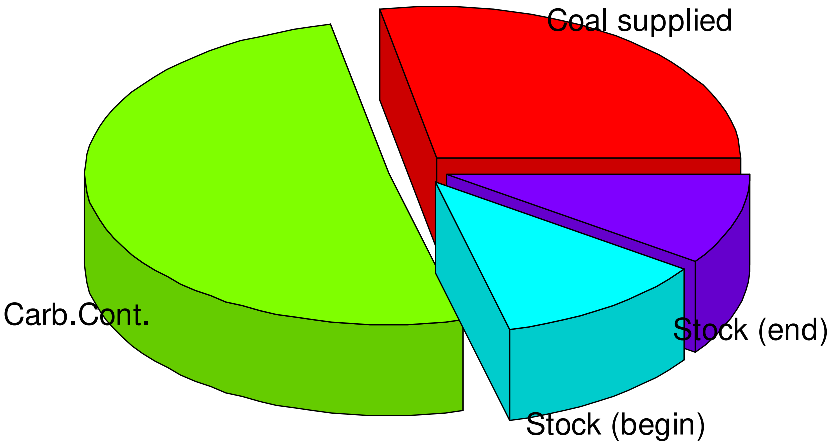
$$\mathbb{V}[Z] = \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{\partial G}{\partial X_i}\bigg|_{X=\mu} \frac{\partial G}{\partial X_j}\bigg|_{X=\mu} \rho_{ij}\sigma_i\sigma_j$$

  - Both Taylor approx. and standard Monte Carlo approach have been used here (with similar results)

Assessing uncertainty in CO_2 emissions [de Rocquigny et al., 2008] (4/4)

- Sensitivity analysis → Spearman ranks' correlations analysis

- Results and industrial feedback
  - Mass Balance method (n. 2) must be preferred
    - It allows to provide an evaluation of the CO_2 emission which complies with Guidelines target uncertainty value
  - Carbon Content is the variable which mostly affects the uncertainty on CO2 emissions estimation



	1 – Beltweigher		2 - Mass balance		3 - Heat rate	
	CO_2 uncertainty	Fuel consum-ption uncertainty	CO_2 uncertainty	Fuel consumption uncertainty	CO_2 uncertainty	Fuel consum-ption uncertainty
Target values (coal)	± 3 %	± 2.5 %	± 3 %	± 2 % (on purchased)	± 3 %	± 2.5 %
Results (coal)	± 3.8 %	± 3.3 %	± 2.8 %	Coal ± 2.0 % Purchased coal ± 1.5 %	± 3.1 %	± 2.5
Importance ranking for most important variables						
Coal suppplied	73.5 %	100%	28.0 %	57.0 %	-	-
Carbon content	26.0 %	-	50.9 %		41.7 %	-
OPHR	-	-	-	-	38.3 %	66.1 %
HHV	-	-	-	-	9.8 %	17.1%
W	-	-	-	-	7.0 %	11.6 %
Hydrogen content d.b.	-	-	-	-	1.6 %	2.8 %
Stock (beginning)	-	-	11.3 %	22.1 %	-	-
Stock (end)	-	-	9.7 %	20.8 %	-	-

Examples.
Hydraulics

Hydraulic simulation: a key issue

- Hydraulic simulation is a key issue for EDF

- Because EDF is a major hydro-power operator
  - mean annual production: 40 TWh
  - 220 dams, 447 hydro-power stations

- Because (sea or river) water plays a key role in nuclear production



Example of UQ in hydraulic simulation: emba-nkment failure hydrograph effects on flooded areas assessment [Arnaud et al., 2010] (1/5)

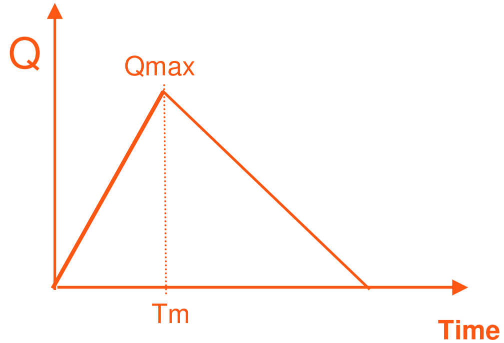
- Context: French regulations for large dams
  - Large dams are considered as potential sources of major risks (Law 22/07/1987)
  - Emergency Response Plans (PPI) must be prepared by the local authority ("Prefet") after consultation of the mayors and operators
  - Risk assessment study :
    - Flooded areas assessment (Maximum water level Z_{max}) and wave front arrival time T_{front} in case of dam failure
    - Seismic analysis
    - Evaluation of the possibility and effect of landslide in the reservoir
    - Hydrology study

- Hypothesis for the dam failure:
  - Concrete dams : the dam collapses instantaneously
  - Earth dams : the dam failure is assumed to be progressive by the formation of a breach due to internal erosion or an overflow → Embankment failure hydrograph

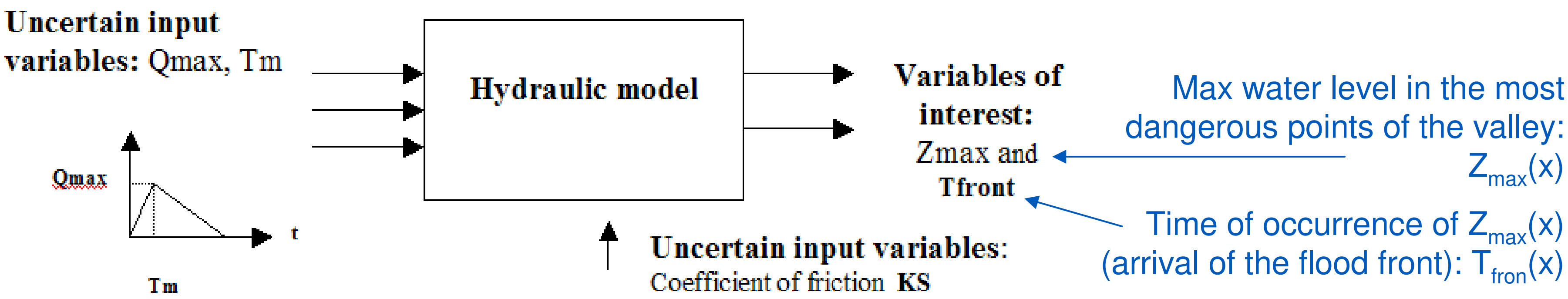
Embankment failure hydrograph effects on flooded areas assessment [Arnaud et al., 2010] (2/5)

- The complex physics at play during the progressive erosion is not well known
  - the emptying hydrograph H is not well known:
    - The maximum discharge Qmax
    - The time of occurrence of the maximum discharge Tmax
  - We assume that the reservoir volume V is known
  - We assume a triangular hydrograph

$$H : t \mapsto Q = H(t)$$
$$\int H(t)dt = V$$



- **Step A**



Embankment failure hydrograph effects on flooded areas assessment [Arnaud et al., 2010] (3/5)

- Known variables:
  - Features of the dam
    - Dam height 123 m, Reservoir volume: V=1200 Mm3
  - Valley features
    - Length : 200 km, no tributaries, No dams downstream
    - Very irregular geometry with huge width variation → Hydraulic jumps

Step B → Uncertainty assessment

- Q_{max} and T_{max} (Hydrograph form)
  - too small amount and imprecise data: the pdf could not be assessed by a statistical procedure
  - According to the experts' advice the following pdf's for Q_{max} and T_m have been proposed:

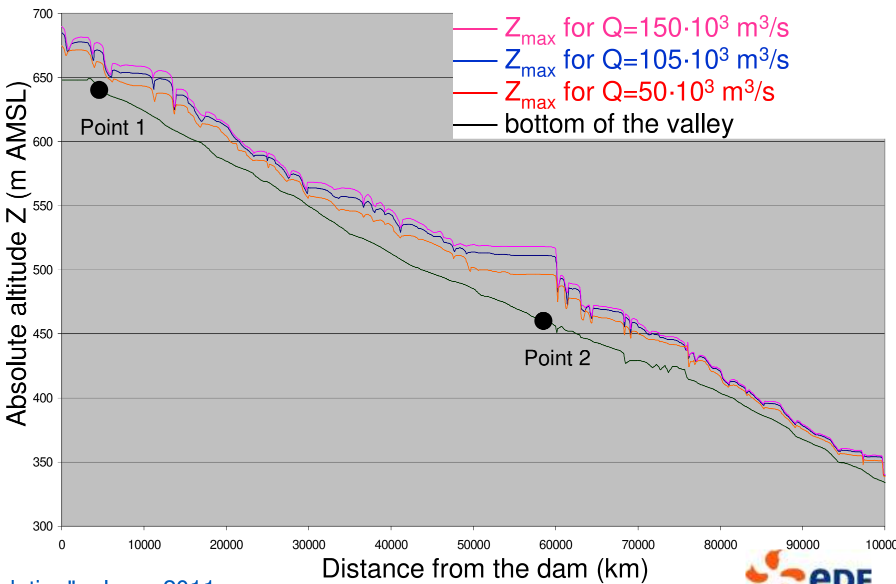
Prob. distr. funct	Q_{max} (m3/s)	T_m (s)
1) Normal :		
Mean	100 000	5 000
Standard dev.	25 000	2 000
2) Uniform :		
Lower bound	50 000	1 000
Upper bound	150 000	7 200

- Friction coefficient Ks
  - Not "measurable" variable
  - Experts' advice, based un valley morphology knowledge

Prob. distr. funct	Ks
1) Truncated Normal:	
Mean	30
Standard dev.	5
Bounds	[17.5, 47.5]
2) Uniform:	
Lower bound	25
Upper bound	35

Embankment failure hydrograph effects on flooded areas assessment [Arnaud et al., 2010] (4/5)

- **Step C → Uncertainty propagation**
- Hydraulics software: "Mascaret" Code (EDF R&D-CETMEF)
  - 1D shallow water modeling based on the De St Venant equations
  - Finite volume scheme with CFL limitation on the time step
- Hydraulic modeling
  - Un-stationary flow conditions, Space discretization: 100 m
  - The time step (1-2 s) is controlled by the CFL condition. Duration of the simulation : 13 000 time steps
- First set of 3 model's runs to look for the more dangerous points
  - 3 values of Q_{max} : 50 000 m³/s, 105 000 m³/s and 150 000 m³/s
  - Mean value of Ks
  - Two points (Point 1 and Point 2) are particularly dangerous with respect to flooding risk. They are both located downstream from a section narrowing → hydraulic jumps
  - We will mainly focus on these two points



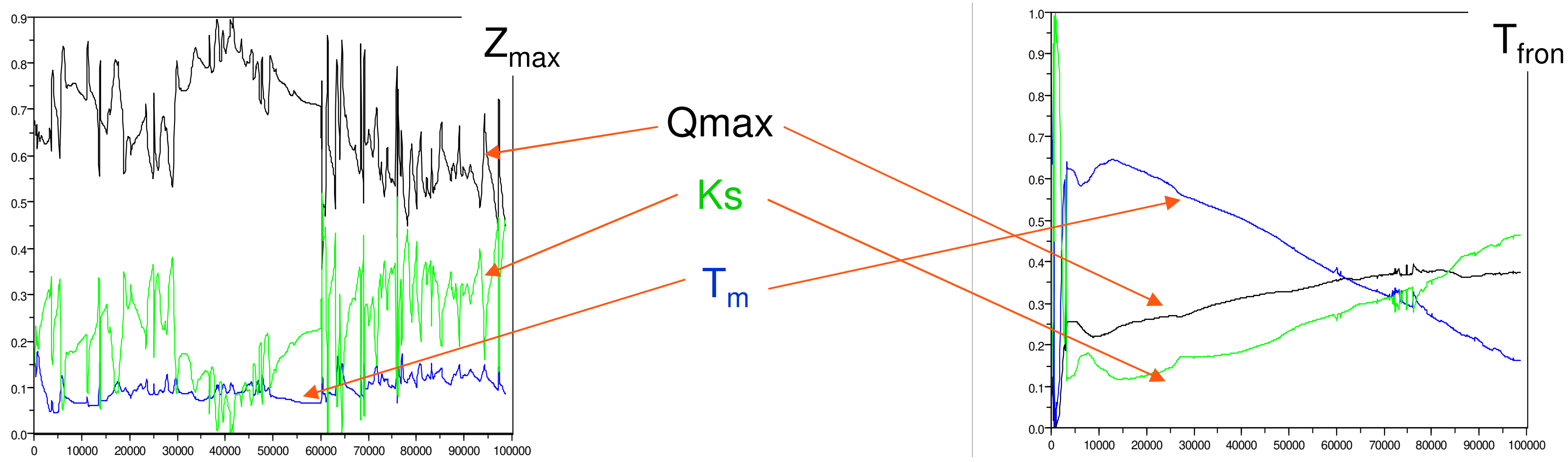
Embankment failure hydrograph effects on flooded areas assessment [Arnaud et al., 2010] (5/5)

- Propagation method: Surface response + Monte Carlo
- Some results
  - Extreme Quantiles of Z_{max} in points 1 and 2 (flood risk assessment)

Z_{max} (m ASML)	Point 1		Point 2	
	Pdf 1	Pdf 2	Pdf 1	Pdf 2
Quantile 99.9%	676.66	676.64	517.14	515.04
Quantile 99%	675.57	675.52	516.49	515.57
Quantile 95%	673.67	674.25	513.71	514.14

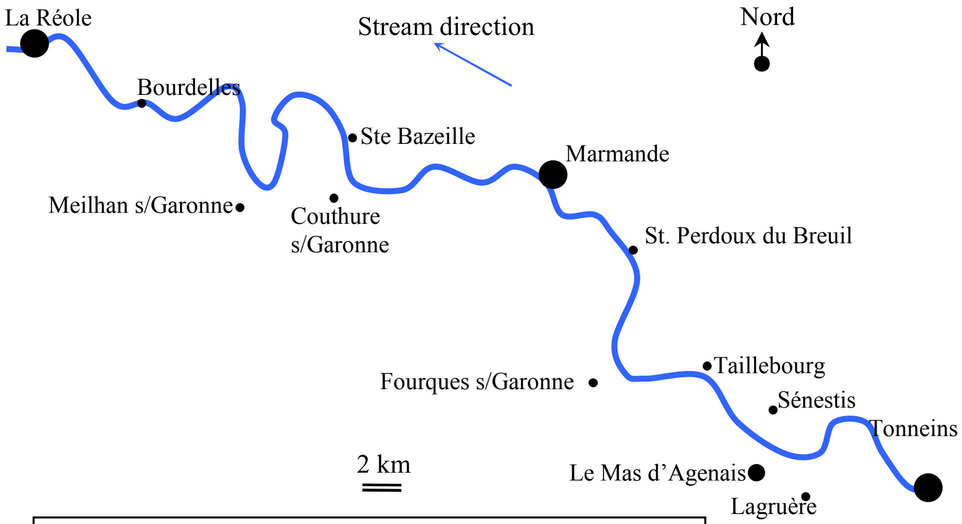
  - Sensitivity analysis → evaluation of the Spearman ranks' correlation coefficients for all values of the abscissa x

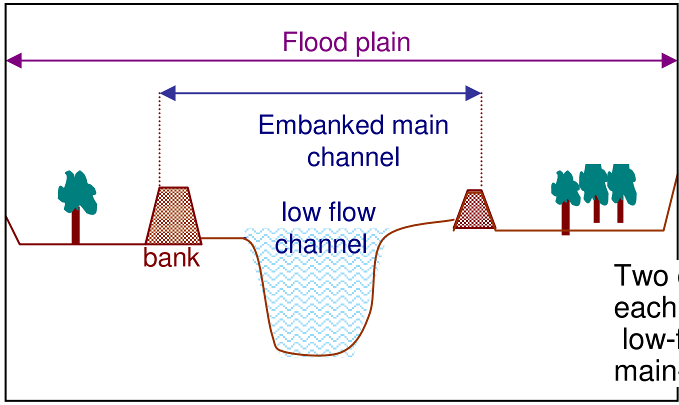
$\rho^S(Q_{max}, Z_{max}), \rho^S(T_m, Z_{max}), \rho^S(Ks, Z_{max})$ $\rho^S(Q_{max}, T_{fron}), \rho^S(T_m, T_{fron}), \rho^S(Ks, T_{fron})$



An hydraulic benchmark: the Garonne case-study

- Hydraulic modeling of a 50 km long part of the Garonne river (SW France) → "Mascaret" Code
- Case study shared between the partners of the OPUS project
- Two examples:
  - Inverse modeling to assess the pdf of the roughness Strickler's coefficient Ks
    - K_s is never directly observed
    - One should estimate the pdf of K_s, given a set of observed coupled data (discharge,water level)
  - Evaluating an extreme quantile of the flood water level at a given abscissa
  - Or evaluating the probability for the flood water level in a given abscissa to be greater than a threshold value

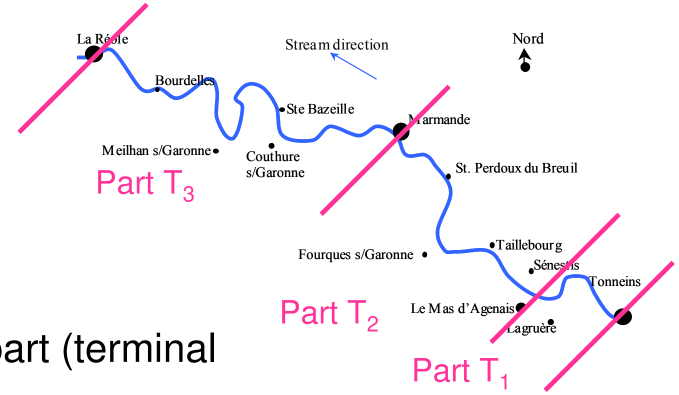




Two different Ks for each section: low-flow Ks and main-channel Ks.

"OPen source platform for Uncertainty treatment in Simulation"
10 partners, Tot. budget: 2.2 M€, Leader: EDF

The Garonne case-study: Inverse modeling of Ks [Couplet, Le Brusquet et al., 2010] (1/2)



- Physical hypothesis
  - 3 parts each one with given values of the 2 Ks

- Statistical problem: assessing the pdf of K_s
  - In this example, we will assess the pdf of the Ks of the T_3 part (terminal part between Marmande and La Réole)
  - Data: couples (discharges Q_i, water levels Z_i) at Mas d'Agenais and Marmande

$$\mathbf{Z}_i = G(Q_i, \mathbf{Ks}) + \mathbf{U}_i, \quad i = 1..n, \quad \mathbf{Z}_i, \mathbf{U}_i \in \mathbb{R}^2$$

  - Hypotheses:

$$\begin{pmatrix} Ks_1 \\ Ks_2 \end{pmatrix} \sim \mathcal{N}(\mu, \Sigma) \qquad U_i \sim \mathcal{N}(0, R), R = \sigma_\epsilon \cdot I_2$$

→ The vector Ks and observation errors are normal

The standard measurement error is σ_ε

$$\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$$

→ Mean values of Ks

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$

→ Covariance matrix of Ks

$$\beta = (\mu_1, \mu_2, \sigma_1, \sigma_2, \rho)$$

Tricky likelihood expression

$$\mathcal{L}(\mathbf{z}; \beta) = \prod_{i=1}^{n} L_i(z_i; \beta)$$
$$L_i(z_i; \beta) = \iint_{\mathbf{Ks}} f_{Ks,\beta}(\mathbf{Ks}) \cdot f_\epsilon\left(z_i - G(Q_i, \mathbf{Ks})\right) d\mathbf{Ks}$$

Density of **Ks** — Density of $\mathbf{z}_i$, given Q_i and **Ks**

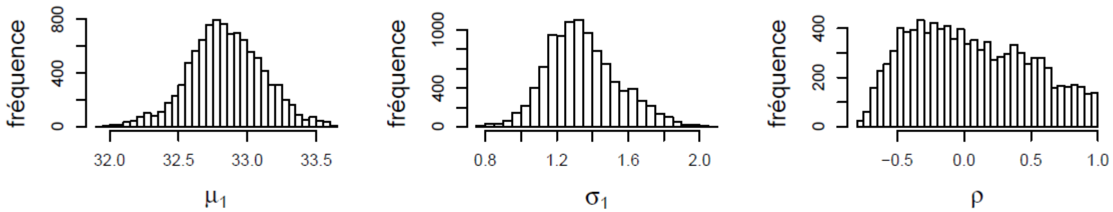
The Garonne case-study: Inverse modeling of Ks [Couplet, Le Brusquet et al., 2010] (2/2)

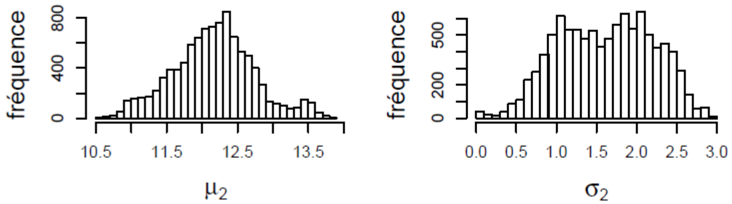
- Some results
- Two solutions
  - Likelihood maximization (variants of the EM algorithm: ECME, SAEM)

$$\hat{\beta}_{\text{ECME}} = (32.84, 12.03, 0.84, 1.33, 0.35)$$

$$\hat{\beta}_{\text{SAEM}} = (32.78, 12.12, 0.85, 1.48, 0.18)$$

  - Bayesian solution: MCMC sampling from the posterior pdf of β: $\pi(\beta|\mathbf{z}) \propto \pi_0(\beta) \cdot \mathcal{L}(\mathbf{z};\beta)$



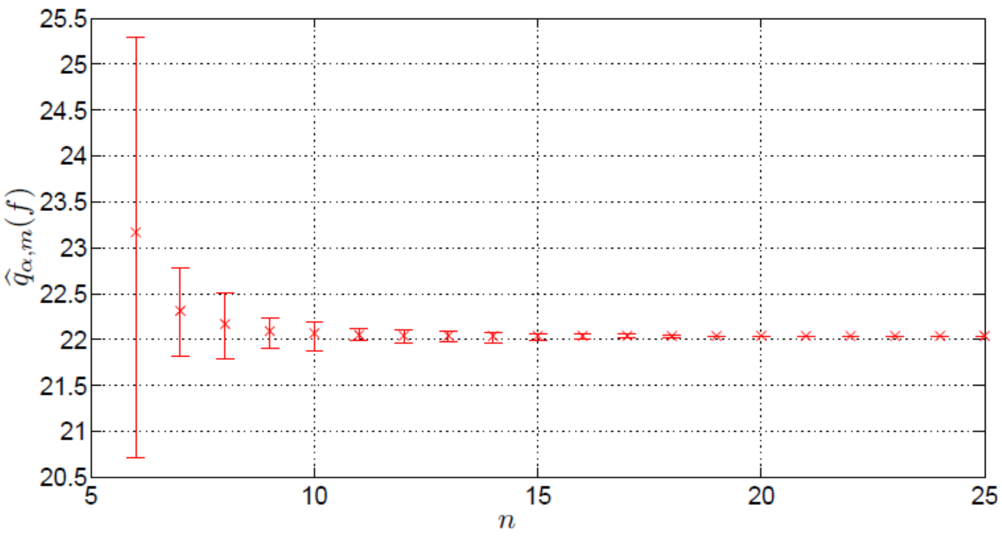


NB: Uniform prior used

The Garonne case-study: Flood risk assessment [Arnaud, Vazquez, Bect et al., 2010]

- Goal: Evaluating the quantile of probability α=0.99 of the water level in a given section
  - Original meta-modeling technique developed within the OPUS project [Vazquez et al, 2010]
  - Empirical estimation of the quantile: $\hat{q}_{\alpha,m} = z^{\lceil m\alpha \rceil}$
  - Building an approximation $\tilde{G}_n(\cdot)$ of $G(\cdot)$ based on the n<<m evaluations: $\{G(x_1), G(x_2), ..., G(x_n)\}$
  - The n points $\{x_1, x_2, ..., x_n\}$ are chosen sequentially in order to minimize a statistical "cost" (e.g. a quadratic loss) between $\hat{q}_{\alpha,m}$ and the empirical estimator built according to the surrogate model

A dozen of model's runs allow to build a "specialized" kriging meta-model for the quantile estimation (here m=2000)



EDF

Examples
Mechanics

A longstanding experience at EDF R&D

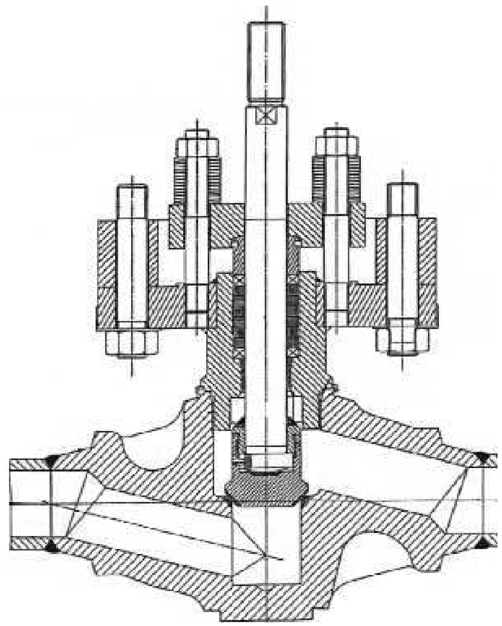
- Several studies in the field of probabilistic mechanics:
  - Reliability analysis
  - Sensitivity analysis
  - Inverse problems → Bayesian updating of the behavior law of the material (e.g. concrete in civil works studies)

- Several research works on polynomial chaos expansion
  - A useful tool to perform high CPU time-consuming calculations above

- Numerous applications
  - Cooling towers, containment structures, thermal fatigue problems, lift-off assessment of fuel rod ...
  - We will focus about an application concerning globe valves reliability and sensitivity analysis

Globe valve reliability and sensitivity analysis [Berveiller et al., 2010] (1/5)

- Industrial globe valves are used for isolating a piping part inside a circuitry
- Harsh operating conditions: water temperature, pressure, corrosion problems ...
- Reliability assessment: the tightness of the valve has to be assured even with a maximum pressure of the water

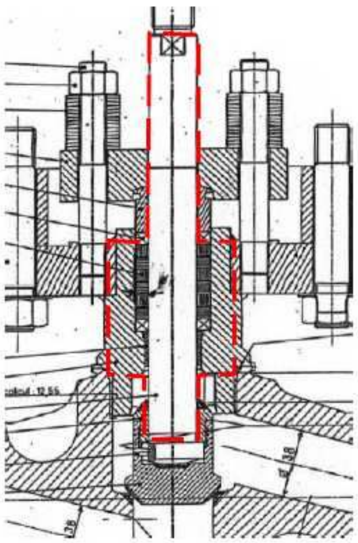


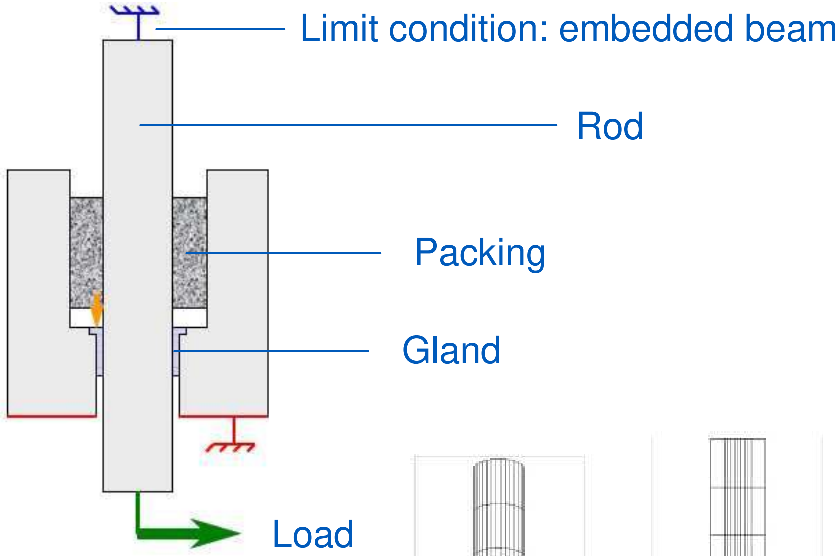
- Several uncertain variables
  - Material properties
  - Functional clearances
  - Load

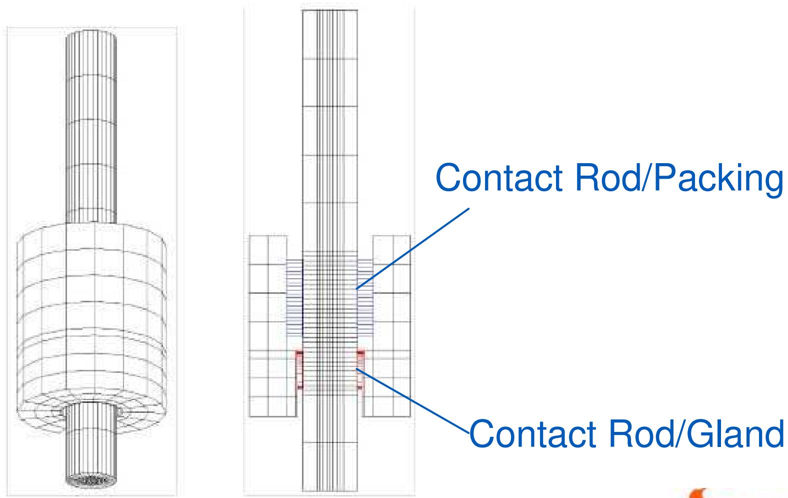
To insure the reliability of the mechanism the contact pressures and de max displacement of the rod must be lower than given values

Globe valve reliability and sensitivity analysis [Berveiller et al., 2010] (2/5)

- The modeling problem is very complex. We will work here on a simplified mechanical modeling







Case-study of the OPUS project

Globe valve reliability and sensitivity analysis [Berveiller et al., 2010] (3/5)

Step A

- Variables of interest:
  - Contact pressures
  - Max displacement of the rod

- 6 Uncertain input variables:
  - Packing Young's modulus
  - Gland Young's modulus
  - Beam Young's modulus
  - Steel (Rod) Young's modulus
  - Load
  - Clearance

- Deterministic model G(·):
  - FEM Numerical model of the simplified scheme using *Code_Aster* software (www.code-aster.org)

- Goal of the study:
  - assessing the sensitivity of the variable of interest with respect to the uncertain inputs

- Quantities of interest: Sensitivity indices
  - Reminder: Sobol' variance decomposition*

$$\mathbb{V}[Z] = \sum_i V_i[Z] + \sum_{i<j} \mathbb{V}_{ij}[Z] + \ldots$$

$$V_i[Z] = \mathbb{V}\left[\mathbb{E}[Z|X_i]\right]$$
$$V_{ij}[Z] = \mathbb{V}\left[\mathbb{E}[Z|X_i, X_j]\right] - V_i[Z] - V_j[Z]$$

*X_i's independent

  - Sobol' indices:

$$S_i = \frac{V_i}{\mathbb{V}[Z]} \qquad S_{ij} = \frac{V_{ij}}{\mathbb{V}[Z]} \qquad S_i^T = S_i + \sum_{i \neq j} S_{ij} + \ldots$$

First order — Second order — "Total" index

  - They measure the "part" of the global variance explained by a single input (or a set of inputs)
  - Monte Carlo calculation is CPU expensive, as many model runs are needed → Meta-modeling approach

Globe valve reliability and sensitivity analysis [Berveiller et al., 2010] (4/5)

Step B

- Uncertainty modeling of input variables:

Variable	Prob. density	Mean	Coefficient of Variation
Packing Young's modulus (MPa)	LogNormal	100 000	20%
Gland Young's modulus (MPa)	LogNormal	207 000	10%
Beam Young's modulus (MPa)	LogNormal	6 000	10%
Steel (Rod) Young's modulus (MPa)	LogNormal	200 000	10%
Load (N)	Normal	10 000	10%
Clearance (mm)	$\text{Beta}_{[0,0.1]}$	0.05	50%

Steps C,C'

- Non intrusive polynomial chaos approximation
  - Isoprobabilistic transformation of the input vector: $\xi_i = T(X_i) \sim \mathcal{N}(0,1) \quad i = 1, \ldots, m$
  - Polynomial chaos (PC) approximation: $Z \approx \widetilde{Z} = \sum_{i=0}^{Q-1} \alpha_j \cdot \psi_j(\boldsymbol{\xi})$ ← PC approx. of order m and degree q

Number if terms of the sum: $Q = \frac{(m+q)!}{m!\, q!}$

coefficients

$\{\psi_j(\boldsymbol{\xi}), j = 1, \ldots, Q\}$ Set of the m-dimensional Hermite polynomials of degree ≤ q

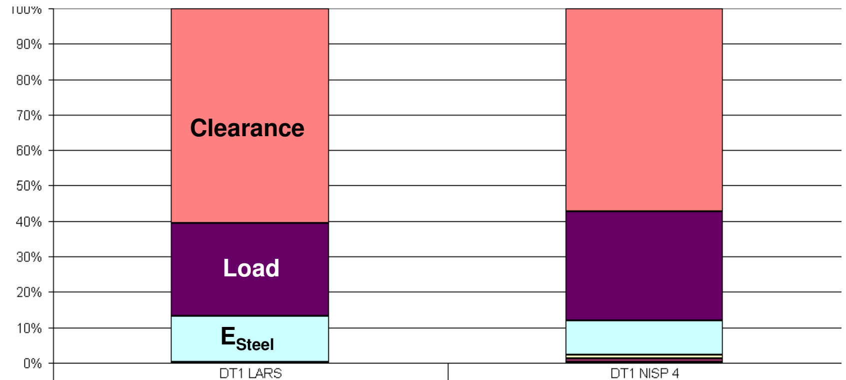
Globe valve reliability and sensitivity analysis [Berveiller et al., 2010] (5/5)

- Benefits of PC approximation
  - Once coefficients are evaluated, PC expansion allows performing quick Monte Carlo simulations, by running the meta-model instead of the expensive numerical code G(·)
  - Moreover, due to the orthogonally of the polynomials, the evaluation of the Sobol' indices is straightforward
  - [Sudret, 2008]: $\mathbb{V}[Z] = \sum_{j=1}^{Q-1} \alpha_j^2 \cdot \mathbb{E}\left[\psi_j^2(\boldsymbol{\xi})\right]$ $\quad V_i[Z] = \mathbb{V}\left[\mathbb{E}[Z|X_i]\right] = \sum_{k \in \mathcal{A}_i} \alpha_k^2 \cdot \mathbb{E}\left[\psi_k^2(\boldsymbol{\xi})\right]$

    Set of polynomials containing only ξ_i
  - The calculation burden (i.e. running several times the code G) is focused on the estimation of the coefficients
    - Several techniques: projection, regression, simulation, sparse PC expansion (LARS) [Blatman & Sudret, 2010]

Example of results

- Sobol' indices for rod displacement
  - PC approximation built by two different methods & tools: LARS, NISP (CEA)
  - Most influent variables : clearance, load, Steel Young's modulus



	DT1 LARS	DT1 NISP 4
JEU	58,92%	54,05%
F_1	25,52%	29,01%
E_acier	12,81%	9,11%
E_poutre	0,11%	0,97%
E_bague	0,04%	0,98%
E_gar	0,02%	0,24%

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Thank you for your attention