GdR MASCOT-NUM - Atelier Validation
ONERA's recent activities in V&V and UQ for aerodynamics

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THE FRENCH AEROSPACE LAB

Overview

Validation and Verification (V&V) in the CFD community

- V & V involves:
  - Physical modelling
  - Numerical discretization ⇒ Mesh adaptation
  - Uncertainty Quantification ⇒ Impact of input data lack of knowledge on simulations
- Gathering activities besides considered individually
- Connex topics : Robust design, Metamodelling

A very broad framework

- Overview of V&V for external aerodynamics
- Two specific research topics
  - Goal oriented mesh adaptation
  - Uncertainty propagation based on sparse collocation methods

Verification and validation for external aerodynamics

Specific flow
Exact geometry
M, Re, α
Navier Stokes equations
-- **Ideal aerodynamic Coefficient C_{ideal}**

Wind tunnel – Coefficient C_{expe} not equal to C_{ideal}
Error term:
δ_{expe} Accuracy of measurement devices, aero elasticity effects, inability to produce exact M…

(Remark: wall and support effects supposed to be corrected for global forces wind-tunnel data)

Numerical simulation(s) – Coef. C_{sim} not equal to C_{ideal}
Error terms:
δ_{model} RANS, LES,…
δ_{num} Approximation error
δ_{input} Uncertainty on wall-roughness, physical constants…

Verification and validation for external aerodynamics

- Validation discussion based on ASME V&V 20 (PTC 61)

- Reasons of the discrepancy

  $E = C_{sim} - C_{expe} = (C_{sim} - C_{ideal}) - (C_{expe} - C_{ideal})$

- Terminology *(not shared by all the community)*
  - *Reference ASME V&V20-2008*
  - *An error δ_i is a quantity with a sign and a magnitude (caused by error source i) between a quantity (measured or simulated) and its true value*
  - *An uncertainty u_i is an estimate of an interval +/- u_i that should contain δ_i*

  - *Correspondence with French words* erreur *and* incertitude *not very natural*
  - *Unfortunately other definitions by* AIAA (AIAA-G-077-1998 ASME V&V 10)
    - *Error* a recognizable deficiency in any phase or activity of the modeling process that is not due to the lack of knowledge
    - Uncertainty potential deficiency in any phase or activity of the modeling process that is due to the lack of knowledge

Verification and validation for external aerodynamics

- Discussion from simple linear assumption for errors

  $E = C_{sim} - C_{expe} = (C_{sim} - C_{ideal}) - (C_{expe} - C_{ideal}) = \delta_{sim} - \delta_{expe}$

  $\delta_{expe} = C_{expe} - C_{ideal}$ $|C_{expe} - C_{ideal}| < u_{mes} + u_{oper} = u_{expe}$

  $\delta_{sim} = d_{disc} + \delta_{param} + \delta_{model} = C_{sim} - C_{ideal}$
  $|C_{sim} - C_{ideal}| < u_{disc} + u_{param} + u_{model} = u_{sim}$

  In general assessment of u_{model}

- Identification of experimental terms
  - δ_{mes} accuracy of measurement instruments
  - δ_{oper} Operational error. Inability to produce the desired flow conditions (*e.g.* 0.001 accuracy for upwind Mach number)

- Wind tunnel
  - Walls and stick effects (corrected for global forces not for local measurements)
  - U_{expe} estimated by short and middle term repetition (hopping no systematic bias…)

Verification and validation for external aerodynamics

- $E = C_{sim} - C_{expe} = (C_{num} - C_{ideal}) - (C_{expe} - C_{ideal}) = \delta_{sim} - \delta_{expe}$

  $\delta_{sim} = \delta_{disc} + \delta_{param} + \delta_{model} = C_{num} - C_{ideal}$

  $|C_{expe} - C_{ideal}| < u_{disc} + u_{param} + u_{model} = u_{sim}$

- Aerodynamic CFD
  - u_{disc} discretisation error
    - Vanishes at the limit of small step sizes
    - Decreasing according to the order of the scheme
    - Does not include error due to too close boundary (part of u_{model})
    - u_{disc} deeply linked with u_{model} for certain models
  - u_{model} according to the model
    - (RANS) inaccurate for transition and massively detached flows
    - (DES) accurate except for phenomena at the scale of the boundary layer width
    - (LES) accurate if the mesh is fine enough
    - (DNS) no modeling error
  - u_{param}
    - unknown wall rugosity for example (satisfactory model, unknown parameter) for example

Verification and validation for external aerodynamics

- $E = C_{sim} - C_{expe} = (C_{sim} - C_{ideal}) - (C_{expe} - C_{ideal}) = \delta_{sim} - \delta_{expe}$

$$\delta_{expe} = C_{exp} - C_{ideal}$$

$$\delta_{sim} = \delta_{disc} + \delta_{param} + \delta_{model} = C_{sim} - C_{ideal}$$

$$\delta_{model} - E = \delta_{expe} - \delta_{disc} - \delta_{param}$$

- Validation discussion for δ_{model} unknown (general case)
  $E - u_{expe} - u_{disc} - u_{param} < \delta_{model} < E + u_{expe} + u_{disc} + u_{param}$
  - Estimation of u_{expe} provided by experimentalists
  - Estimation of u_{param} series of computations, uncertainty quantification
  - Estimation of u_{disc} mesh convergence, theoretical estimations…
- Specific cases. No model error, no parameter error…

Verification and validation for external aerodynamics

Specific flow
Exact geometry
M, Re, α
Navier Stokes equations
-- **Ideal aerodynamic Coefficient C_{ideal}**

Approximation error
Uncertainty propagation

Wind tunnel – Coefficient C_{expe} not equal to C_{ideal}
Error term:
δ_{expe} Accuracy of measurement devices, aero elasticity effects, inability to produce exact M..

δ_{oper} part of δ_{expe} due to inaccurate M (+/-0.001)...

Numerical simulation(s) – Coef. C_{sim} not equal to C_{ideal}
Error terms:
δ_{model} RANS, LES,...
δ_{num} Approximation error
δ_{input} Uncertainty on wall-roughness, physical constants...

Verification and validation for external aerodynamics

- Stronger discrepancy between experiment and abstract mechanical problem
  - Half model in wind-tunnel…peniche effect
  - Masking effect of stick (carrying the weighting device) at model jonction
  - Too low Reynolds number in the wind tunnel for large aeronautical objects

$$\mathrm{Re}_{WT} = \frac{\rho_\infty^{WT} V_\infty^{WT} L^{MODEL}}{\mu(T_\infty^{WT})} \le \frac{\rho_\infty^{FL} V_\infty^{FL} L^{PLANE}}{\mu(T_\infty^{FL})} = \mathrm{Re}_{FL}$$

- CFD calculations for the wind tunnel experiment ?
  - No more wall and stick discrepancy
  - Porous/slotted walls : good for quality of flow (avoids throat effect) not easy for CFD

- In practice. Industrial know-how to associate calculations, too-low Reynolds number wind tunnel tests, exact Reynolds number tests (cryogenic wind tunnel tests ETW) and flight tests

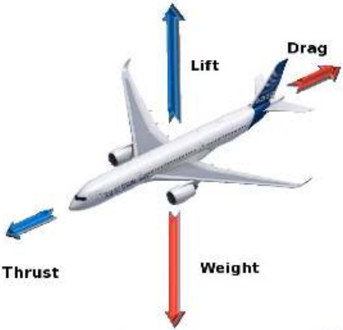
Verification and validation for external aerodynamics

CFD vs wind-tunnel validation
some issues:

-- Reynolds number
-- wall influence
-- sting influence
-- peniche influence

Aerodynamic functions

- "Data acquisition" = Forces and moments calculation for all AoA, M, Re, flap position, rudder position... (without checking flow details) inputs of flight mecanics codes
- Final shape optimization = Mininize drag with constraint on drag and pitching moment



Sensitivity analysis

- **Framework: compressible flow simulation using finite volume method. Discrete approach for sensitivity analysis**

- **Notations**
  - ☐ Volume mesh X, flowfield W (size n_W)
  - ☐ Wall surface mesh S
  - ☐ Residual R, C^1 regular w.r.t. X and W – steady state: $R(W,X) = 0$
  - ☐ Vector of design parameters α (size n_d), $X(\alpha)$ $S(\alpha)$ C^1 regular

- **Asumption of implicit function theorem**
  - ☐ $\forall\ (W_i, X_i)\ /\ R(W_i, X_i) = 0 \quad (\partial R/\partial W)(W_i, X_i) \neq 0$
  - ☐ Unique steady flow corresponding to a mesh

Sensitivity analysis

- **Functions of interest**
  - $\mathcal{J}_k(\alpha) = J_k(W(\alpha), X(\alpha))\ k \in [1, n_f]$
  - Flowfield and volume mesh linked by flow eq. $R(W(\alpha), X(\alpha)) = 0$
- **Sensitivities $d\mathcal{J}_k/d\alpha_i\ \ k \in [1, n_f]\ \ i \in [1, n_d]$ to be computed**
- **Discrete gradient computation methods**
  - Finite differences – $2n_d$ flow computations (non linear, size n_W)
  - Direct differentiation method – n_d linear systems (size n_W)
  - Adjoint vector method – n_f linear systems (size n_W)

Sensitivity analysis

- **Standart design in aeronautics**
  - One objective, few constraints (n_f) *versus* several dozens or hundreds of design parameters (n_d)
  - Adjoint vector method more interesting

- **Equations**

$$(\frac{\partial R}{\partial W})^T \Lambda_k = -(\frac{\partial J_k}{\partial W})^T \qquad \frac{d\mathcal{J}_k}{d\alpha_i} = \frac{\partial J_k}{\partial X}\frac{dX}{d\alpha_i} + \Lambda_k^T(\frac{\partial R}{\partial X}\frac{dX}{d\alpha_i})$$

- **Memory burden of classical discrete adjoint method = storage of $dX/d\alpha_i \;\; i \in [1, n_d]$.**
- **Compute $(\partial R/\partial X)(dX/d\alpha_i)$ as product of two differential (no finite difference for the product) Use the link between wall surface mesh S and volume mesh X (Nielsen E., Park M. $AIAA\ Journal$ 2005)**

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dJ/dX vector field

- Use the link between wall surface mesh S and volume mesh X (Nielsen E., Park M. *AIAA Journal* 2005) to avoid the storage of $dX/d\alpha_i \;\; i \in [1, n_d]$
- Use :

$$\frac{d\mathcal{J}_k}{d\alpha_i} = \frac{\partial J_k}{\partial X}\frac{dX}{d\alpha_i} + \Lambda_k^T\left(\frac{\partial R}{\partial X}\frac{dX}{d\alpha_i}\right) = \left(\frac{\partial J_k}{\partial X} + \Lambda_k^T\frac{\partial R}{\partial X}\right)\frac{dX}{d\alpha_i}$$

- Define $\mathbf{J}_k(X) = J_k(W, X)$ where $R(W, X) = 0$ (from implicit function theorem)
- First compute the term in bracket

$$\frac{d\mathbf{J}_k}{dX} = \left(\frac{\partial J_k}{\partial X} + \Lambda_k^T\frac{\partial R}{\partial X}\right)$$

- Compute the sensitivities from adjoint mesh deformation equation (X and S implicitely linked), or following equation (X function of S) and $\frac{dS}{d\alpha_i} \;\; i \in [1, n_d]$

$$\frac{d\mathcal{J}_k}{d\alpha_i} = \left(\frac{d\mathbf{J}_k}{\partial X}\frac{dX}{dS}\right)\frac{dS}{d\alpha_i}$$

dJ/dX vector field

- **Functional outputs J_k**
  - External flows. Far-field/Near-field drag analysis
  - See D.Destarac. VKI Lecture Series 2003
  - Line integrals in 2D. Surface integrals in 3D

- **Analysis of**

$$\frac{d\mathbf{J}}{dX} = \underbrace{\frac{\partial J}{\partial X}}_{geometric\ gradient} + \underbrace{\Lambda_J^T \frac{\partial R}{\partial X}}_{aerodynamic\ gradient}$$

- **Analysis of total derivative formula**
  - $(\partial J/\partial X_l)$ direct dependency of J on location of node l
  - $\Lambda(\partial R/\partial X_l)$ changes of the flow field on the support of J due to change of node l location, to satisfy $R(W, X) = 0$

dJ/dX vector field

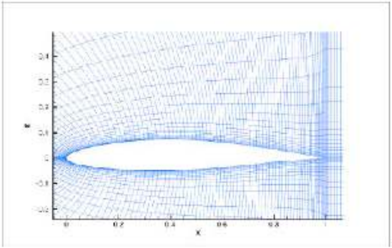
- **ONERA $elsA$ code (see AIAA Paper 2008-664)**

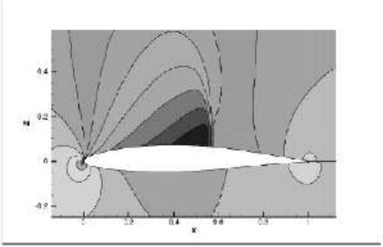
Discrete adjoint vector and discrete direct differentiation method

- **NACA64A212.** $M_\infty = 0.75\ AoA = 2,5^o$

257×33 structured mesh. Roe's flux MUSCL approach. van Albda's limiting function

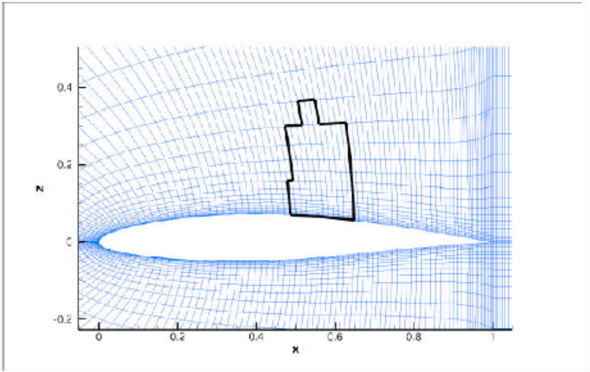
mesh **iso-ρ lines**





dJ/dX vector field

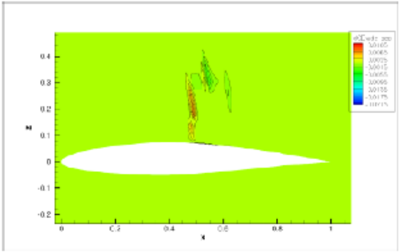
contour for CDw (see Destarac, VKI Lecture Series, 2003)



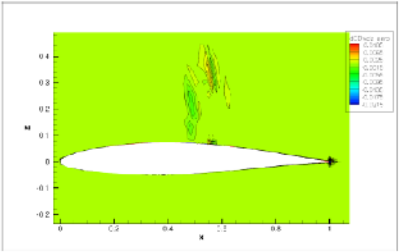
dJ/dX vector field

$$\frac{d\textbf{CDw}}{dX} = \frac{\partial CD_w}{\partial X} + \Lambda^T_{CD_w}\frac{\partial R}{\partial X}$$

iso-$(\partial CD_w/\partial z)$ lines

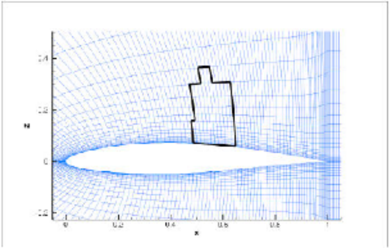


iso-$\Lambda(\partial R/\partial z)$ lines

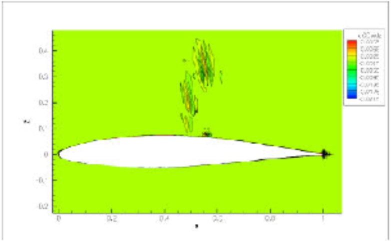


dJ/dX vector field

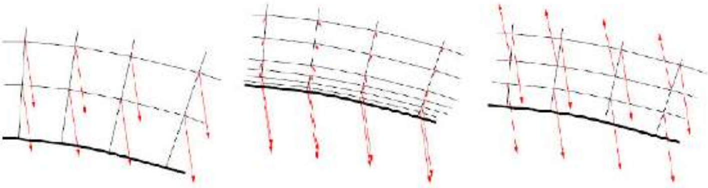
contour for CDw estimation



iso-dCDw/dz lines



dJ/dX vector field



- Visualization of $d\mathbf{J}/dX$ (vector field) or $||d\mathbf{J}/dX||$ (scalar field)
- analysis based on $\mathbf{J}(X + dX) - \mathbf{J}(X) \simeq (d\mathbf{J}/dX).dX$
  - ☐ Mesh (a) not well-suited for J calculation
  - ☐ Mesh (b) possibly well-suited for J calculation
  - ☐ Mesh (c) for J calculation. Questionable

$\mathcal{P}(dJ/dX)$ vector field

- **Some components of dJ$/dX$ not usable for mesh adaptation**

  Components orthogonal to wall

  Components orthogonal to function support

- **Definition of a projected gradient $\mathcal{P}_{(dJ/dX)}$ for mesh adaption**

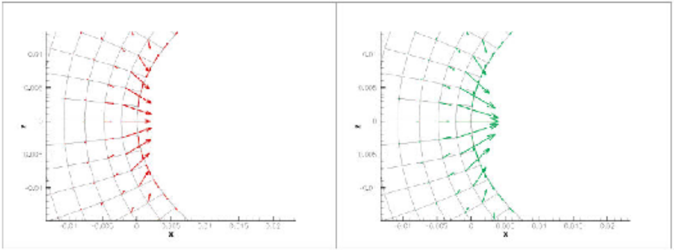
  $\mathcal{P}_{(dJ/dX)} = dJ/dX$ outside walls and function support

  $\mathcal{P}_{(dJ/dX)} = dJ/dX - (dJ/dX \cdot n)n$ along walls and function support

  $\mathcal{P}_{(dJ/dX)} = 0$ at a corner of the function support

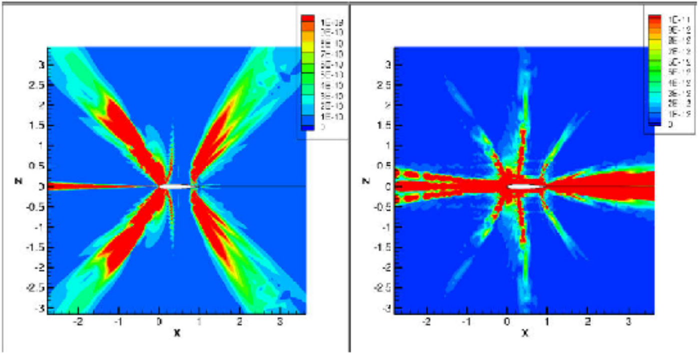
$\mathcal{P}(dJ/dX)$ vector field

- Example : NACA0012 AoA=0, M=0.5 Preliminary examination of $-\mathcal{P}(dCDp/dX)$ (left) and $\mathcal{P}(dPi/dX)$ (right)



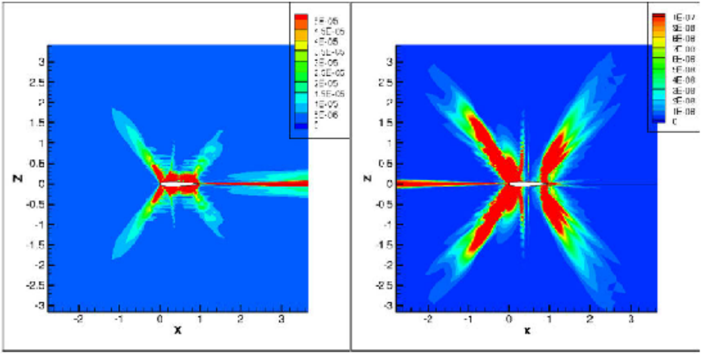
Comparison with Venditti and Darmofal's error estmator

- Left: $||\mathcal{P}(dCDp/dX)|| \times r$ (r local caracteristic cell size)
- Right: Venditti and Darmofal's error estimator



Comparison with a feature based indicator

- Left: feature-based indicator ($||grap(p)|| \times r$)
- Right: $||\mathcal{P}(dCDp/dX)||$



References

- J. Peter, M. Nguyen-Dinh, P. Trontin. Goal-oriented mesh adaptation using total derivative of aerodynamic functions w.r.t. mesh coordinates – With Application to Euler flows. *Computers & Fluids* 66 194–214. 2012.
- Mesh quality assessment based on aerodynamic functional output total derivatives. Maxime Nguyen-Dinh, Jacques Peter, Renaud Sauvage, Matthieu Meaux, Jean-Antoine Désideri. *European Journal of Mecanics B/Fluids. (acceptance submitted to minor changes)*
- AIAA paper 2011-30, AIAA paper 2012-158

NODESIM CFD (2006 - 2010)

- **NO**n-**DE**terministic **SIM**ulation for **CFD** - based design methodologies
- European Consortium with 19 partners
- References at ONERA: Jacques PETER and Marc LAZAREFF

ITN ANADE (2012 - 2015)

- **A**dvances in **N**umerical and **A**nalytical tools for **DE**tached flow prediction.
- European Consortium with 9 partners
- ANADE PhD fellow at ONERA on UQ: Andrea RESMINI
  - Main objective: Mesh adaptation and uncertainty quantification may bring a deeper understanding and an improved prediction of the detachment phenomenon.
- http://www.anade-itn.eu

UMRIDA (2013 - 2016)

- **U**ncertainty **M**anagement for **R**obust **I**ndustrial **D**esign in **A**eronautics
- Consortium of 21 EU and 1 US partners
- References at ONERA: Jacques PETER and Eric SAVIN
- Monte Carlo & surrogates or Polynomial choas for joint variying uncertain parameters

Uncertainty quantification in CFD

- Deterministic (*exact*) VS Stochastic (*most probable*) aerodynamics
- Most exploited uncertain inputs: AoA and Ma
- Low stochastic dimension due to the high computational costs of CFD simulations

Objectives

1. Increase the dimension of the stochastic problem (>2D).
2. Assess the efficiency of different methods.
3. Identify the effects on attached and detached flow on global aerodynamic function.
4. Do some steps towards robust design

Sources of uncertainties

- Input data:
  a Geometrical (surface imperfections, junctions, ice ...)
  b Operational (fluctuations of streamflow velocity, incidence, temperature ...)

Data

- Wind-tunnels (ONERA, ETW ...)
- Real flight conditions (*e.g.* for helicopters: hoovering, forward flight, wind gust ...)

NB

The order of magnitude differs in the two cases. It is important to know **what** one is looking for...

Stochastic approximation

Methods

- Intrusive: the *deterministic* code has to be modified
- Non-intrusive: the *deterministic* code is seen as a black-box by the stochastic approximation
  1. Monte Carlo (MC)
  2. (generalized) Polynomial Chaos (gPC/PC) [Wiener 1938], [Ghanem 1991]
  3. Stochastic Collocation (SC) [Tatang 1995]

Stochastic Collocation

Interpolation method in multi-D: parallelization of decoupled computations.

Collocate the equation $\mathcal{R}(\mathbf{x}, \xi) = 0$ in a nodal set $\Xi_N = \{\xi_k\}_{k=1}^N$.

Stochastic approximation

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  3. Stochastic Collocation (SC) [Tatang 1995]

  $\Rightarrow$ flexibility of sampling method (MC) + regularity of the solution.

Stochastic Collocation

Interpolation method in multi-D: parallelization of decoupled computations.
Collocate the equation $\mathcal{R}(\mathbf{x}, \xi) = 0$ in a nodal set $\Xi_N = \{\xi_k\}_{k=1}^{N}$.

Adaptive quadrature sparse grid

Key point of SC

The complexity of SC is the choice of quadrature points!

- 1D: Clenshaw-Curtis (CC) & Fejér nested formulae

$$\Lambda_l \subset \Lambda_{l+1}, \quad \deg(\mathcal{Q}_l[f]) = n(l) - 1 \quad \forall f \in \mathbb{P}^1_{n(l)-1}$$

  But for $f \notin \mathbb{P}$ ([Trefethen 2008]), $\deg(\mathcal{Q}_l^G[f]) \approx \deg(\mathcal{Q}_l^{CC}[f])$
  Explicit formulae for nodes x and weight w.
- Multi-D: Smolyak algorithm & HPC [Smolyak 1963]. But the grid is **isotropic**, it assumes that the *influence* of each parameter is **equivalent**.
- **Anisotropy**: refine in the dimension where the Sobol' indices [Sobol' 2001] are high.

Test function

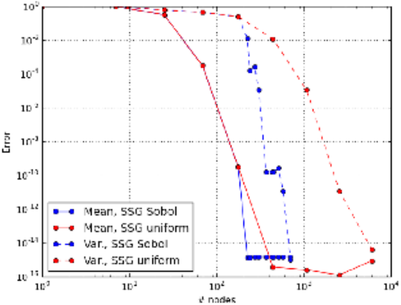
Ishigami [1990] function - smooth in C^∞, x, y, z : *iid* U[0,1]

$$f(\mathbf{x}) = \sin(2\pi x - \pi) + 7\sin^2(2\pi y - \pi) + 0.1(2\pi z - \pi)^4 \sin(2\pi x - \pi)$$

- Benchmark specifically designed to be challenging for global sensitivity analysis.
- Stepwise convergence pattern ⇔ *Physiological* with Sobol'

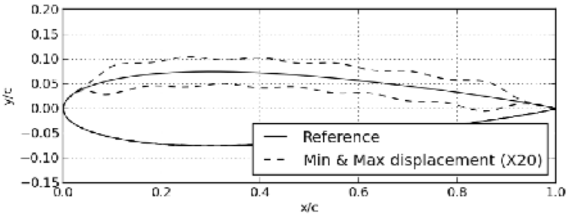
Asset SSG Sobol' adaptivity

10 times less evaluations of f w.r.t. SSG Uniform



NACA0015 case study - RANS+SA, C-mesh, $Re = 1.95 \cdot 10^6$

Uncertain param. *iid* U[-1,1]	Attached (WT)	Detached (RF)
Ma	$0.291 \pm 0.3\%$	$0.291 \pm 6.25\%$
AoA	$5^\circ \pm 0.4\%$	$16^\circ \pm 6.25\%$
Hicks-Henne bumps		
$h(x) = A\left[\sin\left(\pi x^{\frac{\log 0.5}{\log t_1}}\right)\right]^{t_2}$	± 0.15 mm (x 12)	± 1.5 mm (x 6)
Stochastic dimension	14	8
Objective	elsA sensitivity	SA separation



Some results...

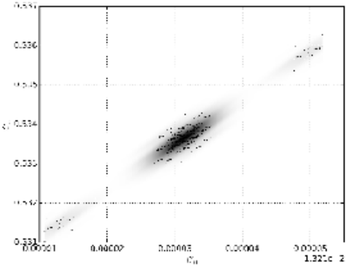
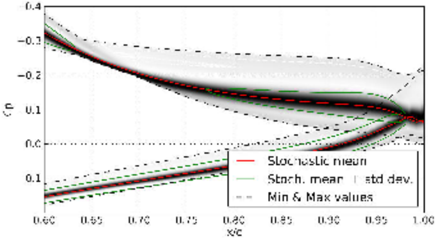


Figure: $N = 14$, attached, C_D vs C_L

- Identify important parameters and cross effects with Sobol' indices
  $\Rightarrow$ Refinement to improve stochastic approx.
- Effect on separation...



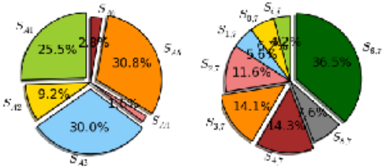


Figure: $N = 8$, C_p at $0.6 \leq x/c \leq 1$ and Sobol' indices.

Perspectives

Goal oriented mesh adaptation

- Application to RANS flows
- Taylor analysis of dJ/dX
- Extension to unstructured meshes

UQ

- Assess other methods for CFD computations saving and adaptive nested quadrature formulae
- Polynomial Chaos method for aerodynamic applications

THANK YOU!

Questions?