Island particle algorithms and application to rare event analysis

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Plan

Introduction to island particle models : a way to parallelize sequential Monte Carlo (SMC) algorithms

An example of island particle algorithm for rare event analysis

Conclusion

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Feynman-Kac models

- $\{Q_n\}_{n\in\mathbb{N}}$: a set of unnormalized transition kernels on the measurable space $(\mathsf{X}, \mathcal{X})$
- η_0 : a probability distribution

Definition :

A sequence $\{\eta_n\}_{n\in\mathbb{N}}$ of *Feynman-Kac measures* is defined, for all $h \in \mathrm{F_b}(\mathsf{X})$ by

$$\eta_n h = \frac{\int \cdots \int h(x_n)\, \eta_0(\mathrm{d}x_0) \prod_{p=0}^{n-1} Q_p(x_p, \mathrm{d}x_{p+1})}{\int \cdots \int \eta_0(\mathrm{d}x_0) \prod_{p=0}^{n-1} Q_p(x_p, \mathrm{d}x_{p+1})} \qquad (n \in \mathbb{N}).$$

- The Feynman-Kac measures satisfy the nonlinear recursion

$$\eta_{n+1} = \eta_n Q_n / \eta_n Q_n \mathbb{1}_{\mathsf{X}}.$$

- Numerous applications (nonlinear filtering, rare event sampling, hidden Markov chain parameter estimation, stochastic control problems, financial mathematics...)

Interacting particle systems (IPS)

Approximation of Feynman-Kac measures $\{\eta_n\}_{n\in\mathbb{N}}$:

- Kalman filter : exact simulation for linear Gaussian models.
- Otherwise, SMC methods : empirical approximation of η_n thanks to an interacting particle systems $\{(\xi_n^N(i), \omega_n^N(i))\}_{i=1}^N$. The weighted sample evolves recursively according to selection and mutation steps :

$$\{(\xi_n^N(i), \omega_n^N(i))\}_{i=1}^N \xrightarrow{\text{Selection}} \{(\xi_n^N(I_n^N(i)), \omega_n^N(I_n^N(i)))\}_{i=1}^N \xrightarrow{\text{Mutation}} \{(\xi_{n+1}^N(i), \omega_{n+1}^N(i))\}_{i=1}^N$$

$$I_n^N(i) \in [\![1, N]\!]$$

$$\xi_{n+1}^N(i) \sim R_n(\xi_n^N(I_n^N(i)), \cdot)$$
$$\omega_{n+1}^N(i) = \omega_n^N(I_n^N(i))\, w_n(\xi_n^N(I_n^N(i)), \xi_{n+1}^N(i))$$

where R_n is a proposal kernel and w_n is the importance function such that $Q_n(x, \mathrm{d}y) = w_n(x, y) R_n(x, \mathrm{d}y) \quad ((x, y) \in \mathrm{X}^2)$.

- Different kinds of selection may be considered :
  - systematic resampling : $\forall i, I_n^N(i) \sim \text{Mult}(\{\omega_n^N(k)\}_{k=1}^N)$ $\rightsquigarrow$ bootstrap algorithm [Ref : N.J. Gordon, D. J. Salmond, A. F. M. Smith (1993)]
  - resampling of all the particles only when their weights are skewed (Effective Sample size, coefficient of variation) [Ref : J. Liu, R. Chen (1993)]

The bootstrap algorithm

/* Initialization */

for $i \leftarrow 1$ **to** N **do**

  $\xi_0^N(i) \sim \eta_0$;

  $\xi_1^N(i) \sim R_0(\xi_0^N(i), \cdot) \quad \omega_1^N(i) \leftarrow w_0(\xi_0^N(i), \xi_1^N(i))$;

end

for $p \leftarrow 1$ **to** $n-1$ **do**

  /* Selection step */

  Sample $\{I_p^N(i)\}_{i=1}^N \sim_{i.i.d} \mathrm{Mult}(\{\omega_p^N(k)\}_{k=1}^N)$;

  /* Mutation step */

  for $i \leftarrow 1$ **to** N **do**

    Sample conditionally independently

    $\xi_{p+1}^N(i) \sim R_p(\xi_p^N(I_p^N(i)), \cdot)$;

    Update the weights $\omega_{p+1}^N(i) \leftarrow w_p(\xi_p^N(I_p^N(i)), \xi_{p+1}^N(i))$;

  end

end

Analysis of the bootstrap algorithm

Proposition : [Ref : P. Del Moral, *Feynman-Kac formulae*, 2004] [Ref : R. Douc and E. Moulines, 2008]

For all $n \in \mathbb{N}$ and $h \in \mathrm{F_b}(\mathrm{X})$, set $\eta_n^N h \triangleq \sum_{i=1}^N \omega_n^N(i) h(\xi_n^N(i)) / \sum_{k=1}^N \omega_n^N(k)$. Then,

$$\eta_n^N h \xrightarrow[N \to +\infty]{} \eta_n h \quad \text{almost surely,}$$

$$\sqrt{N}(\eta_n^N h - \eta_n h) \xrightarrow[N \to +\infty]{\mathcal{D}} \mathrm{N}(0, V_n(h)), \quad \text{where}$$

$$V_0(h) = \eta_0\{(h - \eta_0 h)^2\} \quad \text{and} \quad V_n(h) = \sum_{\ell=0}^{n-1} \frac{\eta_\ell R_\ell \{w_\ell^2 Q_{\ell+1} \cdots Q_{n-1}(h - \eta_n h)^2\}}{(\eta_\ell Q_\ell \cdots Q_{n-1} \mathbb{1}_{\mathrm{X}})^2}.$$

$$N \, \mathbb{E}\left[\eta_n^N h - \eta_n h\right] \xrightarrow[N \to +\infty]{} B_n(h).$$

- The precision of the estimation depends upon the size N of the particle swarm $\Rightarrow$ critical for online applications
- Develop new SMC methods to reduce the size of the particle swarm, while ensuring good estimates $\Rightarrow$ parallelization of SMC methods.

Parallelization of SMC methods

- Spread the total number $N \triangleq N_1 N_2$ of particles into N_1 batches of N_2 particles each.
- Each batch is called an island. Each island evolves independently as a standard SMC algorithm with N_2 particles.
- The N_1 islands may be considered in a parallel architecture or may interact through a selection step on the island level, when assigning as island weight, the average of the particle weights in an island.
  - N_1 independent bootstraps
  - Double bootstrap (B^2)
  - Double bootstrap with adaptive selection on the island level (B^2ASIL)

The B²ASIL algorithm

/* Initialization */

For all $i \in [\![1, N_1]\!]$ and $j \in [\![1, N_2]\!]$,

$\xi_0^N(i,j) \sim \eta_0$; $\xi_1^N(i,j) \sim R_0(\xi_0^N(i,j), \cdot)$; $\omega_1^N(i,j) \leftarrow w_0(\xi_0^N(i,j), \xi_1^N(i,j))$;

$\Omega_1^N(i) \leftarrow \sum_{j=1}^{N_2} \omega_1^N(i,j)/N_2$;

for $p \leftarrow 1$ **to** $n-1$ **do**

  /* Island selection */

  if $\mathrm{CV}^2(\{\Omega_p^N(i)\}_{i=1}^{N_1}) \triangleq N_1 \sum_{i=1}^{N_1} \left(\Omega_p^N(i)/\sum_{i'=1}^{N_1} \Omega_p^N(i')\right)^2 - 1 > \tau$ **then**

    For all $i \in [\![1, N_1]\!]$, $I^N(i) \sim \mathrm{Mult}(\{\Omega_p^N(i')\}_{i'=1}^{N_1})$;

  else

    For all $i \in [\![1, N_1]\!]$, $I^N(i) \leftarrow i$;

  end

  /* Island mutation */

  for $i \leftarrow 1$ **to** N_1 **do**

    /* Individual selection */

    For all $j \in [\![1, N_2]\!]$, $J^N(i,j) \sim \mathrm{Mult}(\{\omega_p^N(I^N(i), j')\}_{j'=1}^{N_2})$;

    /* Mutation */

    For all $j \in [\![1, N_2]\!]$, $\xi_{p+1}^N(i,j) \sim R_p(\xi_p^N(I^N(i), J^N(i,j)), \cdot)$;

    $\omega_{p+1}^N(i,j) \leftarrow w_p(\xi_p^N(I^N(i), J^N(i,j)), \xi_{p+1}^N(i,j))$;

    $\Omega_{p+1}^N(i) \leftarrow \sum_{j=1}^{N_2} \omega_{p+1}^N(i,j)/N_2$;

  end

end

Analysis of the B^{2}ASIL algorithm

Denote by $\eta_n^N h = \sum_{i=1}^{N_1} \frac{\Omega_n^N(i)}{\sum_{i'=1}^{N_1} \Omega_n^N(i')} \sum_{j=1}^{N_2} \frac{\omega_n^N(i,j)}{\sum_{j'=1}^{N_2} \omega_n^N(i,j')} h(\xi_n^N(i,j))$ the estimators returned by the B^{2}ASIL algorithm.

Theorem : [Ref : C. Vergé, P. Del Moral, E. Moulines, J. Olsson, *preprint*]

Let $n \in \mathbb{N}$ and $h \in \mathrm{F_b}(\mathsf{X})$. Then, $\eta_n^N h \xrightarrow[N\to+\infty]{\mathbb{P}} \eta_n h$.

Impose that for all $\beta > 0$, $N_1 \exp(-\beta N_2) \xrightarrow[N\to+\infty]{} 0$. Then, for all $n \in \mathbb{N}$, the random variable $\mathbb{1}\{\mathrm{CV}^2(\{\Omega_n^N(i)\}_{i=1}^{N_1}) > \tau\}$ has a deterministic limit ε_n in probability. Moreover,

$$\sqrt{N}(\eta_n^N h - \eta_n h) \xrightarrow{\mathcal{D}} \mathrm{N}(0, V_n(h) + \widetilde{V}_n(h)), \quad \text{where}$$

$$V_0(h) = \eta_0\{(h - \eta_0 h)^2\},\ \widetilde{V}_0 = 0, \text{ and } V_n h = \sum_{\ell=0}^{n-1} \frac{\eta_\ell R_\ell\{w_\ell^2 Q_{\ell+1} \cdots Q_{n-1}(h - \eta_n h)^2\}}{(\eta_\ell Q_\ell \cdots Q_{n-1}\mathbb{1}_\mathsf{X})^2},$$

$$\widetilde{V}_n h = \sum_{\ell=0}^{n-1} \sum_{p=\ell+1}^{n-1} \varepsilon_p \frac{\eta_\ell R_\ell\{w_\ell^2 Q_{\ell+1} \cdots Q_{n-1}(h - \eta_n h)^2\}}{(\eta_\ell Q_\ell \cdots Q_{n-1}\mathbb{1}_\mathsf{X})^2}.$$

Proof (sketch)

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The proof is by induction on n.
We decompose one iteration of the B^2ASIL algorithm into elementary operations

- selection on the island level
- selection on the particle level
- mutation,

and show that each of them preserves a law of large numbers, a Hoeffding-type inequality and a central limit theorem.

This general framework allows to derive a law of large numbers and a central limit theorem for any algorithm that may be decomposed into these elementary operations.

A criterion to determine when islands should interact

	Bootstrap	N_1 independent bootstraps	B^{2}ASIL
Bias	$\frac{B_n}{N}$	$\frac{B_n}{N_2}$	$\frac{B_n + \widetilde{B}_n}{N_1 N_2}$
Variance	$\frac{V_n}{N}$	$\frac{V_n}{N_1 N_2}$	$\frac{V_n + \widetilde{V}_n}{N_1 N_2}$

Explicit expressions of $B_n, \widetilde{B}_n, V_n$ and $\widetilde{V}_n$ can be found in :

[Ref : C. Vergé, C. Dubarry, P. Del Moral, E. Moulines, *Statistics and Computing*, 2015]
[Ref : C. Vergé, P. Del Moral, E. Moulines, J. Olsson, *preprint*]

Use the mean squared error to make a compromise between bias and variance : island interaction is beneficial when

$$\frac{V_n}{N_1 N_2} + \frac{B_n^2}{N_2^2} > \frac{V_n + \widetilde{V}_n}{N_1 N_2} \quad \Leftrightarrow \quad N_2 < \frac{B_n^2}{\widetilde{V}_n} N_1 .$$

When $N_2 << N_1$, the interaction is beneficial, but prevents a total parallelization.

Stability of the double bootstrap (B^2)

Note that the B^2 algorithm, which selects systematically the islands, is a particular case of B^{2}ASIL algorithm for which $\tau = 0$ (and hence $\varepsilon_n = 1$ for all $n \in \mathbb{N}^*$). We may hence furnish the asymptotic variance of the B^2 algorithm :

$$\sigma_n^2(h) = \sum_{\ell=0}^{n-1} (n-\ell) \frac{\eta_\ell R_\ell \{ w_\ell^2 Q_{\ell+1} \cdots Q_{n-1}(h - \eta_n h)^2 \}}{(\eta_\ell Q_\ell \cdots Q_{n-1} \mathbb{1}_\mathsf{X})^2}.$$

Theorem [Ref : C. Vergé, P. Del Moral, E. Moulines, J. Olsson, preprint]

Suppose the standard *strong mixing conditions :*

(i) There exist constants $0 < \sigma_- < \sigma_+ < \infty$ and $\varphi \in \mathsf{M}_1(\mathsf{X})$ such that for all $p \in \mathbb{N}$, $x \in \mathsf{X}$, and $\mathsf{A} \in \mathcal{X}$, $\sigma_- \varphi(\mathsf{A}) \le M_p(x, \mathsf{A}) \le \sigma_+ \varphi(\mathsf{A})$.

(ii) $w_+ \triangleq \sup_{p \in \mathbb{N}} \| w_p \|_\infty < \infty$.

(iii) $c_- \triangleq \inf_{(p,x) \in \mathbb{N} \times \mathsf{X}} Q_p \mathbb{1}_\mathsf{X}(x) > 0$.

Then for all $n \in \mathbb{N}$ and $h \in \mathrm{F_b}(\mathsf{X})$, $\sigma_n^2(h) \le w_+ \dfrac{\mathrm{osc}^2(h)}{(1-\rho)^2(1-\rho^2)^2 c_-}$, where $\rho \triangleq 1 - \sigma_-/\sigma_+$.

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Context

- Consider a black box model :

- A rare event is often modeled by the exceedence of a threshold S : $\{\phi(X) > S\}$ such that $\mathbb{P}(\phi(X) > S) < 10^{-4}$.
  - Risk analysis is not just evaluating a risk or a probability of failure, but estimating the law of random phenomena that leads to a critic event.
- Some parameters θ of the model or density parameters of the input random variables X may be fixed by the experimenter and can influence the output random variable Y.
  - We want to determine the impact of such tuning of parameters on the realization of the critic event, i.e. to compute the law of the parameters Θ conditionally on the rare event, denoted by $\pi \triangleq \text{Law}(\Theta \mid \phi(X) > S)$.

The splitting algorithm : a way to evaluate $\mathbb{P}(\phi(X) > S|\Theta = \theta)$

[Ref : S-K. Au and J.L. Beck, *Estimation of small failure probabilities in high dimensions by subset simulation*, 2001]

Consider an increasing sequence of thresholds

$$-\infty \triangleq S_0 < S_1 < \ldots < S_m \triangleq S,$$

and decompose, using Bayes' formula,

$$\mathbb{P}(\phi(X) > S|\Theta = \theta) = \prod_{p=0}^{m-1} \mathbb{P}(\phi(X) > S_{p+1}|\phi(X) > S_p, \Theta = \theta).$$

Approximation using an SMC with N_2 particles :

$$\begin{bmatrix} X_n(1) \\ \vdots \\ X_n(N_2) \end{bmatrix} \xrightarrow[\mathbb{1}_{\{.\geq S_{n+1}\}}]{\text{Selection}} \begin{bmatrix} \widehat{X}_n(1) \\ \vdots \\ \widehat{X}_n(N_2) \end{bmatrix} \xrightarrow{\text{Mutation}} \begin{bmatrix} X_{n+1}(1) \\ \vdots \\ X_{n+1}(N_2) \end{bmatrix}$$

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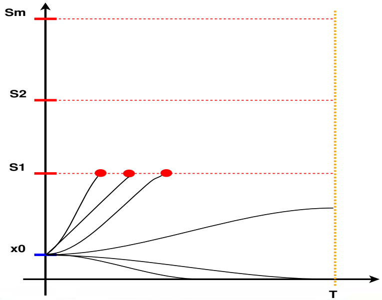

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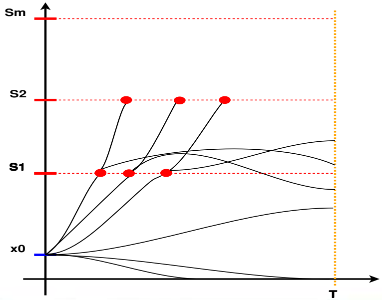

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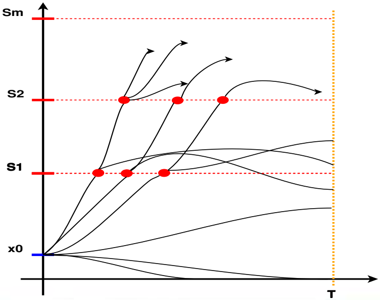


The SMC^2 algorithm

- One way to sample from $\pi \triangleq \text{Law}(\Theta|\phi(X) > S)$, is to use a standard SMC algorithm with N_1 particles $\{\theta_n(i)\}_{i=1}^{N_1}$.
- For that purpose, we create a dynamic introducing intermediary thresholds $S_1 < \ldots < S_m \triangleq S$, and defining the distributions $\{\pi_n\}_{n\in\mathbb{N}^*}$ by
$$\pi_n(\mathrm{d}\theta) \triangleq \text{Law}(\Theta|\phi(X) > S_n).$$
- Instead of trying to sample directly from π, sample successively from $\pi_1, \ldots, \pi_m \triangleq \pi$.
- The particles $\{\theta_n(i)\}_{i=1}^{N_1}$ evolve according to usual selection and mutation steps :
  - **Selection :** Multinomial resampling with weights proportional to : $\{\mathbb{P}(\phi(X) > S_{n+1}|\phi(X) > S_n, \Theta = \theta_n(i))\}_{i=1}^{N_1}$,
  - **Mutation :** An acceptance / rejection step involving the probabilities $\mathbb{P}(\phi(X) > S|\theta_n(i))$, which are not computable.
- For each parameter $\theta_n(i)$, we run a splitting with N_2 particles $\{X_n(i,j)\}_{j=1}^{N_2}$, in order to replace every incalculable quantity by an unbiased estimator.
We then have 2 embedded SMC algorithms $\hookrightarrow$ the SMC^2 algorithm.

[Ref : N. Chopin, P. Jacob and O. Papaspiliopoulos, *JRSSB*, 2013]

[Ref : C. Vergé, J. Morio, P. Del Moral, preprint]

Analysis of SMC^2

Theorem :

The SMC^2 algorithm converges to the target law π when $N_1 \to \infty$, for any fixed N_2.

Sketch of proof : The SMC^2 algorithm can be viewed as an SMC algorithm on an extended state space.

Toy case : threshold exceedence for a Gaussian tail. We can compute explicitly Law$(\Theta | X \geq 5)$ when $X \sim \mathcal{N}(\Theta, 1)$ and $\Theta \sim \mathcal{N}(0, 1)$.
In this simulation, we use 2000×20 particles for the SMC^2 algorithm.

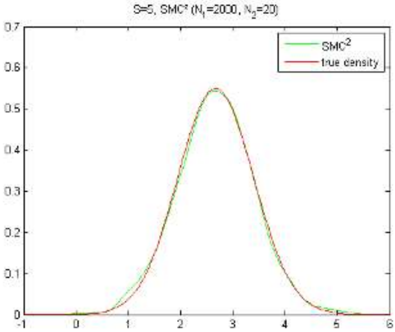
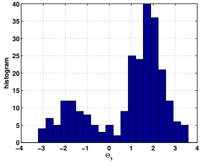


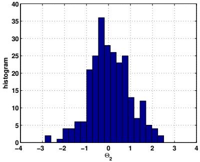
FIGURE: Law$(\Theta | X \geq 5)$

Application of SMC^2 algorithm to the fallout zone of a launch vehicle

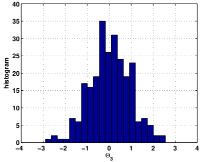
We apply SMC^2 algorithm where ϕ simulates the distance between the true position of the fallout zone of a stage rocket and its predicted position.
X is a Gaussian vector with covariance matrix equal to I_4 and mean $\theta = (\theta_1, \theta_2, \theta_3, \theta_4)^t$ with a Gaussian *prior* i.e. for all $i \in [\![1,4]\!], \theta_i \sim \mathcal{N}(0,1)$.
The critic event is when the output distance $\phi(X)$ exceeds 0.72 km.



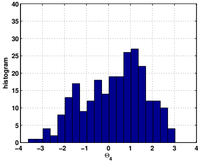
Law(θ_1|critic event)



Law(θ_2|critic event)



Law(θ_3|critic event)



Law(θ_4|critic event)

θ_1 : wind direction ; θ_4 : descent angle.

Influence of the parameters on the probability of interest

Monte Carlo estimates for different sets of parameters :

θ_1	θ_2	θ_3	θ_4	$\hat{\mathbb{P}}(\phi(X) > S\|\theta)$
0	0	0	0	$8.5\ 10^{-4}$
1	0	0	1	$1.05\ 10^{-2}$
-1	0	0	1	$1.02\ 10^{-2}$
-1	0	0	-1	$1.14\ 10^{-2}$

- A bad tuning of the parameters can imply a large increase of the probability of the critic event and an underestimation of the associated risk $\Rightarrow$ security matter.

[Ref : C. Vergé, J. Morio, P. Del Moral, preprint]

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Conclusions and perspectives

Conclusions :

- Introduction to island particle models,
- Definition of operations on islands,
- Establishment of a criterion to determine when islands should be considered in parallel or may interact,
- Study of asymptotic properties of island particle models,
- Transposition of an existing island particle model for rare event analysis.

Application :

- Reliability analysis of a launch vehicle stage fallout.

Perspectives and future application :

- Application to reliability analysis for collision between a space debris and a satellite,
- Study of the SMC^2 algorithm.

Publications

Book :

P. Del Moral and C. Vergé, *Algorithmes Stochastiques : Modèles et Applications*, Springer Series : Maths & Applications, SMAI, vol.75, 2014, 487 pages, DOI = 10.1007/978-3-642-54616-7 (published).

Book chapters :

Contribution to two chapters of : *Estimation of rare event probabilities in complex (and other) systems - a practical approach*, J. Morio and M. Balesdent, Elsevier-Woodhead Publishing (August 2015).

- Chapter 5 : Simulation techniques
- Chapter 11 : Estimation of collision probability between a space debris and a satellite

Publications

Journal publications :

- **Published**
  1. C. Vergé, C. Dubarry, P. Del Moral, E. Moulines, On parallel implementation of Sequential Monte Carlo methods : the island particle model, *Statistics and Computing*, vol. 25, Issue 2, Mars 2015, pp. 243-260, DOI = 10.1007/s11222-013-9429-x.
  2. J. Morio, M. Balesdent, D. Jacquemart, C. Vergé, A survey of rare event estimation methods for static input-output models, *Simulation Modelling Practice and Theory*, vol. 49, pp 287-304, 2014.
- **Submitted**
  1. C. Vergé, P. Del Moral, E. Moulines, J. Olsson, Asymptotic properties of weighted archipelagos with application to particle island methods.
  2. C. Vergé, J. Morio, P. Del Moral, An island Particle Markov Chain Monte Carlo algorithm for safety analysis.
  3. C. Vergé, C. Ichard, Introduction to labeled island particle models and their asymptotic properties.

Conference publication :

1. P. Del Moral, G. W. Peters, C. Vergé, An introduction to particle integration methods : with applications to risk and insurance, *Monte Carlo and Quasi-Monte Carlo Methods 2012, Springer Proceedings in Mathematics & Statistics*, volume 65, 2013, p.39-81, DOI = 10.1007/978-3-642-41095-6_3.
2. C. Vergé, E. Moulines, J. Olsson, Asymptotic properties of particle island models with application to the double bootstrap filter, *Signal Processing Conference (EUSIPCO), Proceedings of the 22nd European, IEEE* (submitted, march 2015).
3. C. Vergé, E. Moulines, J. Olsson, Fluctuation analysis of island particle models, *18th INFORMS Applied Probability Conference*, Istanbul University, Turkey (submitted, march 2015).

Thank you for your attention !